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TOUSCHEK AND INTRABEAM SCATTERING IN ULTRALOW EMITTANCE STORAGE RINGS

R. Bartolini, DESY, Hamburg, Germany

Abstract

In next-generation synchrotron radiation sources targeting extremely low emittance around the so-called diffraction limit, the Touschek and intrabeam scattering (IBS) effects are important factors determining the performance of the facility. As the emittance decreases, the bunch volume decreases and the Touschek beam lifetime also decreases. However, this downward trend in beam lifetime is expected to turn to increase in the emittance region below a certain threshold. Since this threshold is determined by the emittance at equilibrium including the IBS effect, a self-consistent treatment is necessary for a correct and unified understanding of the beam characteristics. In currently operating facilities, such as MAX-IV, or in next-generation light sources under construction or in the planning stages, it is expected that such effects may be observed depending on the operating conditions. This talk will be reviewing Touschek and IBS Effects in terms of how these effects limit the ring performance.

INTRODUCTION

In the last 10 years the synchrotron light sources community has taken a decisive step towards the construction or the upgrade of facilities operating quasi-diffraction limited storage rings. Design emittance in the order of 100 pm are common to many lattice designs, exploiting the concept of multi-bend achromats (MBA) or variations thereof. The optimisation of such ring design is notoriously difficult in terms of single particle dynamics to guarantee basic operational parameters such as injection efficiency and beam lifetime. At the same time operation at high current poses further challenges in the control of collective effects and IBS. Two basic limitations in the performance of such rings, namely the Touschek lifetime and the emittance growth due to IBS, stem from the same basic physics mechanism, i.e. electron-electron scattering within the electrons in a bunch.

TOUSCHEK AND IBS THEORY

Electron-electron scattering occurring between electrons performing betatron oscillations within a bunch can transfer significant momentum from the transverse to the longitudinal plane. The analysis of the scattering in the CM of the two electrons show that the conservation of momentum allows event where the whole transverse momentum px is scattered in the longitudinal plane. When transferred back to the LAB frame the longitudinal component acquires a multiplicative factor γ, given by the relativistic factor of the electron bunch. This can be a significant portion of the total energy E_0 of the beam. If the acquired longitudinal momentum is outside the momentum aperture ε_{MA} of the lattice, the particles are lost. This is the Touschek scattering. If the momentum acquire is within ε_{MA}, the particles survive the scattering event but the sudden exchange in momentum creates an excitation effect on the dynamics which is the core reason for the IBS effect. In short

$p_x\gamma > \varepsilon_{MA}$	Touschek scattering
$p_x\gamma < \varepsilon_{MA}$	Intrabeam scattering

Touschek Scattering

The analysis of Touschek scattering aims at providing expression for the rate of particle loss and the Touschek lifetime of the beam. Starting from the electron-electron differential scattering cross section, we compute the total cross section for the events that produce a longitudinal momentum outside of the momentum aperture. Since the total cross section refers to a fixed momentum incident flux on a single target, the calculation for a real beam requires its computation over the phase space distribution of the electrons to yield the rate of particle loss. A common expression for the Touschek lifetime is given by the celebrated Bruck formula [1] cast in the form

$$\frac{1}{\tau} = \frac{N r_0^2 c}{8\pi\sigma_x\sigma_y\sigma_s}\frac{1}{\gamma^2\delta_{MA}^3}D(\xi) \qquad (1)$$

where N is the number of electrons per bunch, $\sigma_{x,y,s}$ the rms dimension of the bunch, δ_{MA} the momentum aperture, and

$$\xi = \left(\frac{\delta_{MA}}{\gamma\sigma_{p_x}}\right)^2$$

where $D(\xi)$ is the function

$$D(\xi) = \sqrt{\xi}\left[-\frac{3}{2}e^{-\xi} + \frac{\xi}{2}\int_{\xi}^{\infty}\frac{\ln u \times e^{u}}{u}du + \frac{1}{2}(3\xi - \xi\ln\xi + 2)\int_{x}^{\infty}\frac{e^{-u}}{u}du\right]$$

Equation (1) provides the dependence of the Touschek lifetime over the electron beam parameters and the main machine parameters. It is proportional to the current stored in the bunch via N, to the bunch volume $V = \sigma_x\sigma_y\sigma_s$, to the momentum aperture δ_{MA} appearing also in the argument of D, and on the rms of the transverse momentum σ_{px}. This last term can be expressed in terms of the emittance and the optics functions, e.g. neglecting dispersion it is simply $\sigma_{px} = \sqrt{(\varepsilon_x/\beta_x)}$ whereby $\xi = \delta_{MA}^2\beta_x/\gamma^2\varepsilon_x$. It is interesting to observe that the function $D(\xi)$ reported in Figure 1, has a relatively flat behaviour for values of x that are typical of third generation light sources, while it shows a sharp decrease

