Two-dimensional Beam-Beam invariant with applications to HL-LHC

2D Nonlinear invariant for beam-beam

We present a formula for the geometric distortion of the **two-dimensional** nonlinear (Courant Snyder) invariant valid to lowest order of the beam-beam parameter λ. The n-th collision ($n = 1, .., N_{\mathtt{lr}}$), is described by the coefficients $C_{mk}(a_x, a_y; \theta_{\rm str}^{(n)})$ in the Fourier-expansion of the long-range beam-beam Hamiltonian, written in terms of action-angle coordinates $\vec{J}, \vec{\phi}$ of the unperturbed motion: $H(\vec{a}, \vec{\phi}; \theta_{\rm str}^{(n)})$. Here m, k are integers, $N_{\mathtt{lr}}$ is the total number of `lr` around the ring, $\vec{a} = (a_x, a_y)$, where $a_{x,y} \equiv \sqrt{2J_{x,y}/\epsilon}$, are the test particle normalized amplitudes, ϵ is the emittance and $\theta_{\rm str}^{(n)}$ are the strong-beam lattice parameters at this (longitudinal) location.

Effective Hamiltonian and invariants Wx, Wy

$$h(\vec{J}, \vec{\phi}, \vec{\mu}) = -\mu_x J_x - \mu_y J_y + S(\vec{J}, \vec{\phi}, \vec{\mu}); \tag{4}$$

$$S(\vec{J}, \vec{\phi}, \vec{\mu}) \equiv \lambda \sum_{n=1}^{N_{\mathtt{lr}}} \sum_{mk=-N_{\rm c}}^{N_{\rm c}} C_{mk}(a_x, a_y; \theta_{\rm str}^{(n)}) \times \frac{(m\mu_x + k\mu_y)}{2\sin\frac{1}{2}(m\mu_x + k\mu_y)} e^{i\,m(\frac{\mu_x}{2}+\phi_x+\mu_x^{(n)})+i\,k(\frac{\mu_y}{2}+\phi_y+\mu_y^{(n)})}.$$

$$W_x(J_x, \phi_x) \equiv J_x - S_x(J_x, J_y^0, \phi_x, \frac{\pi}{2}),$$

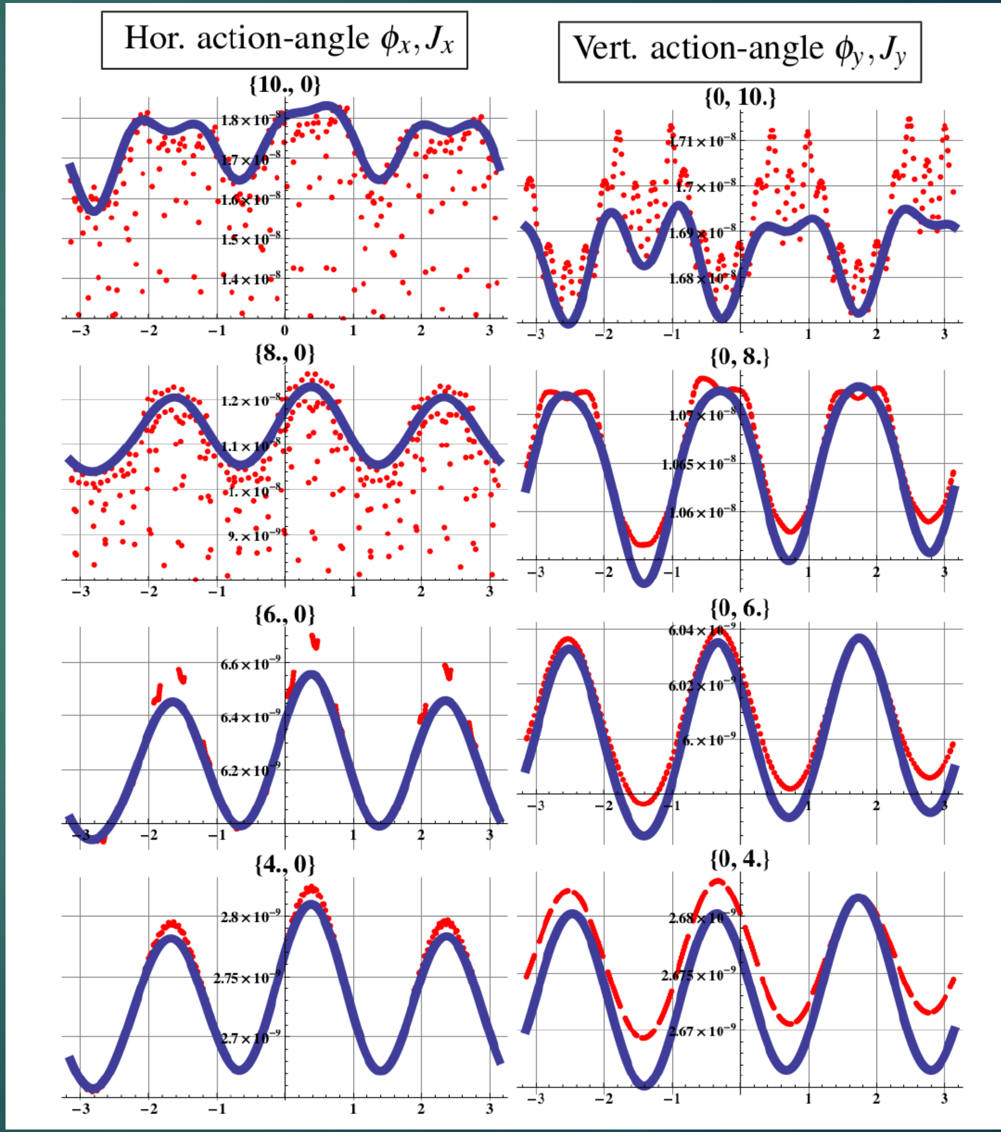
$$W_y(J_y, \phi_y) \equiv J_y - S_y(J_x^0, J_y, \frac{\pi}{2}, \phi_y).$$

Hamiltonian Fourier Coefficients

In previous papers [7], [8], expressions for C_{mk} were presented valid at large amplitudes and large ~ 12 normalized separations, as required by the nominal beam-beam layout and a round collision optics in the HL-LHC

Results

Figure 2: In-plane tracking around the ring ($N_{\mathtt{lr}} = 4 \times 18$) of the weak-beam particle using MadX (red dots) and the projected invariants computed with 6 coefficients $N_{\mathtt{c}} = 6$, Eqn. (5) (blue). The particle is launched in either X (left) or Y (right) planes with $a_{x(y)} = 4$, 6, 8 and 10. It penetrates the strong beam core at $\sim 8\ \sigma$.



HiLumi HL-LHC PROJECT

TRIUMF

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Hamiltonian driving-terms (HDT). Verify wire correctors with HDT

Simultaneous (for all m) cancellation of Hamiltonian driving terms in IR5 in case of the in-plane left-right independent wire correction: