Microbunching Instability in the Presence of Intrabeam Scattering for Single-Pass Accelerators

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Introduction: Micro-Bunching

Microbunching involves evolution of **phase space modulation** during **pure optics transport** and/or in presence of *incoherent* effects and/or *collective* short-range interactions (**high-frequency impedances**).

Microbunching instability has been one of the research focuses in accelerator physics and is expected to remain so in the years to come, as evidenced by x-ray free-electron lasers (FELs), the associated harmonic generation schemes or other advanced light sources based on high-brightness electron beams.

Particle tracking simulation is popular but the results are very **sensitive to numerical noise** in view of microbunching. Better to perform **6-D start-to-end** analysis. Either low dimensional or concatenated analysis would likely **underestimate** microbunching instability phenomena[2].

[2] C.-Y. Tsai, NIMA 940 (2019), pp. 462-474.

Introduction: Micro-Bunching

In view of microbunching dynamics, below are pros and cons for particle tracking simulation vs. kinetic analysis:

- **Particle tracking**: time domain, can be sensitive to numerical noise $\Rightarrow$ time-consuming (huge number of macroparticles, sufficient number of bins, dedicated noise filters), easy to implement different physical effects, many available simulation packages
- **Kinetic analysis**: frequency domain, direct solution of phase space distribution can be avoided $\Rightarrow$ efficient and free from numerical noise, suitable for systematic studies and/or design optimization, not always straightforward to add various physical effects, simulation packages usually not available

Goal: How to evaluate microbunching dynamics *efficiently* and *accurately*?

Motivation: Microbunching with IBS

- Intrabeam scattering (IBS) has long been studied in lepton and hadron storage rings as a slow diffusion process.
- In single-pass accelerators, IBS studies are, for example, mainly relevant to beam halo mechanism in high-intensity proton linacs. For electron accelerators, a few studies were on microbunching dynamics with IBS in low-emittance storage rings[3].
- For linac-driven or single-pass electron accelerators, from preliminary analytical estimate of IBS, people did not expect significant impact on microbunching-FEL dynamics[4].
- Recent experiments at FERMI[5], however, indicate that IBS may play a significant role in microbunching dynamics, the observation thanks to high-brightness electron beam (sufficiently high peak bunch current).
- Then followed by a few theoretical studies[6].

[3] M. Venturini, PAC2001, pp. 2958-2960.

[4] Z. Huang, LCLS-TN-02-8. S. DiMitri, PRST-AB **17**, 074401 (2014).

[5] S. DiMitri *et al.*, NJP **22**, 083053 (2020).

[6] C.-Y. Tsai *et al.*, PRAB **23**, 124401 (2020), C.-Y. Tsai and W. Qin, Phys. Plasmas **28**, 013112 (2021), G. Perosa and S. Di Mitri, Sci. Rep. **11**, 7895 (2021)

Order of magnitude estimate

	Storage ring light source	Middle-energy single-pass accelerator
Beam energy	$\sim$GeV	$\sim$100 MeV
Particles per bunch	10^{10} or more	$10^8 \sim 10^9$
Peak current	50$\sim$100 A	100 $\sim$ a few kA
Normalized emittances	$\sim \mu$m	1 μm or lower
Fractional energy spread	$10^{-3} \sim 10^{-4}$	10^{-4} or smaller
Effective distance	∞	100 m$\sim$a few km

IBS growth $$\tau_{\text{IBS}}^{-1} \left(\equiv \frac{1}{\left(\epsilon_\perp^N, \sigma_\delta\right)} \frac{\mathrm{d}\left(\epsilon_\perp^N, \sigma_\delta\right)}{\mathrm{d}s} \right) \propto \frac{N_b}{\gamma^2 \epsilon_x^N \epsilon_y^N \sigma_z \sigma_\delta}$$

$$\Rightarrow \tau_{\text{IBS,single-pass}}^{-1} \approx 10^{2\sim3} \times \tau_{\text{IBS,storage-ring}}^{-1}$$

Energy chirp & bunch compression $\Rightarrow$ another factor of $10 \sim 10^2$ enhancement

Kinetic analysis: Vlasov-Fokker-Planck equation

$$\frac{\mathrm{d}f}{\mathrm{d}s} = -\sum_{i=x,y,z} \frac{\partial}{\partial p_i}\left(D_i f\right) + \frac{1}{2}\sum_{i,j=x,y,z} \frac{\partial^2}{\partial p_i \partial p_j}\left(D_{ij} f\right)$$

If the friction and diffusion are neglected, VFP equation reduces to Vlasov equation (or collisionless Boltzmann equation). In usual situations, the time scale for the collective dynamics is faster than that of the diffusion dynamics. For long-term dynamics and/or high-peak current, one may need to include RHS to base the analysis on VFP equation.

Direct, 6-D solution can be too complicated ⇒ Apply perturbation technique, $f = f_0 + f_1$ with $|f_1| \ll f_0$

- 0th order solution ⇒ pure optics transport and/or incoherent effects (e.g., IBS, ISR), PWD (for storage ring)
- 1st order solution ⇒ collective dynamics

Phase space microbunching involves the dynamical evolution of **the characteristic functions of** f_1, see later.

Zeroth order: pure optics & IBS

Zeroth order equation for $f_0(\mathbf{X}; s)$

$$\frac{\mathrm{d}f_0}{\mathrm{d}s} = -\sum_{i=x,y,z} \frac{\partial}{\partial p_i}\left(D_i f_0\right) + \frac{1}{2}\sum_{i,j=x,y,z} \frac{\partial^2}{\partial p_i \partial p_j}\left(D_{ij} f_0\right)$$

For pure linear optics without IBS, RHS vanishes and the general solution gives $f_0(\mathbf{X}; s) = f_0(\mathbf{M}^{-1}\mathbf{X}; 0)$. For the case with IBS, the solution gives IBS growth rates [see, for example, K. Kubo *et al.*, PRST-AB **8**, 081001 (2005)]

$$\frac{1}{\sigma_\delta}\frac{\mathrm{d}\sigma_\delta}{\mathrm{d}s} = \tau^{-1}_{\mathrm{IBS},\delta} + \frac{1}{C}\frac{\mathrm{d}C}{\mathrm{d}s} \text{ with } \tau^{-1}_{\mathrm{IBS},\delta} = 2\times 2\pi^{3/2} A\left[\frac{\sigma_H^2}{\sigma_\delta^2}\left([\mathrm{Log}]_x \frac{g\left(\frac{b}{a}\right)}{a} + [\mathrm{Log}]_y \frac{g\left(\frac{a}{b}\right)}{b}\right)\right]$$

$$\frac{1}{\epsilon_x^G}\frac{\mathrm{d}\epsilon_x^G}{\mathrm{d}s} = \tau^{-1}_{x,\mathrm{IBS}} = 4\pi^{3/2} A\left[-a[\mathrm{Log}]_x g\left(\frac{b}{a}\right) + \frac{\mathcal{H}_x \sigma_H^2}{\epsilon_x^G}\left([\mathrm{Log}]_x \frac{g\left(\frac{b}{a}\right)}{a} + [\mathrm{Log}]_y \frac{g\left(\frac{a}{b}\right)}{b}\right)\right]$$

$$\frac{1}{\epsilon_y^G}\frac{\mathrm{d}\epsilon_y^G}{\mathrm{d}s} = \tau^{-1}_{y,\mathrm{IBS}} = 4\pi^{3/2} A\left[-b[\mathrm{Log}]_y g\left(\frac{a}{b}\right) + \frac{\mathcal{H}_y \sigma_H^2}{\epsilon_y^G}\left([\mathrm{Log}]_x \frac{g\left(\frac{b}{a}\right)}{a} + [\mathrm{Log}]_y \frac{g\left(\frac{a}{b}\right)}{b}\right)\right]$$

$$A = \frac{I_b}{\left(2\sqrt{2\ln 2}\right) 64\pi^2\gamma^2\epsilon_x^N\epsilon_y^N\sigma_\delta}\frac{r_e^2}{ce}, \ [\mathrm{Log}]_x = \ln\left(\frac{q^2}{a^2}\right), \ [\mathrm{Log}]_y = \ln\left(\frac{q^2}{b^2}\right)$$

