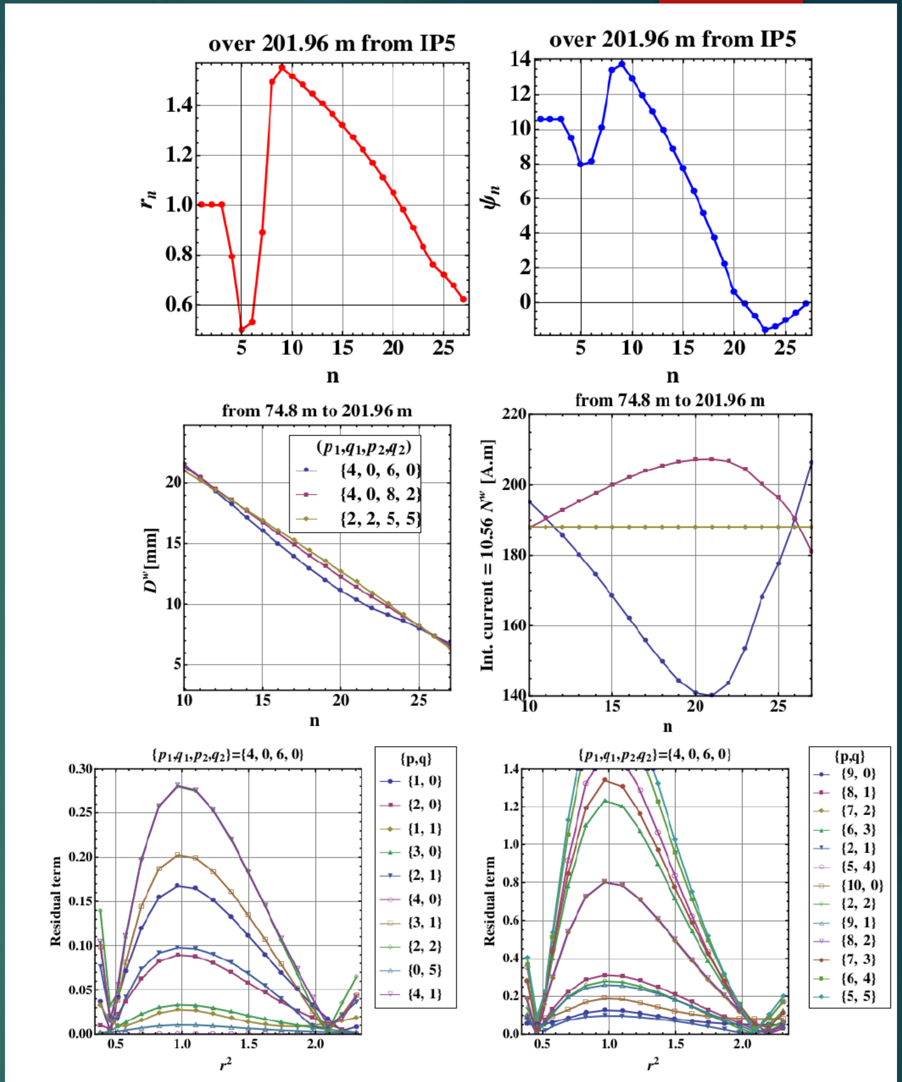
On wire-corrector optimization in the HL-LHC and the appearance of special aspect ratios

WIRE CORRECTION USING RDT

Two-dimensional amplitude-independent Resonance Driving Terms (RDT), based on the coefficients c_{pq} in the multipole expansion of the beam-beam kick delivered to the weak-beam particle have been used in [1] to describe the effects of long-range beam-beam interactions (l.r.bb, or simply bb) in the HL-LHC, and also optimise the wire correctors. An analytic formula is presented to compute the optimum parameters of the wires. The formula follows from imposing the condition for simultaneous cancellation of a *target* pair RDT (indices p_1, q_1, p_2, q_2, where $p_i \geq 0$) and produces, for two wires located left-right symmetrically w.r.t. the IP, two equal optimum parameters: integrated current and distance to the axis. These guarantee that the target pair RDT, and also the symmetric one (q_1, p_1, q_2, p_2), vanish over a single turn. Further, by varying the longitudinal location of the wire, it is found that for some special such locations many, in fact nearly all, other (*residual*) driving terms are eliminated, besides the target pair. These locations correspond to two special values of the *beta*-function aspect ratio $\frac{\beta_{x,y}}{\beta_{y,x}} = 1/2$ and 2. It has been long inferred [2] that particular values of the aspect ratio at the bb collisions, positioned the closest to the opposite beam, specific to the optics, are responsible for the occurrence of special locations. This paper, see also [3], proposes a mathematical formalism (p-norm) to explain the existence of special ratios. The results confirm the above conjecture.