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for larger values of x, corresponding to ultra-low emittance lattices. By way of example we can consider δ_{MA} = 3%, β_x = 10 m, ε_x = 3 nm, E_0 = 3 GeV, giving ξ = 0.1, for typical third generation light sources, while for the same values of a typical low emittance upgrade (using e.g. ε_x = 100 pm) we have ξ = 2.6. In these conditions we can expect that the Touschek lifetime of third generation light sources decrease with the emittance as a consequence of the decrease bunch volume, up to the point where ultra-low emittance lattice generate a transverse momentum distribution σ_{px} so small that where actually the decrease of the emittance produce an increase in the Touschek lifetime.

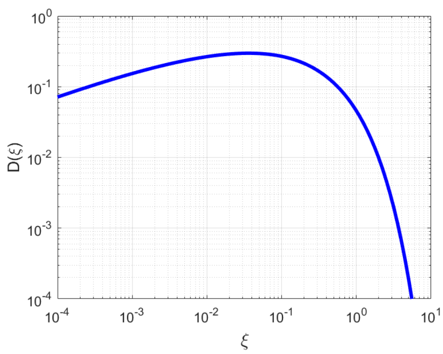

Figure 1: plot of the function D(x) in Eq. (1).

This behaviour was highlighted already in the conceptual design of the NSLS-II [2] and MAX-IV [3]. Figure 2 reports the corresponding plots of Touschek lifetime vs emittance where the lifetime increase for small emittance is clearly visible.

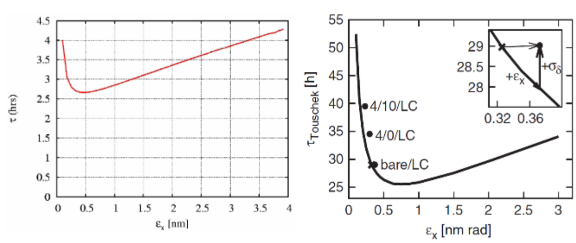

Figure 2: Lifetime vs emittance: for NSLS-II lattice (left) [2] and for MAX IV [3].

New ultra-low emittance designs target ever smaller emittance values and such lattices are expected to benefit significantly for the outlined dependence of the Touschek lifetime on emittance. As an example, PETRA IV aims at operating with an emittance of 20 pm achieved with extensive use of damping wiggler on a bare lattice which provide 43 pm. A H9BA lattice developed for the SSRLX upgrade provide 60 pm with a momentum aperture in the order of ~4% resulting in exceptional Touschek lifetime of 60 h for the operational parameters 480 mA in 500 bunches that exceeds the present operational lifetime of the SSRL [4].

The operation in this large Touschek lifetime regime hinges on the achievement of a large momentum aperture in presence of errors and the preservation of the low emittance. Correct tuning of the lattice in its linear and nonlinear optics are therefore of paramount importance to avoid the rapid degradation of the lifetime with ξ. Likewise, high charge collective effects producing emittance increase must be controlled. In this respect, high-harmonic bunch lengthening cavities are an effective tool in increasing the Touschek lifetime by increasing the bunch length, hence the bunch volume, and in counteracting the collective effects by decreasing the peak charge per bunch. Figure 3 shows the Touschek lifetime computed as a function of the emittance with a complete description of the PETRA IV lattice where the emittance is varied by switching gradually on the damping wigglers. We note that the Touschek lifetime increases while the emittance is reduced. We note also the strong dependence on the momentum aperture. Similar investigation in the context of PEP-X pointed out that in this regime, the Touschek lifetime scales as the fifth power of the momentum aperture rather than a simper third power inferred from Eq. (1).

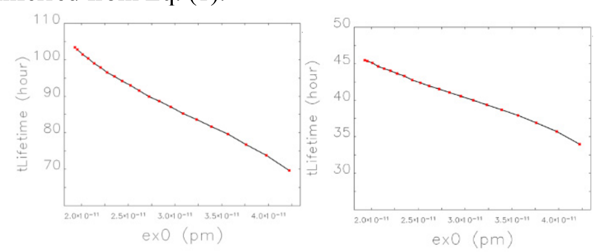

Figure 3: PETRA IV Touschek lifetime dependence with emittance: lattice without errors (left, ε^{acc} ~ 4.5%) with errors (right, ε^{acc} ~ 3.5%).

