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EXCITATION OF THE σ_{l_l} = 90° RESONANCE BY THE CAVITY RF ACCELERATING FIELDS

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Abstract

In rf linacs the longitudinal focalization is done by sinusoidal forces and at high accelerating fields the zero-current longitudinal phase advance per longitudinal focusing period σ_{0l_l} can be high. The nonlinear components of the sinusoidal rf field (sextupolar, octupolar and higher order components) can then excite parametric resonances, including the 4th-order resonance ($\sigma_{l\_l} = 90°$) when σ_{0l_l} is higher than 90°, inducing strong longitudinal emittance growths and acceptance reductions. As pointed out in previous papers, the longitudinal beam dynamics is therefore complex, even when the space-charge forces are ignored. The parametric resonance excitation by the rf field is analyzed before discussing the additional effect of the space-charge forces, in particular to explain why the zero-current longitudinal phase advance per transverse focusing period σ_{0l_t} is not a relevant parameter. Examples are given in the SPIRAL2 linac case.

PARAMETRIC RESONANCES, BASIC DEFINITIONS

As done in [1, 2], the choice done here is to study the parametric resonances in the longitudinal plane independently of the study of the radial-longitudinal coupling resonances in order to avoid unnecessary complications and confusions. As usual (e.g. [3] chapter X, section IV), the resonance zoology is defined considering the linear equation of motion of individual particles perturbed by linear (n = 1) or nonlinear (n > 1) periodic perturbing forces

$$\frac{d^2\delta\varphi}{dz^2} + {\sigma_l}^2\,\delta\varphi + a_{n,k}\,\begin{matrix}[\delta\varphi_0\sin(\sigma_l z)]^n\\ [\delta\varphi_0\cos(\sigma_l z)]^n\end{matrix}\,\cos(k\sigma_p z) = 0 \qquad (1)$$

with σ_l the phase advance per unit of length of the unperturbed particle motion, σ_p and k the phase advance per unit of length and the harmonic number of the perturbing force respectively. Considering that a resonance condition is fulfilled when a component $(n,\,k)$ of the perturbing force has a "frequency" equal to the unperturbed particle oscillation "frequency" σ_l [3], a resonance occurs when

$$(i+1)\,\sigma_l = k\,\sigma_p$$

with i even integer from 0 to n for n even and i odd integer from 1 to n for n odd.

Using $\sigma_{l\_l} = \sigma_l\,L_l$ the longitudinal phase advance per period of the longitudinal focusing system (gaps or cavities), $\sigma_{l\_p} = \sigma_l\,L_p$ the longitudinal phase advance per period of the perturbation and $\sigma_p = 2\pi/L_p$ with L_p the period of the perturbation, the resonance condition is

$$\sigma_{l\_p} = (L_p/L_l)\,\sigma_{l\_l} = \frac{2\pi k}{i+1} \qquad (2)$$

$(i+1)$ is the order of the resonance and k is the harmonic of the perturbation.

For example, particles with $\sigma_{l\_l} = 90°$ can be excited by the $a_{n=3,k=1}$ component of the perturbing force when $L_p = L_l$ (fourth order resonance), or can be excited by the $a_{n=1,k=1}$, $a_{n=3,k=1}$ and $a_{n=3,k=2}$ components of the perturbing force when $L_p = 2L_l$ (half integer and fourth order resonances).

EXITATION BY THE CAVITY RF FIELD

Following [1, 2], the non-linear components of the rf accelerating field must be considered as main perturbing forces in the parametric resonance studies. This can be understood starting from the equation of motion of the phase oscillations around the synchronous particle obtained in smooth approximation ([3], chapter XVI section II-D).

$$\frac{d^2\delta\varphi}{dz^2} + K_{dp}\,\frac{d\delta\varphi}{dz} + \left[\frac{2\pi q\,E_0\,T_{\beta_s}}{m_0c^2\lambda\,\beta_s^3\gamma_s^3}\right][\cos(\Phi_s+\delta\varphi)-\cos\Phi_s] = 0 \qquad (3)$$

