

# FAST ORBIT RESPONSE MATRIX MEASUREMENT VIA SINE-WAVE EXCITATION OF CORRECTORS AT SIRIUS

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## Abstract

Sirius is the new 4th generation storage ring based synchrotron light source built and operated by the Brazilian Synchrotron Light Laboratory (LNLS). In this work, we report on the implementation at Sirius of a fast method for orbit response matrix (ORM) measurement which is based on sine-wave parallel excitation of orbit corrector magnets' strength. This "AC method" has reduced the ORM measurement time from  $\sim 25$  minutes to 2.5–3 minutes and displayed increased precision if compared to the standard serial measurement procedure. When used as input to the Linear Optics from Closed Orbits (LOCO) correction algorithm, the AC ORM yielded similar optics corrections with less aggressive quadrupoles strength changes.

## INTRODUCTION

### Orbit Response Matrix and Sirius' Setup

At Sirius, 160 *beam position monitors* (BPMs) read horizontal and vertical displacements of the electron beam. The BPMs data is arranged in a 320-component vector  $\mathbf{u} = (x_1, x_2, \dots, x_{160}, y_1, y_2, \dots, y_{160})^T$ . A  $\Delta\theta_j$  kick from the  $j$ -th corrector magnet (CM) causes an orbit distortion which is measured by the  $i$ -th BPM as the combination

$$\Delta u_i = \sum_{j=1}^n M_{ij} \Delta\theta_j. \quad (1)$$

$M_{ij}$  are the entries of the orbit response matrix (ORM), which relates the orbit change due to CMs strength variations. At Sirius,  $n = n_x + n_y = 280$  is the total number of CMs, with  $n_x = 120$  and  $n_y = 160$  being the number of horizontal (CHs) and vertical correctors (CVs), respectively. In matrix notation, the orbit distortion reads

$$\begin{bmatrix} \Delta \mathbf{x} \\ \Delta \mathbf{y} \end{bmatrix} = \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix} \begin{bmatrix} \Delta\theta_{\text{CHs}} \\ \Delta\theta_{\text{CVs}} \end{bmatrix}, \quad (2)$$

which highlights the diagonal blocks  $M_{xx}$  and  $M_{yy}$ , and off-diagonal blocks  $M_{xy}$  and  $M_{yx}$  of the ORM.

The ORM is essential to orbit correction, where we wish to minimize  $\chi^2 = |\mathbf{u} - \Delta\mathbf{u}|^2 = |\mathbf{u} - M\Delta\theta|^2$ ,  $\Delta\theta$  being the vector with entries  $\Delta\theta_j$ . The matrix also encodes information about the storage ring linear optics and is the input to the model-based correction algorithm LOCO [1, 2]

### Fast Measurement Procedure: The "AC Method"

If we perform kicks to the beam using only one CM, say, the  $j$ -th CM, Eq. (1) reduces to  $\Delta u_i = M_{ij} \Delta\theta_j$ , giving

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$M_{ij} = \Delta u_i / \Delta\theta_j$ . Therefore, by serially kicking the beam corrector by corrector and measuring the corresponding orbit distortions we can reconstruct the ORM column by column. This is the traditional procedure for measuring the ORM. At Sirius, it usually takes about 25 up to 30 minutes to be completed.

The alternative method we report here is based on the parallel, alternating excitation of the beam. This "AC method" was first implemented at the Diamond Storage Ring [3] and later at ALBA [4] and NSLS-II [5], where it proved to be a faster and reliable ORM measurement method. The general idea is to sinusoidally drive the beam by CMs at different frequencies so the harmonic signature in the BPMs readings holds information about several CMs' excitation at the same acquisition.

In the  $i$ -th BPM time series, we fit the beam motion to harmonic components at the CMs frequencies by solving the linear problem

$$\begin{bmatrix} \cos(2\pi f_1 t_1) & \sin(2\pi f_1 t_1) & \dots \\ \cos(2\pi f_1 t_2) & \sin(2\pi f_1 t_2) & \dots \\ \vdots & \vdots & \vdots \\ \cos(2\pi f_1 t_n) & \sin(2\pi f_1 t_n) & \dots \end{bmatrix} \begin{bmatrix} b_{i1} \\ c_{i1} \\ \vdots \\ b_{im} \\ c_{im} \end{bmatrix} = \begin{bmatrix} u_i(t_1) \\ u_i(t_2) \\ \vdots \\ u_i(t_n) \end{bmatrix}, \quad (3)$$

where the cosines and sines columns at frequencies  $f_j$  repeat up to frequency  $f_m$ . We solve for the Fourier components  $b_{i,j}$  and  $c_{i,j}$  by least-squares, thus extracting the amplitudes  $a_{i,j} = \sqrt{b_{i,j}^2 + c_{i,j}^2}$  and phases  $\phi_{i,j} = \text{atan2}(b_{i,j}, c_{i,j})$  from the beam motion imprinted by the CM oscillating at frequency  $f_j$ . The expected orbit distortions are  $\Delta u_i(t_n) =$