Recently, a lattice proposed to be host in the PEP-II tunnel at SLAC was optimized to reach a momentum aperture up to 8%, with a moderate sensitivity to errors, achieving a Touschek lifetime of ~500h [4].

Intrabeam Scattering

IBS has been recognised as a potential problem as it generates an increase in the emittance of the beam as a function of the stored charge, especially in ultra-low emittance lattice where the charge density is high. The IBS theory has been developed by Piwinsky [5] and Bjorken-Mtingwa [6]. It provides the growth rates for the beam characteristics, based on the analysis of the scattering effect and the redistribution of momenta in the longitudinal and transverse planes. While the Piwinski and Bjorken-Mtingwa growth rates appear rather different in from, Bane [7] has proven their equivalence in the approximation of high beam energy and provided a simpler for the IBS growth rates

$$\frac{1}{T_p} = \frac{N r_0^2 c(\log)}{32\gamma^3 \varepsilon_x^{3/4}\varepsilon_y^{3/4}\sigma_s\sigma_p^3}\left\langle \sigma_H g(a/b)(\beta_x\beta_y)^{-1/4}\right\rangle \qquad (2)$$

$$\frac{1}{T_{x,y}} = \frac{\sigma_p^2 \langle H_{x,y}\rangle}{\varepsilon_{x,y}}\frac{1}{T_p}$$

for the energy spread and the transverse beam size respectively, where

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$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{H_x}{\varepsilon_x} + \frac{H_y}{\varepsilon_y}$$

and $H_{x,y}$ is the dispersion invariant computed in the horizontal and vertical plane respectively. The function g can be approximated as

$$g_{bane}(x) = 2x^{(0.021-0.44\ln x)}$$

The equilibrium emittances in presence of IBS are the stationary solutions of the set of first order differential equations

$$\frac{d\varepsilon_x}{dt} = -2\frac{\varepsilon_x - \varepsilon_{x,0}}{\tau_x} + \frac{2\varepsilon_x}{T_x(\varepsilon_x, \varepsilon_y, \sigma_p)}$$

$$\frac{d\varepsilon_y}{dt} = -2\frac{\varepsilon_y - \varepsilon_{y,0}}{\tau_y} + \frac{2\varepsilon_y}{T_y(\varepsilon_x, \varepsilon_y, \sigma_p)}$$

$$\frac{d\sigma_p^2}{dt} = -2\frac{\sigma_p^2 - \sigma_{p,0}^2}{\tau_p} + \frac{2\sigma_p^2}{T_p(\varepsilon_x, \varepsilon_y, \sigma_p)}$$

where $\varepsilon_{x,0}$, $\varepsilon_{y,0}$ and $\sigma_{p,0}$ are the values for the emittance and energy spread in the limit of zero current. The steady state solutions read

$$\varepsilon_x = \frac{\varepsilon_{x,0}}{1 - \tau_x / T_x(\varepsilon_x, \varepsilon_y, \sigma_p)}$$

$$\varepsilon_y = \frac{\varepsilon_{y,0}}{1 - \tau_y / T_y(\varepsilon_x, \varepsilon_y, \sigma_p)}$$

$$\sigma_p^2 = \frac{\sigma_{p,0}^2}{1 - \tau_p / T_p(\varepsilon_x, \varepsilon_y, \sigma_p)}$$

These expressions show that the impact on the final equilibrium emittance is strong in the case the rise times of the IBS (T_p, $T_{x,y}$) are comparable with the radiation damping times (τ_p, $\tau_{x,y}$). The IBS growth rates depend linearly on the stored charge N and with $1/\gamma^4$ on the beam energy. Their dependence on the beam properties (bunch dimension and energy spread) and lattice functions is rather involved. These considerations allow to define some basic strategies to minimise the impact of IBS on the emittance.

Effort to use the linear optics of the lattice to minimise the IBS growth rates were made in the design of the TME cell for the CLIC damping ring [8]. Semi-analytical calculation of the emittance and IBS growth rates for different phase advance of the TME cell are reported in Figure 4. It is clear that a choice of a low phase advance per cell minimises IBS but can be relatively far from the TME condition. Such studies shown that the use of a combined function gradient dipoles in a TME cell is beneficial in reducing the IBS while maintaining the same emittance. This is ultimately due a better separation of the optics functions β_x, β_y that prevents a local squeeze of the beam dimension at the dipole and the corresponding high IBS local growth rate. In general, a plot of the local growth rate is helpful in identifying the regions where the IBS is most important.