The effect of the damping term K_{dp} which has a great importance on the longitudinal dynamics at high acceleration rates [1, 2] is not discussed here to focus the analysis on the rf field nonlinear focusing force which can be expressed by its Taylor series

$$\begin{aligned}&[\cos(\Phi_s+\delta\varphi)-\cos\Phi_s] =\\ &-[\sin\Phi_s]\,\delta\varphi \quad \text{“quadrupole” (linear focusing)}\\ &-[\cos\Phi_s/2!]\,\delta\varphi^2 \quad \text{“sextupole”}\\ &-[\sin\Phi_s/3!]\,\delta\varphi^3 \quad \text{“octupole”}\\ &+[\cos\Phi_s/4!]\,\delta\varphi^4 \quad \text{“decapole”} + \ldots\end{aligned} \qquad (4)$$

Equation (3) is obtained using the smooth approximation, then averaging the effect of the rf field over the longitudinal period L_l. The effect of the gap or cavity periodicity must be analyzed including the harmonics of the rf field $E_z(z) = E_0 + \sum E_k\,cos(k\,2\pi z/L_l)$ in the longitudinal focusing force of Eq. (3), then to each term of the Taylor series of Eq. (4). The longitudinal focusing of the rf field can then be seen as a nonlinear focusing Eq. (4) and with $k = 0$ in Eq. (1), plus $\delta\varphi^n\,cos(k\,2\pi z/L_l)$ perturbing

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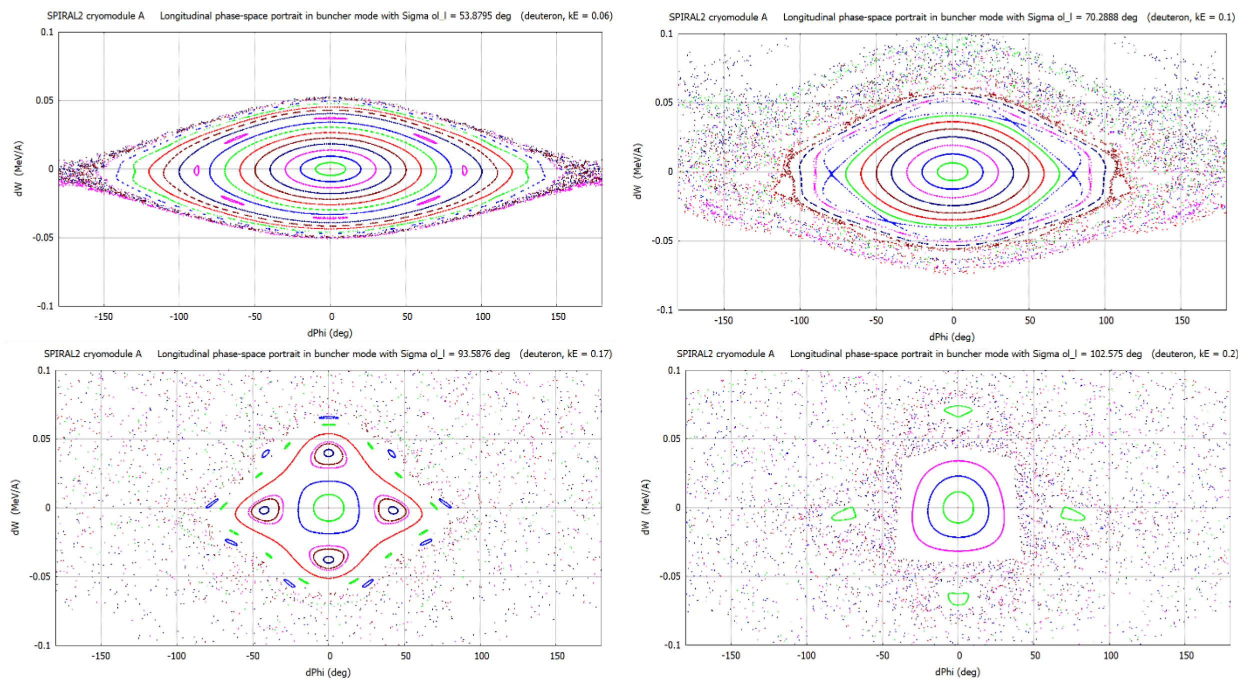

Figure 1: Longitudinal phase-space portrait for SPIRAL2 low beta cavities tuned in buncher mode ($\Phi_s = -90°$) at the RFQ output energy (0.732 MeV/A). Top left : $\sigma_{0l\_l} = 53.9°$ (kE = 0.06), top right $\sigma_{0l\_l} = 70.3°$ (kE = 0.10), bottom left $\sigma_{0l\_l} = 93.6°$ (kE = 0.17), bottom right $\sigma_{0l\_l} = 102.6°$ (kE = 0.20).

terms which induce resonances when the resonance conditions are fulfilled.

This was demonstrated in [1, 2] using theoretical field distributions, Fig. 1 shows that, as expected, this is also true in the case of the SPIRAL2 superconducting linac cavity field [4]. The different plots in Fig. 1 are Poincaré maps done with the SPIRAL2 low beta cavities (L_l = 1.19 m), working at different fields and tuned in buncher mode (no damping) for deuterons at the RFQ output energy (0.732 MeV/A).

With kE = E/E_{max} and E_{max} = 6.5 MV/m the nominal field, for $\sigma_{0l\_l} = 53.9°$ at kE=0.06 Fig. 1 shows that, as expected, there is 8 resonance islands at $\sigma_{l\_l} = 45°$ and a beginning of acceptance reduction due to the high-order resonance overlap in the separatrix area. As expected, the 6th order parametric resonance $\sigma_{l\_l} = 60°$ is present at $\sigma_{0l\_l} = 70.3°$ (kE = 0.10), with a significant acceptance reduction due to higher-order resonance overlap in the separatrix area. As expected, the fourth-order resonance is present at $\sigma_{0l\_l} = 93.6°$ (kE = 0.17) and $\sigma_{0l\_l} = 102.6°$ (kE = 0.20), with the 4 resonance islands present in the stable region around the synchronous particle in the first case and in the chaotic area in the second case. The drastic longitudinal acceptance reduction due to even more high-order resonance overlap in the separatrix region above $\sigma_{0l\_l} = 90°$ is also clearly visible in both cases.

As foreseen by Eq. (4) with $cos\,\Phi_s = 0$, there is only even order resonances present in Fig. 1. Not shown, but in the $\sigma_{0l\_l} > 90°$ cases the chaotic region extends at high energies, particles injected out of the reduced acceptance will then form "high-energy tails" (up to $\delta W > 1.2$ MeV/A above the RFQ output energy of 0.732 MeV/A).

One can finally point out the strong constraint imposed by the $\sigma_{0l\_l} < 90°$ rule that can impose the use of cavity fields much lower than the nominal ones in SC linacs, e.g. a limitation at less than 10% of the nominal field for the nominal tuning in the SPIRAL2 linac low energy part.

All of this pushes to better study the effect of the damping reducing the effect of the resonances at high accelerating rates [1, 2]. Considering that at the linac design stage, the longitudinal acceptance could be sufficiently preserved to leave the possibility to use higher accelerating fields.

EXITATION BY THE SPACE-CHARGE FIELD

With no loss of generality, the model for the space charge forces assuming the bunch to be a three-dimensional ellipsoid of uniform density introduced by Lapostolle [5] can be used. Following Gluckstern [6], using this model the longitudinal space charge force dependency (via the Poisson equation) on the bunch dimensions is usually ($0.8 < \xi < 5$ range in [6]) given by

$$\mu_l \propto \frac{1}{\sqrt{a_x\, a_y}\, b^2} \tag{5}$$

with a_x and a_y the bunch radial sizes and b the bunch longitudinal size.

For matched beams, the longitudinal envelope oscillation has the period L_l of the longitudinal focusing system (gaps or cavities); all the harmonics at the longitudinal focusing period L_l are then always present in the $b^2(z)$ contribution to the space charge "perturbing force" Eq. (5).

In linacs the horizontal and vertical emittances are usually equals and the phase advances in the horizontal and

