Three-Dimensional Space Charge Oscillations in a Hybrid Photoinjector

M. Carillo\textsuperscript{1,4}, M. Behtouei\textsuperscript{2}, F. Bosco\textsuperscript{1,4}, L. Faillace\textsuperscript{1,2}, A. Gribonno\textsuperscript{2}, L. Giuliani\textsuperscript{1,4}, A. Mostacci\textsuperscript{1,4}, M. Migliorati\textsuperscript{1,4}, B. Spataro\textsuperscript{2}, J. Rosenzweig\textsuperscript{3}, C. Vaccarezza\textsuperscript{2}, L. Palumbo\textsuperscript{1,4}

\textsuperscript{1}La Sapienza University of Rome, 00161 Rome, Italy
\textsuperscript{2}INFN-LNF, 00044 Frascati, Italy
\textsuperscript{3}UCLA, Los Angeles, 90095 California, USA
\textsuperscript{4}INFN-Sez.Roma1, 00161 Roma, Italy

**This work is supported by DARPA GRIT under contract no. 20204571 and partially by INFN National committee V through the ARYA project

carillo.159653@studenti.uniroma1.it
1. Introduction: Hybrid Photoinjector

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Charge</td>
<td>250 pC</td>
</tr>
<tr>
<td>Sigma x (cut@1-sigma)</td>
<td>500 um</td>
</tr>
<tr>
<td>Laser Pulse Length</td>
<td>0.5 ps</td>
</tr>
<tr>
<td>E-field at cathode</td>
<td>120 MV/m</td>
</tr>
<tr>
<td>Peak Bz</td>
<td>0.27 T</td>
</tr>
<tr>
<td># particles</td>
<td>50,000</td>
</tr>
</tbody>
</table>

L. Faillace et al., Beam dynamics for a high field C-band hybrid photoinjector, this conference.
2. Slice Analysis:

- Bunch splitting in n=10 slice from cathode
- Slice dynamics observation
- Bunch projection
- Transverse phase space
- Longitudinal phase space

Elliptical shape

Bunch splitting in n=10 slice from cathode

Slice dynamics observation

Bunch projection

Transverse phase space

Longitudinal phase space

Elliptical shape
3. Triple waist: Envelope equation

Drift region: only space charge term is dominant

\[ \sigma''_x = \frac{k}{\sigma_x} \]

where: \( \kappa = \frac{l}{2l_A} \) is the Perveance term
and \( l \) is the bunch current, that is function of \( \sigma_z \):
\[ I \propto \frac{1}{\sigma_z} \]

Triple waist approximation:
\[ \sigma_z \sim \sigma_x \]

\[ \sigma''_x = \frac{k}{\sigma_x^2} \]
3. Triple waist: Envelope solution

- **Exact solution:**

\[
z = \frac{\sigma_0^{3/2}}{\sqrt{2k}} \left[ \ln \left( \sqrt{\frac{\sigma}{\sigma_0}} + \sqrt{\frac{\sigma}{\sigma_0} - 1} \right) + \sqrt{\frac{\sigma}{\sigma_0} - 1} - 1 \right] + z_0
\]

- **Perturbative solution:**

\[
\sigma(z) = \sigma_0 \left[ \sqrt{\frac{k}{2\sigma_0}} \frac{z}{\sigma_0} \tan^{-1} \left( \sqrt{\frac{k}{2\sigma_0}} \frac{z}{\sigma_0} \right) \right]
\]
4. Conclusion:

- Beam dynamics have been studied.
- Through the slice analysis, the properties and physical effects of the beam have been observed, in particular the evolution of the beam shape has allowed us to use the right distribution in analytic studies.
- The beam envelope equation in drift was analysed and compared with simulations, an excellent agreement was found.

**Future goals:**

- Solving the longitudinal equation in drift and so obtaining the exact transverse envelope equation.
- Studying emittance compensation.

Thank you for your attention!