$$\mathcal{H}_{x,y} = \frac{R_{16,36}^2 + \left(\beta_{x,y}R_{26,46} + \alpha_{x,y}R_{16,36}\right)^2}{\beta_{x,y}}, \ q = \sigma_H\beta\sqrt{2d/r_e}, \ d = \min\left\{\sigma_x, \sigma_y, \lambda_D\right\}$$

$$g(w) = \sqrt{\frac{\pi}{w}}\left[P^0_{-1/2}\left(\frac{w^2+1}{2w}\right) \pm P^{-1}_{-1/2}\left(\frac{w^2+1}{2w}\right)\right], \ a = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x^G}}, \ b = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\epsilon_y^G}}, \ \frac{1}{\sigma_H^2} = \frac{1}{\sigma_\delta^2} + \frac{\mathcal{H}_x}{\epsilon_x^G} + \frac{\mathcal{H}_y}{\epsilon_y^G}$$

First order: linearized integral/matrix equations

From definition of the diffusion and friction coefficients in VFP equation, for IBS, they can be derived[7]

$$D_z(s) = -\left(\frac{r_e[\text{Log}]}{\gamma^2\epsilon_{\perp,N}^2}\frac{I_b}{I_A}\right)\text{erf}\left(\frac{\delta}{\sqrt{2}\sigma_\delta}\right)$$

$$D_{zz}(s) = \frac{\sqrt{\pi}}{2}\left(\frac{r_e[\text{Log}]}{\gamma^2\epsilon_{\perp,N}\sigma_\perp}\frac{I_b}{I_A}\right)$$

where $T_\parallel \ll T_\perp$ is assumed. Substituting $f = f_0 + f_1$ into VFP and neglecting higher order terms of f_1, we would obtain the linearized 1st order VFP equation. Expressed in terms of the density and energy modulations,

$$\mathbf{b}_{k_z} = b(k_z;s) = \frac{1}{N}\int f_1(\mathbf{X};s)e^{-ik_z z_s}\text{d}\mathbf{X}$$

$$\mathbf{p}_{k_z} = p(k_z;s) = \frac{1}{N}\int (\delta_s - hz_s)f_1(\mathbf{X};s)e^{-ik_z z_s}\text{d}\mathbf{X}$$

we would obtain a set of **linear coupled** integral equations.

[7] Following similar procedures by G. Stupakov, FEL 2011 (MOPB20)

First order: linearized integral/matrix equations[8]

$$\begin{pmatrix} \mathcal{P} & \mathcal{Q} \\ \mathcal{R} & \mathcal{S} \end{pmatrix} \begin{bmatrix} \mathbf{b}_{k_z} \\ \mathbf{p}_{k_z} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{k_z}^{(0)} \\ \mathbf{p}_{k_z}^{(0)} \end{bmatrix}$$

where the matrix elements are

$$\mathcal{P} = \mathcal{I} - i\mathcal{K}_{Z_\parallel}^{(1)} - \mathcal{K}_{\mathrm{IBS},z}^{(1)} + 2\mathcal{K}_{\mathrm{IBS},zz}^{(2)}, \qquad \mathcal{Q} = -i\mathcal{K}_{\mathrm{IBS},z}^{\perp(0)} - i\mathcal{K}_{\mathrm{IBS},zz}^{(3)}$$

$$\mathcal{R} = \mathcal{K}_{Z_\parallel}^{(0)} - \mathcal{K}_{Z_\parallel}^{(2)}\sigma_{\delta\tau}^2 - i\mathcal{K}_{\mathrm{IBS},z}^{(0)} - 2i\mathcal{K}_{\mathrm{IBS},z}^{(1)} + 4i\mathcal{K}_{\mathrm{IBS},zz}^{(1)} - 2i\mathcal{K}_{\mathrm{IBS},zz}^{(3)}\sigma_{\delta\tau}^2$$

$$\mathcal{S} = \mathcal{I} + \mathcal{K}_{\mathrm{IBS},z}^{\perp(0)} - \mathcal{K}_{\mathrm{IBS},z}^{\perp(2)} + 3\mathcal{K}_{\mathrm{IBS},zz}^{(2)} - \mathcal{K}_{\mathrm{IBS},zz}^{(4)}\sigma_{\delta\tau}^2$$

with $\mathcal{I}$ identity matrix

- $\mathcal{K}_{Z_\parallel}$ from collective interaction
- $\mathcal{K}_{\mathrm{IBS},zz}$ from IBS diffusion
- $\mathcal{K}_{\mathrm{IBS},z}$ from IBS friction

In absence of IBS, $\mathcal{Q} = 0$, the well-known integral equation $\left(\mathcal{I} - i\mathcal{K}_{Z_\parallel}^{(1)}\right)\mathbf{b}_{k_z} = \mathbf{b}_{k_z}^{(0)}$ is recovered.

[8] C.-Y. Tsai *et al.*, PRAB **23**, 124401 (2020)

Estimate of slice energy spread (SES)[9]

An important figure of merit, SES, may originate from

- pure-optics bunch compression $\Rightarrow C(s)\sigma_{\delta 0}$
- IBS $\tau^{-1}_{\mathrm{IBS},\delta} \Rightarrow \sigma_{\delta,\mathrm{IBS}}(s)$
- short-wavelength collective interaction $\Rightarrow C(s)\sigma_{\delta,\mathrm{coll}}(s)$

$$\sigma_{\delta,\mathrm{coll}}(s) \approx \sqrt{\frac{8}{n_b}\int_0^{\lambda^*}\frac{\mathrm{d}\lambda}{\lambda^2}\left|\int_0^s \mathrm{d}\tau \frac{C(\tau)I_b(\tau)}{\gamma I_A} Z_\parallel(\lambda;\tau)\tilde{G}(\lambda;\tau)\right|^2}$$

with microbunching gain factor $\tilde{G}(\lambda;s) = \mathbf{b}_{k_z}(s)/\mathbf{b}^{(0)}_{k_z}(0)$

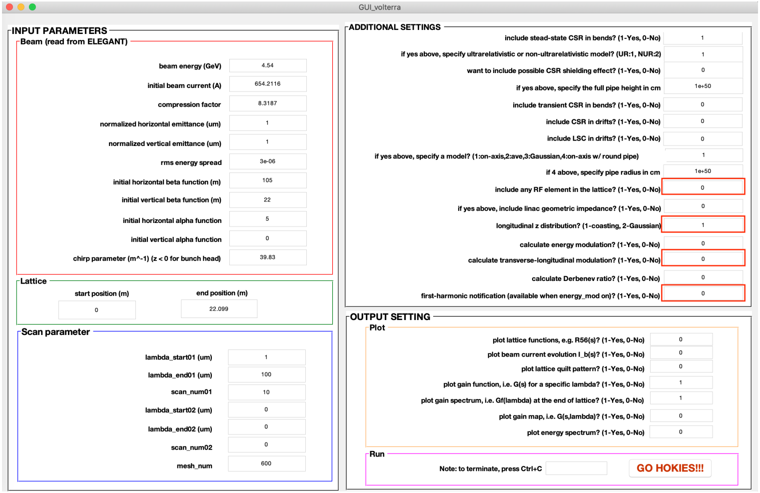
The total SES is evaluated as a quadrature sum

$$\sigma_{\delta,\mathrm{tot}}(s) \approx \begin{cases} \sqrt{C^2(s)\sigma^2_{\delta 0} + C^2(s)\sigma^2_{\delta,\mathrm{coll}}(s)}, & \text{without IBS} \\ \sqrt{\sigma^2_{\delta,\mathrm{IBS}}(s) + C^2(s)\sigma^2_{\delta,\mathrm{coll}}(s)}, & \text{with IBS} \end{cases}$$

★ $\sigma_{\delta,\mathrm{coll}}$ and $\sigma_{\delta,\mathrm{IBS}}$ are not independent! $\sigma_{\delta,\mathrm{IBS}}$ may *heat* the beam, while it may also help *mitigate* $\sigma_{\delta,\mathrm{coll}}$, leading to $\sigma_{\delta,\mathrm{tot}}$.

[9] S. DiMitri *et al.*, NJP **22**, 083053 (2020). C.-Y. Tsai and W. Qin, Phys. Plasmas **28**, 013112 (2021).

Semi-analytical, GUI-based tool for MB analysis

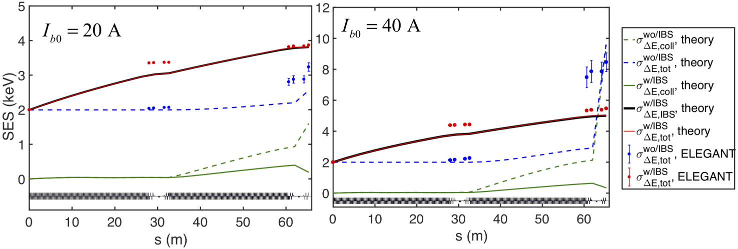


Input files: `elegant *.ele & *.lte`
Available on Github: `https://github.com/jcytsai/volterra_mat`. Most updated version available upon request

More refined, friendly GUI is under development

Example 1: FODO-BC-FODO-BC transport line[10]

Name	Value	Unit
Beam energy	150	MeV
Peak current	5∼40	A
Initial energy spread	1.33×10^{-5}	
Normalized emittances	0.4	μm
Momentum compaction per each BC	24.45	cm
Initial energy chirp	0∼1.92	m^{-1}



Below and above a certain *current threshold* (25 A), we see quite different SESs at the end of the beamline when IBS is included.

[10] C.-Y. Tsai and W. Qin, Phys. Plasmas **28**, 013112 (2021).

elegant tracking simulation

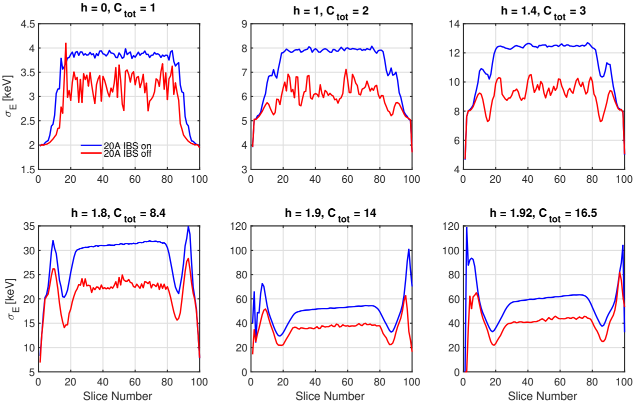


Figure: Slice energy spread for $I_{b0} = 20$ A for different energy chirps. In the simulation only LSC is included.

elegant tracking simulation

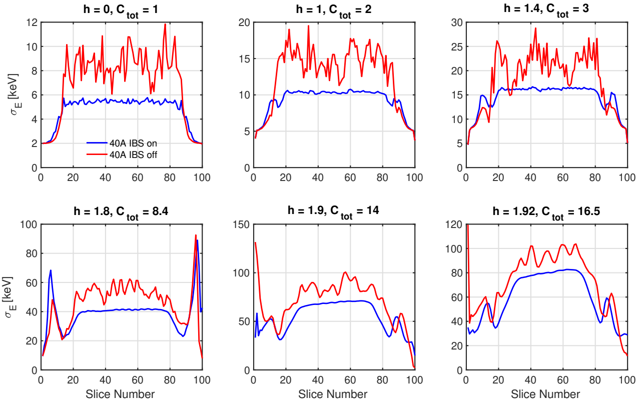


Figure: Slice energy spread for $I_{b0} = 40$ A for different energy chirps. In the simulation only LSC is included.

Using the semi-analytical VFP analysis, we can find the threshold condition, *above* which IBS plays a beneficial role to mitigate microbunching instability; *below* which IBS merely heats the beam. The following contour plot draws $\sigma_{\Delta E.\text{tot}}^{\text{wo/IBS}} - \sigma_{\Delta E.\text{tot}}^{\text{w/IBS}}$

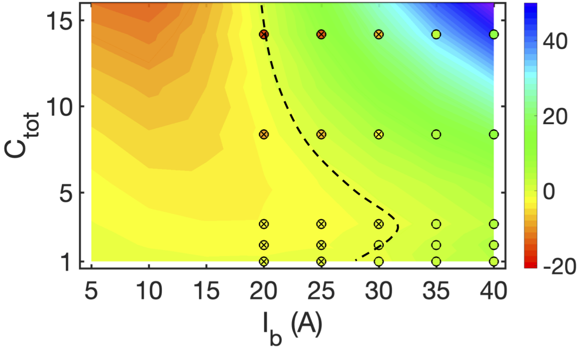


Figure: ◯ and ⊗ are `elegant` tracking. Background are results from VFP calculation. Dashed line refers to the case $\sigma_{\Delta E,\text{tot}}^{\text{wo/IBS}} = \sigma_{\Delta E,\text{tot}}^{\text{w/IBS}}$.