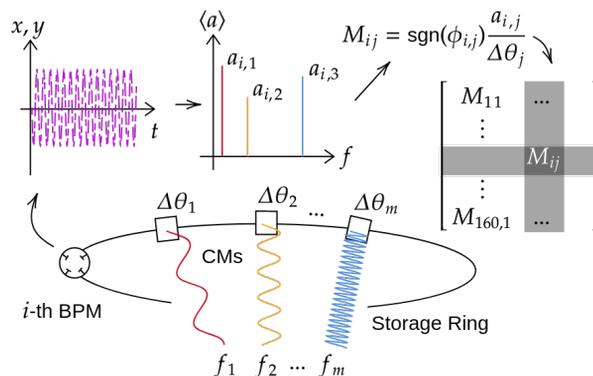


Figure 1: Parallel AC measurement of ORM: beam is excited by different CMs, each one at a different frequency. Spectral signature in beam motion reveals the amplitudes  $a_{i,j}$ , induced by the  $j$ -th CM to the beam as read by the  $i$ -th BPM.

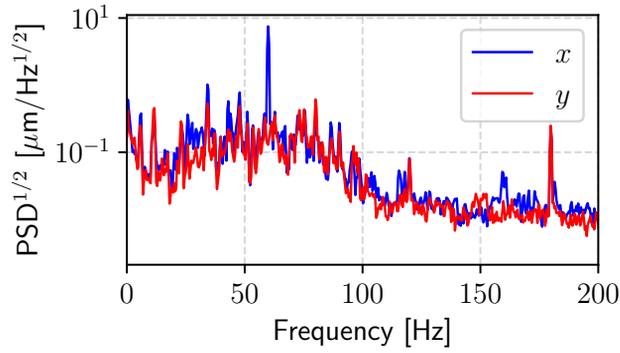


Figure 2: An example of PSD estimated for BPMs' readings.

$\sum_j a_{i,j} \sin(2\pi f_j t_n + \phi_{i,j})$ , so the ORM entries read

$$M_{ij} = \text{sgn}(\phi_{i,j}) \frac{a_{i,j}}{\Delta\theta_j}, \quad (4)$$

where  $\phi_{i,j} \in (-\pi, \pi]$  and  $\text{sgn}(\cdot)$  is the sign function. In summary, we condense the measurement process to the frequency domain, as illustrated by Fig. 1.

## BEAM RESPONSE

In order to choose adequate driving frequencies, we sought to characterize the beam frequency response compared to the BPMs noise baseline by constructing a signal-to-noise ratio vs. frequency. We implemented scripts to set one CM to operate with alternating sine-wave excitations while other CMs remain static. One CH drove the beam at fixed strength of  $\Delta\theta = 5 \mu\text{rad}$  with frequencies ranging from 1 to 200 Hz, one frequency at a time, with steps of 5 Hz. Subsequent orbit distortions were captured by all the BPMs, whose acquisition trigger was synced with the CMs' trigger. We repeated this procedure for one CV. The BPMs were set to read 5500 points at  $\sim 1$  kHz sampling rate after receiving the trigger event, i.e. they collected positions for about 5.5 s. In the time series for each BPM, we fitted the recorded beam displacements to harmonic components and solved for the amplitude  $a_{i,j}$  of the  $i$ -th BPM orbit readings excited by the CM at the  $f_j$  frequency.

To determine noise at BPMs readings we acquired  $T_{\text{acq}} = 9.9$  s of CM excitation-free orbits, at BPMs sampling rate of  $\sim 1$  kHz and estimated the Power Spectral Density (PSD) of the BPMs readings (Fig. 2). Noise at frequency  $f_j$  on the  $i$ -th BPM readings was evaluated as the square-root of the signal variance  $\sigma_{i,j} = \sqrt{\text{PSD}(f_j) \times \delta f}$ , with  $\delta f = T_{\text{acq}}^{-1}$  being the frequency resolution. The average ratio between amplitude and noise at a given frequency,  $\langle a_{i,j} / \sigma_{i,j} \rangle_i$ , was adopted as the signal-to-noise ratio (SNR) in units of dB (the last  $i$  subscript indicates the average over BPMs). The  $xx$  ( $yy$ ) line in Fig. 3 represents the SNR for  $x$  ( $y$ ) orbit distortions due to CH (CV) excitations, while the  $xy$  ( $yx$ ) line indicates the SNR for  $x$  ( $y$ ) orbit distortions due to CV (CH) excitations. The constructed SNRs discourage the use of frequencies multiples of 60 Hz.

