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TUNE CONTROL IN FIXED FIELD ACCELERATORS

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Abstract

Fixed Field Alternating Gradient Accelerators have been proposed for a wide range of challenges, including rapid acceleration in a muon collider, and large energy acceptance beam transport for medical applications. A disadvantage of these proposals is the highly nonlinear field profile required to keep the tune energy-independent, known as the scaling condition. It has been shown computationally that approximately constant tunes can be achieved with the addition of nonlinear fields which do not follow this scaling law. However the impacts of these nonlinearities are not well understood. We present a new framework for adding nonlinearities to Fixed Field Accelerators, seeking a constant normalised focusing strength over the full energy range, and verify the results by simulation using Zgoubi. As a model use case, we investigate the degree of tune compensation that can be achieved in a Fixed Field Accelerator for ion cancer therapy.

INTRODUCTION

In synchrotrons, the tune working point is dominated by linear quadrupole focusing, and is kept approximately independent of energy by ramping the fields of all magnets up during acceleration. The tune shift can be taken to be small, in part because the energy deviation δ is small. In many cases, corrections can be performed by the placement of low-field sextupole or octupole magnets around the lattice [1], giving overall tune shifts that are small enough for machine operation. However, this is not feasible in Fixed Field Accelerators (FFAs) [2, 3], where momentum acceptance can be as much as $\pm 50\,\%$, and small corrections to chromaticity are insufficient. In addition, the orbit excursions in FFAs are generally large, and the closed orbit can vary significantly between the lowest and highest energy: this can complicate attempts at tune compensation, as the field gradient varies with amplitude when nonlinearities are included.

Multiple methods have been proposed to control the tune in FFAs. The most widely known approach is for the field to follow a 'scaling law' [3], a highly nonlinear profile which was first proposed before the formulation of linear beam dynamics. Although this will produce a lattice where the tune is not a function of rigidity, a downside is that all closed orbits must be scale enlargements of one other. Also, to ensure strong focusing, either the magnets must have a large spiral angle (as in spiral-sector cyclotrons [4]), or the lattice must include reverse-bending defocusing magnets. Although these challenges make scaling FFAs unfavourable when compared with synchrotrons or cyclotrons, there are other Fixed Field Accelerator designs available.

One attempt to control the tune in an FFA without strictly adhering to the scaling law was used for PAMELA [5], which began with a scaling lattice, but truncated the multipole field expansion to decapole order and used rectangular rather than sector magnets. However, the design is still similar to a scaling FFA, with the same issues as previously discussed. Another method, which fixes tunes by modifying orbit shapes directly [6], works well for constant-field isochronous cyclotrons but imposes restrictions (such as no orbit crossing) that make synchrotron-like lattices difficult.

An alternative option is to begin with a simple linear lattice with rigidity-dependent tunes, and to add nonlinearities to combat this. Previous studies [7, 8] have achieved moderate success by performing numerical optimisations to flatten the tune, at the expense of a reduction in dynamic aperture and high error sensitivity. Results from this purely numerical approach are difficult to interpret, with no clear link between the tunes and field strengths. Instead of optimising the tunes themselves, we propose a lattice where the focusing strength is approximately constant for every energy, for each magnet individually: this is determined by the local magnetic field gradient along the closed orbit trajectories. An ion therapy accelerator is used to demonstrate this method, using the PyZgoubi wrapper for the tracking code Zgoubi [9, 10]. The resulting lattice has dynamics that are almost energy-independent, with tunes that are robust against magnet errors.