[1] S. Fartoukh, A. Valishev, Y. Papaphilippou, D. Shatilov, *Compensation of the long-range beam-beam interactions as a path towards new configurations for the high luminosity LHC*, Phys. Rev. ST Accel. Beams 18, 121001 (2015)

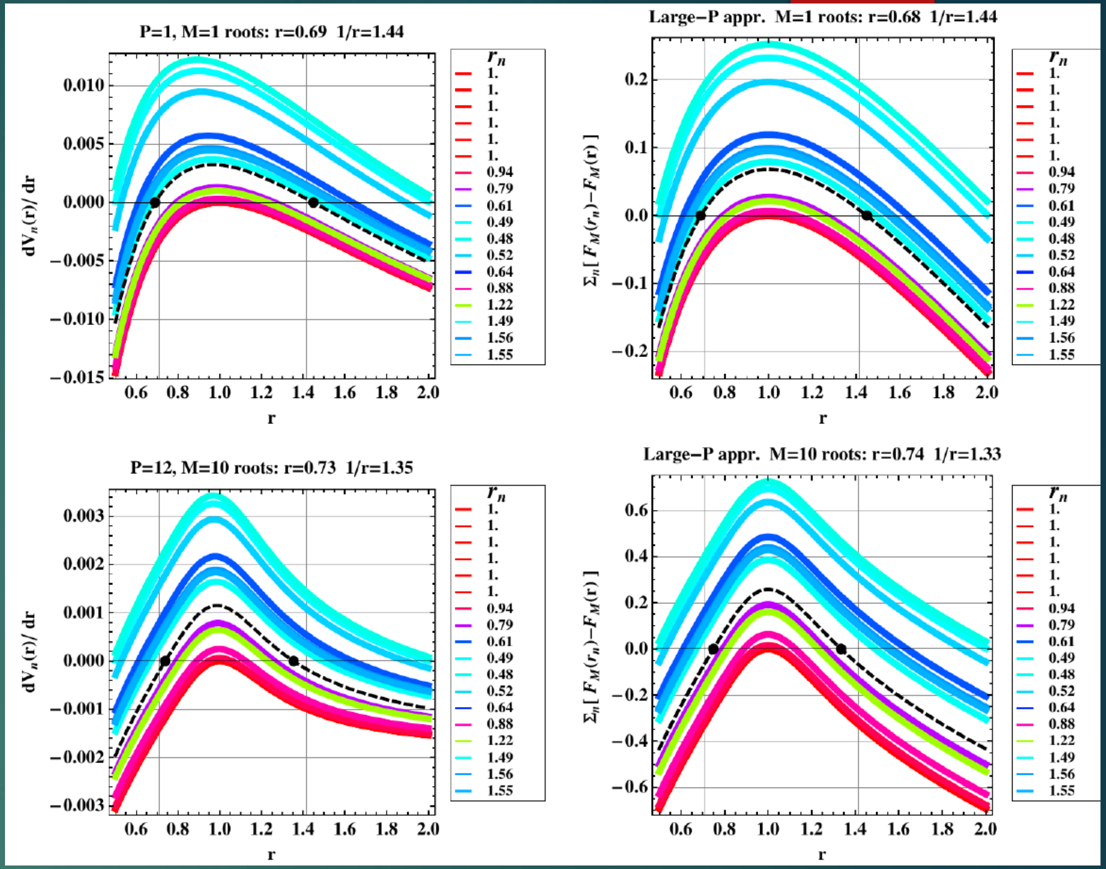
EQUATION FOR THE SPECIAL RATIOS

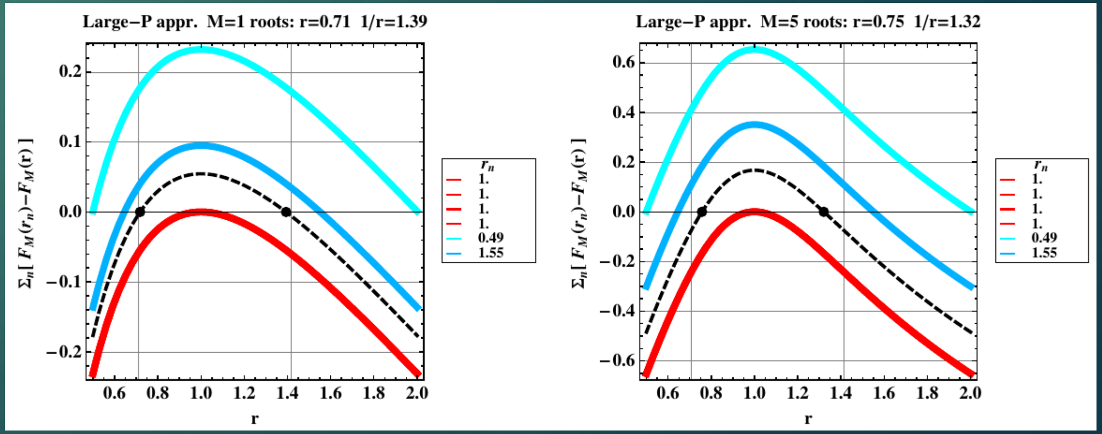
Special aspect ratios r, for which the residual term may also (besides the target pair) be canceled, satisfy $R_{pq}(r) = 0$, hence from $\boxed{7}$ and $\boxed{8}$, these are roots of the equation

$$A_1(r)^{-\frac{P_2-P}{P_1-P_2}}\, A_2(r)^{\frac{P_1-P}{P_1-P_2}} = A(r). \qquad (9)$$

via P-norm of vector V

$$\sum_{n\in \mathrm{bb}} \frac{dV_n}{dM} = 0.$$





P=p+q M=p-q

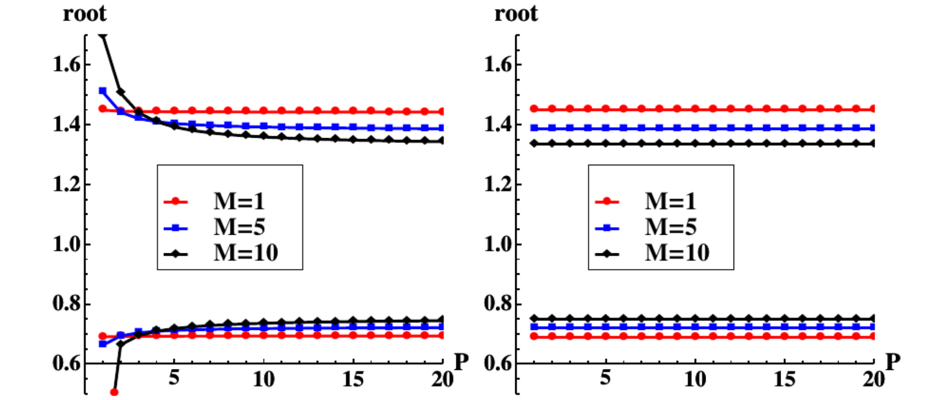


Figure 4: Solutions of the exact Eqn $\boxed{13}$ (left) and the approximate Eqn $\boxed{15}$ (right). Only the range $P \geq M$ is meaningful.

Summary: The findings in [1]: the two locations at which multiple driving terms are cancelled correspond to the magic values $\sim \sqrt{2}$ (and inverted) of the sigma aspect parameter r are confirmed and explained with properties of the p-norm. It is further found that: **1)** depending on how far r is from the magic value, low- and high-order terms are canceled to a slightly different degree because of the spread occurring for large $p - q$ (weaker coupling), Fig 4. Namely, higher order terms are better cancelled a little below $\sqrt{2}$ and a little above $1/\sqrt{2}$. The spread is larger for the location with $r \sim \sqrt{2}$ (beta aspect ratio ~ 2), as also observed in [1]. The other location, near Q5, is therefore preferable. **3)** For the optics considered, the magic values can be explained with the min/max values of r in the beam-beam region being $\approx \sqrt{2}$ and $\approx 3\sqrt{2}$.

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[1] S. Fartoukh, A. Valishev, Y. Papaphilippou, D. Shatilov, *Compensation of the long-range beam-beam interactions as a path towards new configurations for the high luminosity LHC*, Phys. Rev. ST Accel. Beams 18, 121001 (2015)