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# RECONSTRUCTION OF BEAM PARAMETERS FROM BETATRON RADIATION USING MAXIMUM LIKELIHOOD ESTIMATION AND MACHINE LEARNING

S. Zhang[1]*, M. Yadav[1,2]†, C. Hansel[3], M. H. Oruganti[1], B. Naranjo[1], P. Manwani[1], G. Andonian[1], O. Apsimon[1,2], C. Welsch[1,2], James Rosenzweig[1]
[1]Department of Physics and Astronomy, University of California Los Angeles, CA, USA
[2]Department of Physics, University of Liverpool, Liverpool, UK
[3]Department of Physics, University of Colorado Boulder, CO, USA

*Abstract*

Betatron radiation that arises during plasma wakefield acceleration can be measured by a UCLA-built Compton spectrometer, which records the energy and angular position of incoming photons. Because information about the properties of the beam is encoded in the betatron radiation, measurements of the radiation can be used to reconstruct beam parameters. One method of extracting information about beam parameters from measurements of radiation is maximum likelihood estimation (MLE), a statistical technique which is used to determine unknown parameters from a distribution of observed data. In addition, machine learning methods, which are increasingly being implemented for different fields of beam diagnostics, can also be applied. We assess the ability of both MLE and other machine learning methods to accurately extract beam parameters from measurements.

## INTRODUCTION

In plasma wakefield acceleration, a dense drive beam generates a linear focusing force by repelling the plasma electrons away from its path while leaving the much heavier plasma ions uniformly distributed. Subject to this focusing force in plasma frequency $k_\beta$, electrons inside the witness beam then undergo harmonic transverse betatron oscillations, giving rise to betatron radiation. Because information about the properties of the beam is encoded in betatron radiation, measurements of this radiation can be used to reconstruct beam parameters, allowing devices which record information about betatron radiation, such as the UCLA-built Compton spectrometer, to be used for beam diagnostics. A variety of beam diagnostic devices and techniques already exist, such as the beam current transformer, used to measure beam intensity and charge, and LASER-Compton scattering, used to measure beam emittance and spot size[1]. Machine learning (ML) methods are also implemented for different fields of beam [2]. For example, the application of convolutional neural networks at FAST is able to produce a prediction for various downstream beam parameters from simulated datasets[3], and ML may also have the potential to be applied to betatron radiation diagnostics. Another method of extracting information about beam parameters from measurements of radiation is maximum likelihood estimation (MLE), a statistical technique used to determine unknown parameters from a given distribution of observed data. The goal of this work is to assess the ability of both maximum likelihood estimation and machine learning as methods for accurately extracting a beam parameters from measurements of betatron radiation.

* s.ziqianzhang@gmail.com
† monika.yadav@liverpool.ac.uk

## MAXIMUM LIKELIHOOD ESTIMATION

The method of maximum likelihood estimation involves some probability distribution function $f(x|\sigma)$, which specifies the probability of observing a data vector $x$ given the parameter $\sigma$. The probability distribution function is related to a likelihood function $L(\sigma|x)$ by $L(\sigma|x) = f(x|\sigma)$, where $L(\sigma|x)$ specifies the likelihood of $\sigma$ given $x$. Given a set of N observations of data vectors, the overall likelihood is the product of the likelihoods for each individual data vector [4], and the value of the parameter $\sigma$ which is most likely to have produced the set of observed data is determined by maximizing the likelihood with respect to $\sigma$. Because working with log-likelihood, rather than raw likelihood, avoids possible problems with arithmetic underflow [5], this work performs MLE with the log-likelihood, which is given by

$$\ln L(\sigma|x_1, x_2, ..., x_N) = \sum_{n=1}^{N} \ln L(\sigma|x_n), \qquad (1)$$

where the product of likelihoods has been converted into a sum of log-likelihoods.

## BEAM PARAMETER RECONSTRUCTION USING MLE

The first task tackled by this work was to correctly identify a beam's spot size from its radiation spectrum using MLE. The process described here is easily applied to other beam parameter reconstruction tasks.

First, several simulations of betatron radiation from beam particles in a plasma wakefield accelerator were run for beams of different spot sizes. The results of these simulations, plotted as 1D energy spectra, are shown in Fig. 1. This work uses a simulation in which particles are sampled from a beam and tracked through idealized fields. Betatron radiation was computed for a single particle using Lié-

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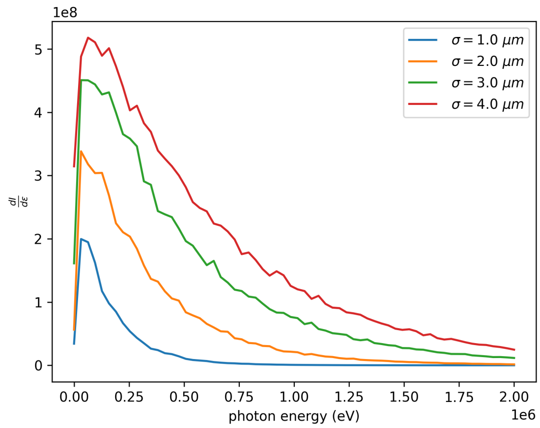


Figure 1: Radiation spectra for several spot sizes.

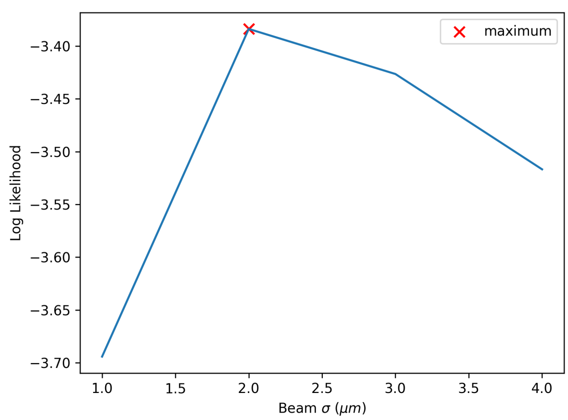


Figure 2: The log-likelihood function $lnL(\sigma|x)$ reaches a maximum at the test spot size, 2 μm.

nard–Wiechert potentials, and the radiation generated by the a was computed by summing the contribution from each particle. Some known "test" spot size was then arbitrarily chosen, and an additional radiation spectrum was obtained for this beam. In this example, the test spot size was chosen to be 2 μm. Each of the $dI/dt$ spectra are then converted into a probability distribution with photon energy treated as a discrete variable, together forming a probability distribution function $f(x|\sigma)$, where x is photon energy and $\sigma$ is spot size. In authentic MLE, the test spectrum's y-axis should be in terms of a concrete number of objects (i.e. photons), but, because it does not affect MLE results, this work also converts the test spectrum to a probability distribution $f_{test}(x)$ for ease of comparison. Now, the likelihood that the probability distribution $f(y|\sigma)$ models the test spectrum $f_{test}(x)$ for different values of $\sigma$, can then be calculated by Eq. (1). That is, for an test spectrum of discrete photon energies $x_1, x_2, ..., x_N$,

$$\ln L(\sigma|x_1, x_2, ..., x_N) = \sum_{j=1}^{J} f_{test}(x_j) \ln f(x_j|\sigma). \quad (2)$$

Figure 2 shows the log-likelihood function plotted and correctly identifying the test spot size of 2 μm.

Furthermore, this same MLE model can also be expanded to analyze data for two-dimensional distributions, such as

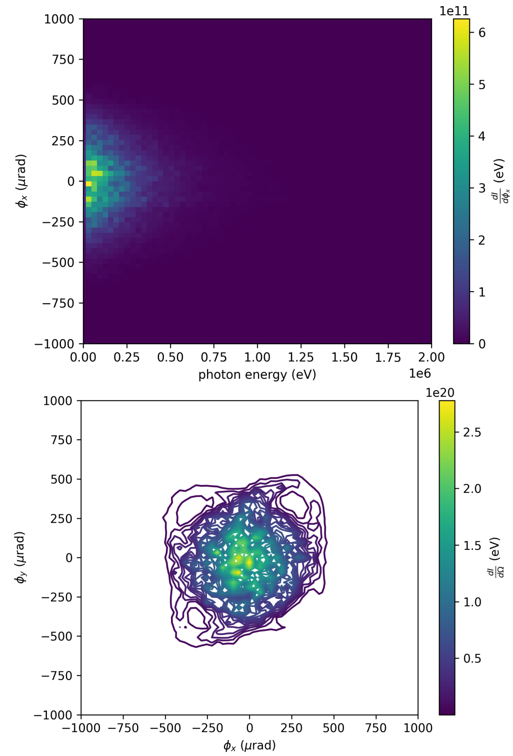


Figure 3: Two types of two-dimensional distributions plotted from same radiation data. Top: Double Differential Spectrum. Bottom: Angular Spectrum.

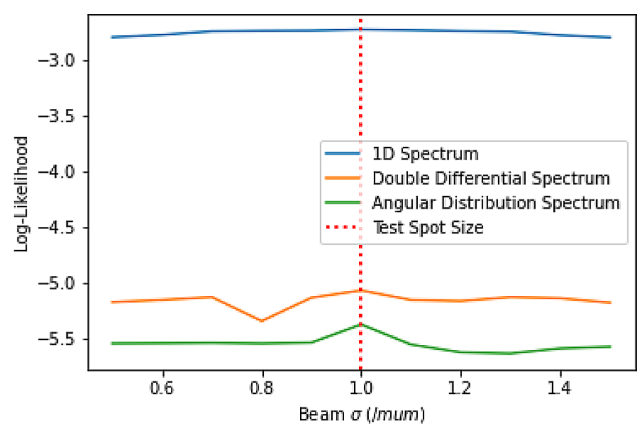


Figure 4: Comparing likelihood plots for the MLE algorithm using three different formulations of the radiation data, all correctly identifying an test spot size of 1 μm.

double differential spectrum distributions and angular spectrum distributions, shown in Fig. 3. For 2D distributions, the MLE algorithm works very much in the same way, with the expression in Eq. (2) being summed over all points (now ordered pairs) in the distribution. The results of MLE using the 2D distributions, displayed in Fig. 4, show that all methods are able to correctly identify the test spot size of 1 μm. This makes the 1D radiation spectrum a more attractive choice for use with the MLE algorithm (as well with machine learning) because it delivers similarly reliable results with less computation.

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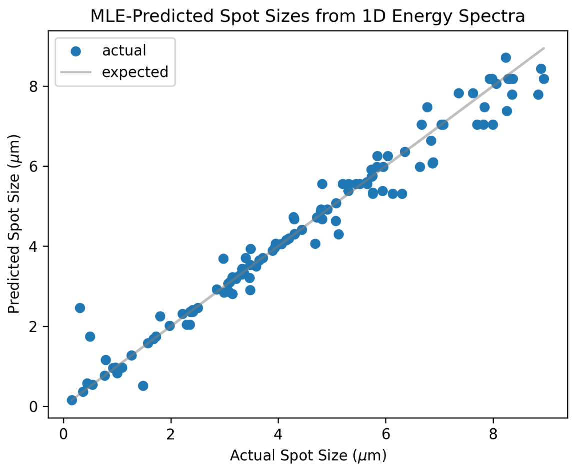


Figure 5: Overall spot size prediction results for 1D radiation spectrum MLE with 310 sets of reference data and 120 test cases.

In addition to being expanded to different plot types, the MLE algorithm can also be expanded to identify different beam parameters. The results present in Fig. 4 were able to be analogously replicated to identify beam energy, emittance and beam charge by simply repeating the above methods and replacing spot size with the appropriate parameter.

While the results in Fig. 4 show success of the MLE algorithm at a basic level, more precise and accurate results can also be achieved with the same methods. To show this, a total of 310 training simulations and 210 "test" simulations were run, each with a spot size chosen randomly from a uniform distribution between the values 0.1 µm and 9.0 µm. The overall results of using the 310 initial simulations to predict the spot sizes of all 120 "test" simulations by performing MLE with their 1D radiation spectrum data is displayed in Fig. 5, where the "expected" line represents perfect predictions. At a mean-squared error (MSE) of 0.186 µm$^2$, the prediction results appear relatively accurate, except in the region below 1 µm, where a few predictions are significantly greater than the actual spot sizes.

## BEAM PARAMETER RECONSTRUCTION USING MACHINE LEARNING

While the MLE method of beam parameter reconstruction was able to identify several beam parameters from different types of betatron radiation data at a basic level, this method is limited in its prediction ability because it cannot predict parameter values not included in the values for which simulation data is already provided. Therefore, machine learning is also explored as another method of extracting beam parameters from betatron radiation data.

We began again by attempting to build a model to predict spot size. Radiation data in the form of the 1D spectra was used to train and test a fully connected neural network with no bias and ReLU activations. Simulations were run to generate 310 training data sets and 120 test cases data for different spot sizes ranging from 0.1 µm to 9.0 µm in the form of both 1D energy spectra.

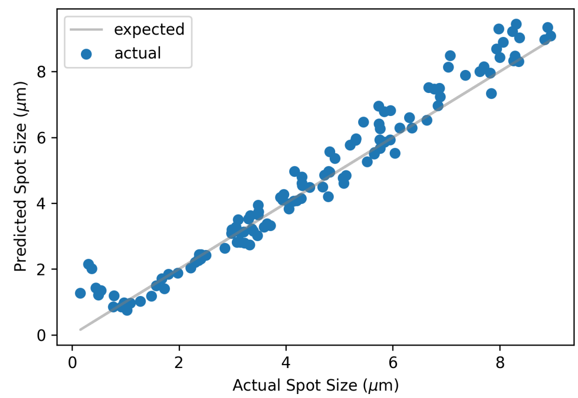


Figure 6: Spot size prediction results for 1D radiation spectrum ML with 400 sets of training data and 120 test cases. There is a clear inaccurate "tail" in the data at low spot sizes.

The results for ML predictions displayed in Fig. 6, show a "tail" below ∼1 µm, where the predictions all tend to be higher than the actual spot size values. The persistence of this problem between the MLE and ML methods suggests either that the simulation has difficulty resolving differences between the spectra of beams with extremely small spot sizes or that neither method is sensitive enough to differences in the spectra at low spot sizes. However, the predictions of the spot sizes appear otherwise reasonably accurate for Fig. 6, with a MSE of 0.2638.

## CONCLUSION

This work demonstrates that both MLE and ML can both effectively use betatron radiation data in order to wield betatron radiation as a tool for beam diagnostics, specifically in order to identify beam spot size, emittance, charge, and energy. While spot size is the most thoroughly tested of these parameters, both ML and MLE have difficulty accurately identifying small beam spot sizes.

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