TUNES IN LINEAR FFAS

Where there is no overall bending – either from dipole fields, or from a rotation of the reference axis – all closed orbit trajectories go directly through the magnet centres, with the same path length. For a synchrotron, increasing the magnetic fields keeps the normalised gradient constant: this in turn leads to constant tune and dynamics. However, this is not the case when fields are fixed, as higher rigidity (P/q) beams feel a weaker focusing force: this was observed in the linear FFAs EMMA and CBETA [11, 12]. In the straightforward case of a FODO cell with drift length l_d, quadrupole length l_q, normalised quadrupole strengths $k_\pm = \pm\frac{g}{(P/q)}$ for gradients $\pm g$, and negligible fringe fields, we find from the the overall linear transfer matrix that

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$$\begin{aligned}2\cos(2\pi\nu) &= 2\cos\left(\sqrt{k}l_q\right)\cosh\left(\sqrt{k}l_q\right)\\ &+ 2l_d\sqrt{k}\cos\left(\sqrt{k}l_q\right)\sinh\left(\sqrt{k}l_q\right)\\ &- 2l_d\sqrt{k}\sin\left(\sqrt{k}l_q\right)\cosh\left(\sqrt{k}l_q\right)\\ &- kl_d^2\sin\left(\sqrt{k}l_q\right)\sinh\left(\sqrt{k}l_q\right)\\ &\approx 2 - \frac{1}{3}\frac{g^2 l_q^2}{(P/q)^2}\left(3l_d^2 + 4l_d l_q + l_q^2\right) + \mathcal{O}\left(\frac{1}{(P/q)^4}\right) \quad (1)\end{aligned}$$

where tune ν is the same in both the horizontal and vertical planes. The first two terms in the expansion give a good approximation: using the linear lattice in the next section for a rigidity of 5 T m, the tune error is less than 0.01 %. This allows us to find bounds on the acceptance of linear FODO cells for fixed magnets. As the range of $\cos\theta$ is $[-1, 1]$, the upper bound is the limit where $(P/q) \to \infty$: this suggests that there is no maximum rigidity, although the tune becomes vanishingly small. Conversely, the lower bound gives

$$(P/q)_{\mathrm{min}} \approx \frac{g l_q}{2}\sqrt{\frac{3l_d^2 + 4l_d l_q + l_q^2}{3}} \quad (2)$$

which serves as an approximate guide to initiate lattice design. This result is an overestimate, so the rigidity range is slightly larger than predicted. In a more general linear lattice – where there are independent dipole and quadrupole fields but higher order multipoles are not included – the leading-order dependence on rigidity still follows

$$\cos(2\pi\nu) \sim \frac{1}{(P/q)^2} \quad (3)$$

due to the normalisation of the field gradient. Although linear constant tune FFAs can be designed by varying path length as a function of rigidity using magnet edge angles, modelling suggests that this is highly sensitive to the resulting fringe fields [13]. As such, tune control by introducing nonlinearities may prove more feasible.

TUNE CONTROL IN NONLINEAR FFAS

In the nonlinear case, the scaling law can be used, but the disadvantages of reverse bending or complex magnet shapes are significant. As an alternative, we propose to control the tune indirectly, manipulating higher order multipoles so the integrated focusing strength KL along the closed orbit is the same for every rigidity. Assuming that the trajectory lies in the midplane of the i^{th} magnet, we require for all rigidities

$$(KL)_i = \int_0^L \frac{\left(\frac{\partial B_i}{\partial x}\right)}{(P/q)}\, ds = \mathrm{const} \quad (4)$$

where $\hat{x}$ is the direction locally perpendicular to the closed orbit trajectory. Here, we approximate the path length s as the distance along the magnet axis, and take the derivative perpendicular to this: although the difference in this case is small, it is not negligible in general. In the limit of thin magnets without edge-focusing, the linear dynamics will be the same for all rigidities: not only does this ensure constant tune, but also energy-independent Courant-Snyder parameters. However, this will not be the case if local focusing strength K(Eq. 3 without the integral) varies in a magnet.

Table 1: Multipole fields of the lattice before and after optimisation. After fixing the lengths and bending field, the initial quadrupole strengths were set using Eq. 1.

Pole Order	F-magnet		D-magnet	
	Initial	Final	Initial	Final
2 [T]	1.346	1.346	1.346	1.346
4 [$\mathrm{Tm^{-1}}$]	72.50	68.75	-72.50	-61.72
6 [$\mathrm{Tm^{-2}}$]	0.0	498.5	0.0	-576.9
8 [$\mathrm{Tm^{-3}}$]	0.0	744.1	0.0	780.6
≥10	0.0	0.0	0.0	0.0

There are limitations to this method of controlling tunes. One is in the optimisation procedure: unlike in synchrotrons – where the closed orbit is fixed – changes to the magnetic field alter the particle trajectories, making it difficult to predict the impact on the integrated focusing strength. In addition, it is unclear whether there is a unique optimal set of multipoles to obtain a given working point: this makes it hard to know how much a proposed lattice can be improved. Also, where orbits cross inside a magnet, it is more difficult to ensure that $(KL)_i$ is the same for all rigidities.

Ion Therapy Machine Study

As an example, we have applied this method of tune compensation to a FODO lattice with rectangular magnets, suitable for acceleration or beam transport in an ion therapy accelerator: the rigidity range is 2.63–6.63 T m (±43 %), corresponding to 80–430 MeV/u for He^{2+}, C^{6+}, and O^{8+}. The drift between magnets is fixed at 0.1 m and magnet lengths at 0.2 m, with a total of 54 cells giving a radius of approximately 5 m. For simplicity, we neglect fringe fields for this study. Parameters for the initial and final field strengths are given in Table 1. The beam experiences fields between −3 to 6 T, which could be achieved by superconducting Canted–Cosine–Theta magnets [14, 15], like those proposed for the NIMMS medical synchrotron study [16].

Beginning with the linear lattice, multipole strengths are varied manually to set the working point while reducing variation in the integrated focusing strength with energy. The nonlinear lattice is then further optimised using a standard Nelder-Mead downhill simplex algorithm [17]. The initial optimiser only minimises the standard deviation of KL in both magnets, however this is insufficient as our focusing strength calculation neglects edge focusing and changes in path length with energy: although tune shifts are much reduced, improvements are possible. As such, the final optimiser step directly considers the tunes calculated by Zgoubi and minimises the sum of their standard deviations: we find

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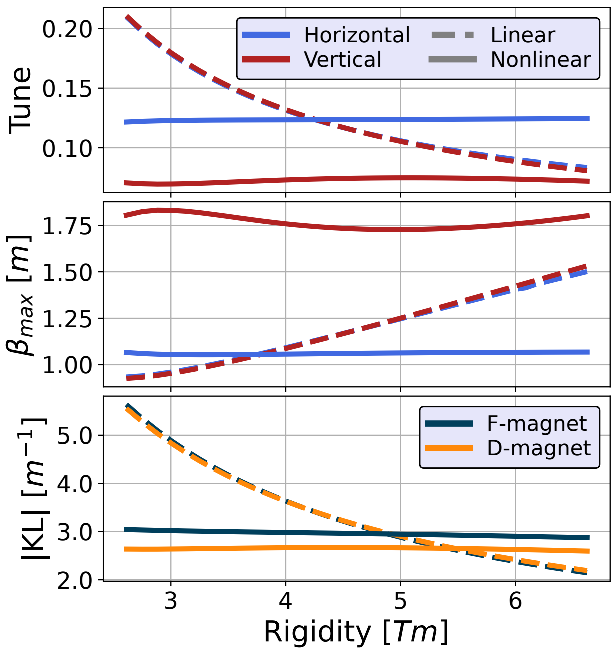

Figure 1: Comparison of tunes, maximum beta functions, and integrated focusing strengths in the initial and final (linear and nonlinear) lattices. In all cases, variation is much smaller in the final lattice, as is desired. Small residual fluctuations in K in the nonlinear lattice lead to some energy dependence for the beta functions.

that multipoles up to only octupole order are not enough to completely flatten the tunes. Nevertheless, this study demonstrates in Fig. 1 that a cell with significantly reduced tune variation over a large rigidity range can be achieved.

To further see how the focusing strength varies through the cell, the closed orbit trajectories and variation in K for the final lattice are presented in Fig. 2. Here, the local focusing strength deviates by less than ± 5 % in the lattice: in contrast, it is greater than ± 30 % for each magnet in the linear case.

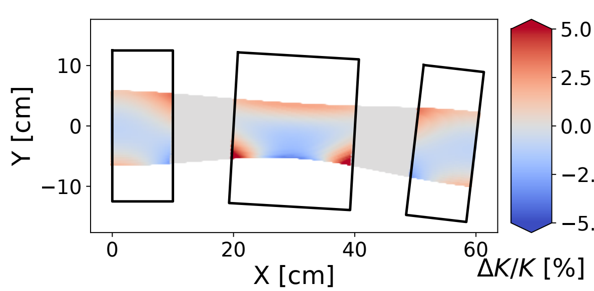

Figure 2: Fluctuations in focusing strength along the midplane closed orbits, for all rigidities. Tracking begins at the 'F' magnet centre. Outlines give magnet boundaries, with arbitrary aperture. Axes are 'global' top-down coordinates.

Previous studies have suggested that the addition of nonlinearities can significantly reduce dynamic aperture in the presence of realistic errors [7]. As this lattice lacks detailed fringe fields, changes in the dynamic aperture are not explored: instead, we investigate how much larger the tune footprint becomes when random errors are assigned to all multipole fields. As CBETA operated with magnet errors in the order of one part in 10^4 [18], this is chosen as our cutoff. In Fig. 3, the increase in tune footprint due to these errors is small, indicating that our method of tune compensation in nonscaling FFAs is less susceptible to errors than previous designs: this is an important step towards designing an accelerator that could operate under realistic conditions. Low rigidities are more affected, as predicted by Eq. 2: field errors have a smaller impact on high rigidities.

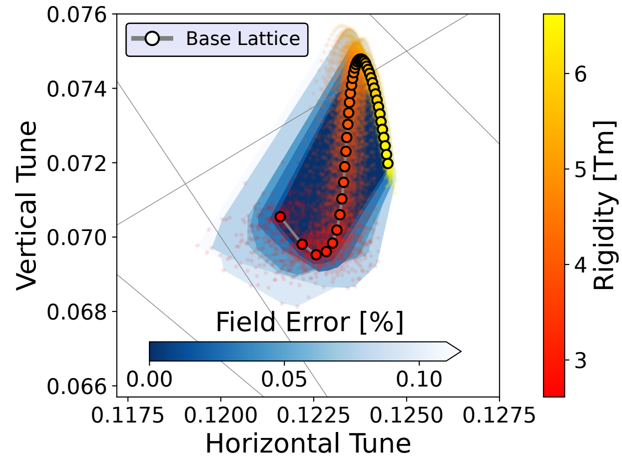

Figure 3: Cell tunes, with field errors up to $\sigma(B) = 0.1$% applied separately to all multipoles. The footprint is the convex hull of the tunes calculated for 25 random multipole settings, set using a normal distribution truncated at $\pm 3\sigma$. Potentially destructive resonances are also shown.

DISCUSSION

As the tune in linear Fixed Field Accelerators can't be controlled without difficult path length manipulations, we have presented a framework for creating constant tune FFAs with nonlinear fields. Our approach does not rely on the scaling law: instead, we optimise individual multipoles so the normalised focusing strength KL is energy-independent. This has been applied to an accelerator for ion therapy, achieving a variation of less than 1.5 % in the horizontal tune and 3.7 % vertically: these could be further improved by the addition of decapole or higher order fields. We have confirmed that the small tune footprint for this lattice is achievable in a realistic accelerator by applying random field errors.

There are still improvements to pursue in future work. For the theory, it is not clear whether a given working point has a unique optimal solution, nor is it known how the multipoles will impact the longitudinal dynamics. Including realistic fringe fields in future simulations should also improve the utility of our modelling. Further, we plan to apply this work to matching sections, as controlling linear dynamics is vital for transition regions and beam transfer lines.

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