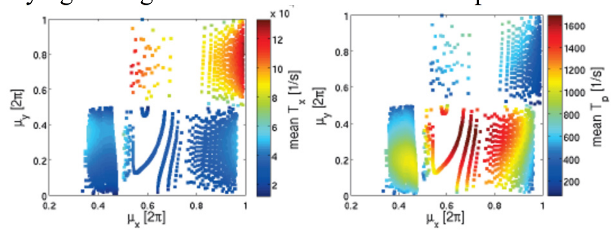

Figure 4: IBS growth rates T_x (left) T_p (right) as a function of the TME cell phase advance.

Another way of minimising the relative impact of IBS is to use damping wigglers in dispersion free straight sections. Damping wigglers allow reducing the ratio between the radiation damping time ($\tau_{x,y,s}$) with respect to the IBS rise time ($T_{x,y,s}$) therefore reducing the increase of the IBS equilibrium emittance. The damping wiggler parameters can be chosen to reduce the zero current emittance with moderate impact on the energy spread, while at the same time, reducing the synchrotron radiation damping time. The energy loss due to a wiggler with period λ_W, length L_W and wiggler parameter K is

$$U_{DW} = \frac{8\pi^2 r_0 mc^2 \gamma^2 K^2}{3\lambda_W^2} L_W$$

The corresponding change in emittance can be expressed as a function of the corresponding change of the radiation integrals due to the damping wigglers as

$$\varepsilon_{x,DW} = C_q \gamma^2 \frac{I_5 + \Delta I_5}{I_2 + \Delta I_2 + (I_4 - \Delta I_4)}$$

$$\sigma_{\varepsilon,DW}^2 = C_q \gamma^2 \frac{I_3 + \Delta I_3}{2(I_2 + \Delta I_2) + (I_4 - \Delta I_4)}$$

$$\tau_{x,DW} = \frac{3}{r_0} \frac{T_0}{\gamma^3} \frac{1}{I_2 + \Delta I_2 - (I_4 + \Delta I_4)}$$

$$U_{0,DW} = \frac{e^2 \gamma^4}{6\pi\varepsilon_0}(I_2 + \Delta I_2)$$

with

$$C_q = \frac{55}{32\sqrt{3}} \frac{h}{mc} = 3.84 \times 10^{-13}\,\mathrm{m}$$

Assuming that the dispersion in the straight section is zero, and that we can neglect the variation of I_4 and the self-dispersion generated by the damping wigglers, we have that both the emittance ε_x and the radiation damping time τ_x scale with $1/I_2$, i.e. inversely proportional to the energy loss per turn, i.e. $\varepsilon_x \sim 1/U_0$ and $\tau_x \sim 1/U_0$.

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The corresponding analysis of the IBS growth rate in the Bane formulation can be carried observing that from Eq. (2).

$$\frac{1}{T_x} = \frac{N r_0^2 c(\log)}{32\gamma^3 \varepsilon_x^{7/4} \varepsilon_y^{3/4} \sigma_s \sigma_p} \left\langle H_x \sigma_H g(a/b)(\beta_x \beta_y)^{-1/4} \right\rangle \quad (3)$$

As the energy loss per turn increases and the emittance decreases, the energy spread and bunch length tend to grow. Assuming operation at constant coupling, approximating the combined growth of $\sigma_s\sigma_p$ with the energy loss per turn as $U_0^{1/4}$, i.e. $\varepsilon_x^{-1/4}$, and finally noting that $\sigma_H \sim \varepsilon_x^{1/2}$, we can infer that the IBS growth rate is also scaling with the inverse of the emittance, i.e. $T_x \sim 1/\varepsilon_x$. In these assumptions, the IBS equilibrium emittance does not depend on the zero current emittance as the energy loss per turn is increased. In this way we can reduce the emittance with damping wiggler with a limited IBS emittance blow up independent on the emittance.