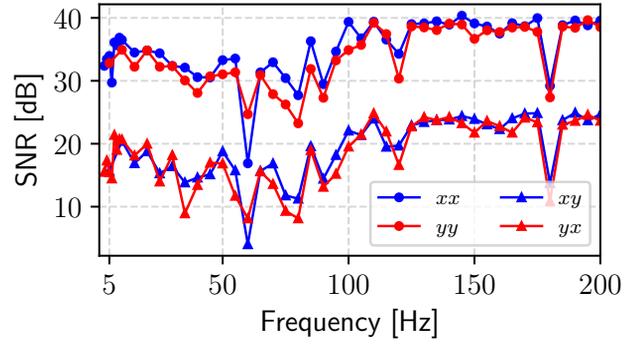


Figure 3: SNR for orbit distortions due to kicks on the same axis ( $xx$ ,  $yy$ ) and on different axes ( $xy$ ,  $yx$ ).

## FAST ORM MEASUREMENTS

We configured parallel alternating excitation of CMs and acquired data of multi-frequency excitation in order to reconstruct the ORM by the AC method. We excited the beam by driving the CMs of each of the 20 sectors present in the Sirius storage ring. At each sector, the 6 CHs drove frequencies  $f_x = 3, 7, 13, 19, 29, 37$  Hz, while the 8 CVs drove frequencies  $f_y = 5, 11, 17, 23, 31, 41, 47, 59$  Hz. We chose prime numbers to avoid problems with harmonics which might arise from non-linearities in the beam response function.

The start of BPM acquisition and the activation of CMs were synced by the same triggered event from the timing system, and a delay of 25 ms was set for the CMs to start driving the beam. Since the chosen frequencies are integers in units of Hz, all the CMs performed an integer number of oscillations for 4 s at  $5 \mu\text{rad}$  while the BPMs recorded positions for  $\sim 4.1$  s. This was done to be sure we knew exactly when the beam excitation started and ended in the time series. The fitting algorithm was restricted to fit data within the time window in which CMs were actually driving the beam, and the integer number of oscillations guarantee the orthogonality of the data vectors at the frequencies which we used to fit the data. Measurements with these settings took around 2.5–3 min to be completed.

To evaluate the resemblance between AC and DC ORMs we performed the following analysis: the cosine  $\cos\theta_j$  between the  $j$ -th column vectors  $\mathbf{v}_{\text{AC},j}$ ,  $\mathbf{v}_{\text{DC},j}$  of the AC and DC ORMs is an estimator of these vectors' signature correlation: the resemblance between the columns signature is higher when this estimator is closer to 1. In our measurements, the correlation residue  $1 - \cos\theta_j$  between the AC and DC ORMs columns is on average  $\sim 3\%$  for off-diagonal blocks ( $M_{xy}$ ,  $M_{yx}$ ) and  $\sim 0.03\%$  for diagonal blocks ( $M_{xx}$ ,  $M_{yy}$ ), indicating a good agreement between the matrices signature.

### The Method's Precision

For comparing consecutive ORM measurements we defined a variance matrix

$$\sigma_{ij}^2 = \frac{1}{N-1} \sum_{k=1}^N (M_{ij}^k - \langle M_{ij} \rangle)^2 \quad (5)$$