Using multi-stage coefficient[11], a semi-analytical expression of the threshold current can be found.

[11] C.-Y. Tsai, NIMA **943**, 162499 (2019).

Example 2: LCLS from LH to undulator entrance

A longer, more practical beamline will take much more computing time/resources for microbunching analysis using particle tracking simulation. It becomes effective using semi-analytical VFP solver.

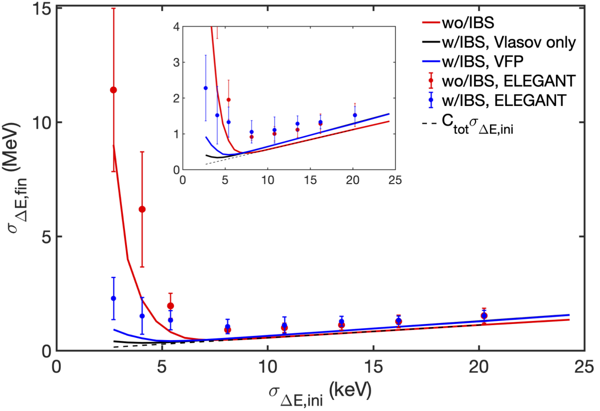
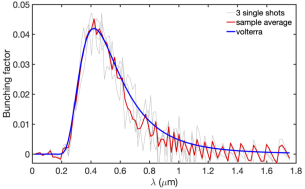
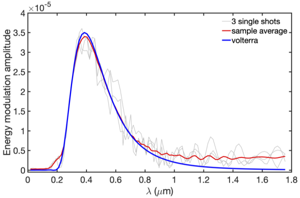
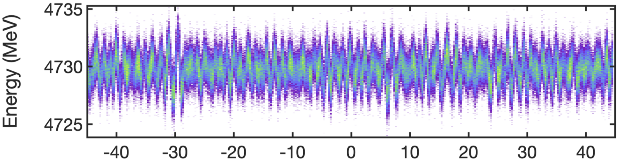


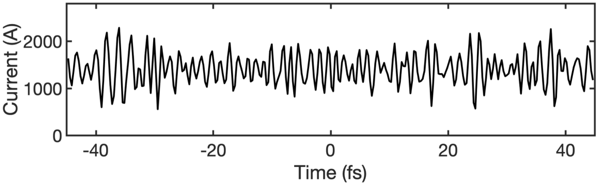
Figure: IBS plays an important role for small initial SES. Vlasov analysis valid for $\sigma_{\Delta E,\text{ini}} > 6$ keV. In the simulation only LSC is included.

Use gain curves $\tilde{G}(\lambda)$ (and/or energy modulation spectrum) to generate modulated beam phase space for downstream FEL analysis (W. Qin, work in progress)







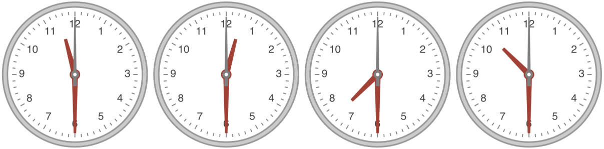


Summary and Discussion

- We have extended the existing microbunching analysis to include the incoherent IBS effects into the analysis
  - 0th order: IBS growth rate in $\sigma_\delta, \epsilon_{x,y} \Rightarrow$ enhance Landau damping
  - 1st order: diffusion D_z and friction D_{zz} effects
  - In case $\sigma_\delta, \epsilon_{x,y}$ are very small, D_z and D_{zz} should be taken into account.
- Our analysis is generally applicable for linear optics transport, with IBS, with bunch compression or decompression, with beam acceleration or deceleration, and with inclusion of collective effects (LSC, CSR, etc).
- We have applied the analysis to a simple FODO-BC transport line to illustrate the impact of IBS to microbunching dynamics and find out the threshold condition when IBS helps mitigate microbunching.
- Work in progress: further extension of the analysis to systematic evaluation of FEL performance.

Acknowledgements

Thank you for your attention



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