Emittance reduction based on the control of I_5, related to the dispersion invariant, will not change significantly the energy loss per turn, hence will not change the radiation damping time and will be more prone to IBS emittance increase. It should be noted, however, that the use of strong damping wigglers will eventually increase the energy spread reducing the brightness of the radiation generated by the undulators, eventually overriding the brightness gain due to a smaller emittance. At larger field the effect of self-dispersion will eventually increase also the zero current emittance of the ring. An example of this behaviour is reported from the studies of the PETRA IV ring. Figure 5 shows the change in emittance due to damping wiggler with λ_W = 7.3 cm. The emittance is reduced from 43 pm (U_0 = 1.3 MeV/turn) to 20 pm (U_0 = 4.2 MeV/turn). Figure 6 shows the relative increase of emittance and energy spread for PETRA IV operating in brightness mode, consisting of 200 mA in 1600 bunches (0.9 nC per bunch).

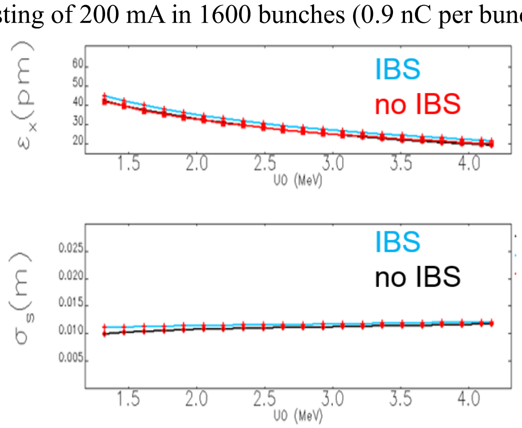

Figure 5: Emittance (top) and bunch length (bottom) as a function of the energy loss per turn due to damping wiggler in PETRA IV. IBS equilibrium emittance and bunch length computed for 0.9 nC per bunch.

The impact of IBS on the equilibrium emittance remains below 5 pm across despite a reduction of the emittance by a factor 2 without any significant blow up at smaller emittances. Figure 7 show the corresponding analysis for the timing mode 80 mA in 80 bunches with a substantially large charge per bunch (7.7 nC). The impact of IBS is now significant although still roughly independent emittance.

It should be noted that including the effect of the machine impedance, collective effects dominate completely the beam dynamics. The bunch lengthening due to the longitudinal impedance elongate the bunch, therefore reducing the impact of the IBS on the equilibrium emittance.

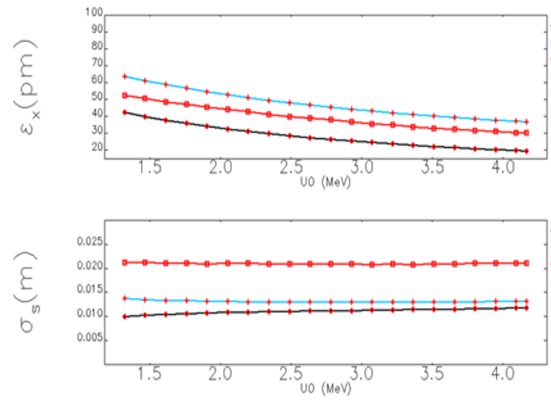

Figure 6: Emittance (top) and bunch length (bottom) as a function of the energy loss per turn due to damping wigglers in PETRA IV. In cyan the IBS equilibrium emittance and bunch length computed for 7.7 nC per bunch. In red the same data computed including the machine impedance.

Another clear way to minimise the IBS growth rate is indeed to generate longer bunches. In this sense bunch lengthening cavity are an effective tool to reduce the IBS equilibrium emittance. Example of the impact is shown in Fig. 7 for the MAX IV lattice where the impact of IBS is reduced from 46% to 13% at the nominal emittance of 330 pm.

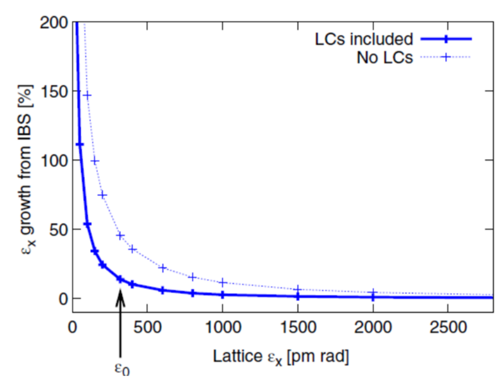

Figure 7: MAX IV impact of IBS with/without Landau cavities [9].

CONCLUSION

Ultra low emittance ring promise to operate in condition of very large Touschek lifetime provided the emittance and the momentum aperture such that the parameter $\xi \geq 1$.

In the case of wiggler dominated rings, the reduction of the emittance with damping wiggler is also very effective in controlling the IBS growth rates. When this approach is not possible operation with bunch lengthening cavities or round beams can alleviate the impact of IBS.

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