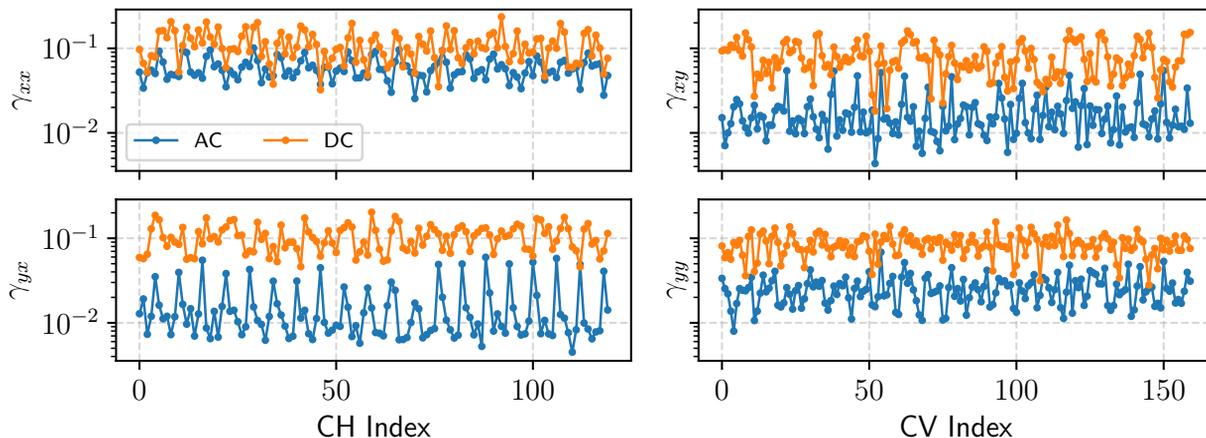


Figure 4: Mean column deviation for four consecutive AC and DC ORM measurements: matrices are measured and the variance of the entries are averaged for each one of its blocks. The square-root of the average is taken, resulting in a quantity  $\gamma_j$  condensing a column's deviation. The AC methods presents lower deviations across measurements.

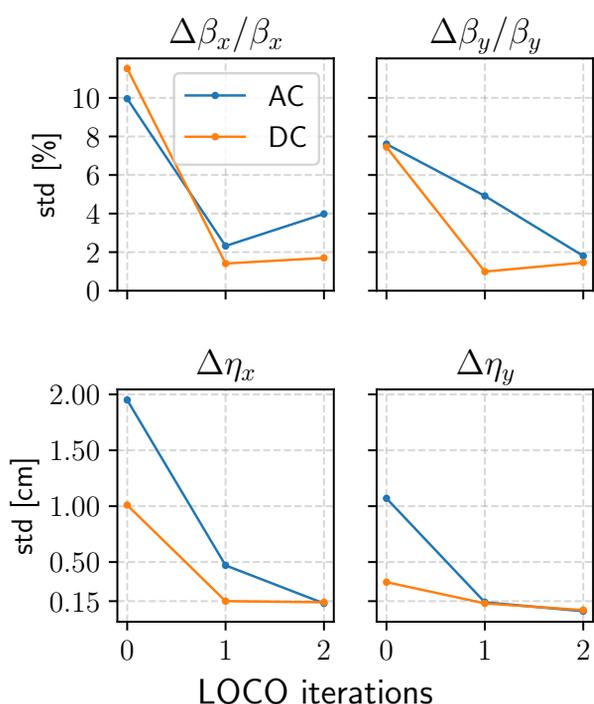


Figure 5: Comparison between LOCO fitted models: standard deviation (STD) of horizontal and vertical beta-beatings and dispersion function errors for the AC and DC ORM methods as LOCO iterates.

with entries indicating the variance between the  $ij$  entries of the measured  $M^k$  matrices with respect to the average matrix  $\langle M \rangle$ . For each corrector, i.e. for each ORM column, we considered the average  $\langle \sigma_{ij}^2 \rangle_i$ , which condenses the mean variance for a given corrector's column in the ORM. Four consecutive AC and DC measurements were carried and the variance matrix was calculated for each block  $uv \in \{xx, xy, yx, yy\}$  of the ORMs. We also defined a  $\gamma$

mean deviation vector, with entries  $\gamma_j = \sqrt{\langle \sigma_{ij}^2 \rangle_i}$ . Figure 4 shows  $\gamma_{uv}$ , the  $\gamma$  vector for the  $uv$  ORM blocks. The higher precision of the AC method is evident from the lower values of the  $\gamma$  vectors.

### Performance at Linear Optics Correction

The LOCO algorithm fed with an AC ORM provided optics corrections similar to those achieved previously at Sirius with DC-measured ORMs, as Fig. 5 shows. The ‘‘AC LOCO’’ was able to deliver similar optics with subtler changes. In the DC ORM LOCO, the STD of percentual changes in quadrupoles' trim coils integrated gradient is 0.67%, while it is 0.42% for the AC LOCO.

## CONCLUSIONS

The ‘‘AC ORM’’ method sped up ORM measurements at Sirius by almost ten times, while also displaying increased precision and delivering similar optics correction when used as input to the LOCO algorithm. Our measurement process still needs further improvements so it can be used for orbit correction. A proper characterization of the beam transfer function would elucidate the calibration of amplitudes needed to determine the ORM entries with correct scale factors and reveal additional phases introduced by the vacuum chamber and magnets. This will allow a more accurate determination of the beam motion phases and the use of higher frequencies to drive the beam, e.g., in the range [120 – 180] Hz, for which the SNR is well-behaved.

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