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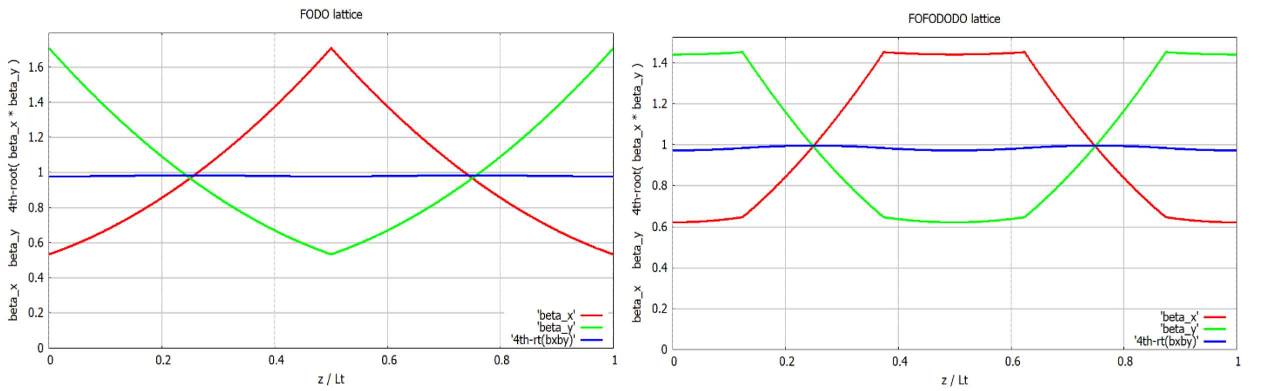

Figure 2: Evolution of the Courant-Snyder beta functions (red and green) and of the contribution of the radial envelope oscillations in the longitudinal space-charge perturbing force (blue) for matched beams in FODO (left) and FOFODODO (right) lattices with thin lenses tuned such that σ_{t_t}~ 63°.

vertical planes are traditionally chosen equals. In the case of matched beams in a solenoid or quadrupole doublet focusing (SPIRAL2 case), all the harmonics at the $L_t = L_l$ transverse period are also always present in the $a_x a_y$ contribution to the space charge "perturbing force".

In the case of matched beams in an alternating focusing with quadrupoles (FODO or FOFODODO) and choosing z = 0 such that the lattice has an even symmetry, the bunch radial dimensions can be expressed by their Fourier series

$$\begin{aligned} a_x(z) &= a_0 + \sum_{k=1}^{\infty} a_k \cos\left(k\frac{2\pi z}{L_t}\right) \\ a_y(z) &= a_x\left(z+\frac{L_t}{2}\right) \\ &= a_0 + \sum_{k=1}^{\infty} (-1)^k\, a_k \cos\left(k\frac{2\pi z}{L_t}\right) \end{aligned} \tag{6}$$

The product $a_x\, a_y$ inducing the variations of the longitudinal space charge force with respect to the radial bunch size variations Eq. (5) is then of the form

$$\begin{aligned} a_x a_y = &\left[{a_0}^2 + \sum_{k=1}^{\infty} (-1)^k\, \frac{{a_k}^2}{2}\right] \\ &+ \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} A_1(k,l)\cos\left(2k\frac{2\pi z}{L_t}\right) \\ &+ \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} A_2(k,l)\cos\left(2(k+l)\frac{2\pi z}{L_t}\right) \end{aligned} \tag{7}$$

There is therefore no "space-charge excitation" in the longitudinal plane at a period equal to the radial focusing period, the first component ($k = 1$) is at $L_t/2 = L_l$ in the FODO case and at $L_t/2 = 2\,L_l$ in the FOFODODO case.

Figure 2 also shows that the oscillatory part of the radial envelope contribution to the longitudinal space-charge force (blue curve, $\sqrt{a_x\, a_y}$ term in Eq. (5)) is very low, then has a small contribution compared to the one of the longitudinal envelope oscillations (b^2 term in Eq. (5)). One can then also conclude that in the case of the retained assumptions, the radial envelope oscillations have week effects on the excitation of parametric resonances in the longitudinal plane in both FODO and FOFODODO cases.

SUMMARY

Many publications systematically discuss the excitation of the fourth-order parametric resonance $\sigma_l = 90°$ by the space-charge forces. After [1, 2], it is again pointed out here that the first source of excitation of this resonance (as well as the other parametric resonances in the longitudinal plane) is the rf field of the cavities.

Concerning the space-charge effects, we have shown that in the linac case (equal tune and emittance in the horizontal and vertical planes) there is no excitation in the longitudinal plane at a period equal to the radial focusing period in the FODO and FOFODODO cases, and that the radial envelope oscillations have week effects in both cases.

The periodicity of the main longitudinal motion perturbations (rf field and space-charge) is the longitudinal lattice. As pointed out several times, it is a mistake to consider the σ_{l_t} longitudinal phase advances per transverse focusing period speaking of parametric resonances in the longitudinal plane.

For linac tunings at high acceleration rates, the strong effect of the longitudinal phase oscillation damping [1, 2] would need to be taken into account.

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