Shielding of CSR wake in a drift

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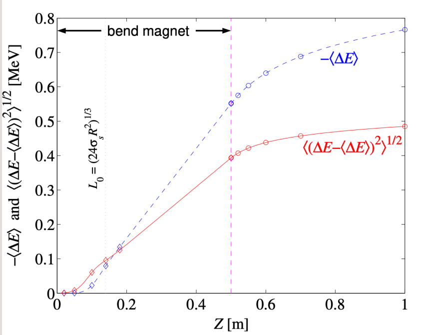
IPAC

Introduction-1

Coherent synchrotron radiation (CSR) wakefield was in a 1D model is used in several computer codes for simulation of relativistic electron beams. It includes transient effects at the entrance and exit from a bending magnet of finite length. In the ultra-relativistic limit, $v = c$, the exit CSR wake decays inversely proportional to the distance from the magnet end. To calculate the total energy loss of the beam one needs to integrate this wake to infinity, but the integral diverges. The physics behind this divergence is the edge radiation at the exit from the magnet that in the limit $\gamma = \infty$ and in the absence of metal walls carries an infinite energy at small angles. Naturally, the integral of the CSR wake that is responsible for the energy balance in this process, takes an infinite value. This means that one has to either drop the assumption $\gamma = \infty$ or take into account the shielding effect of the metal walls in the system in order to get a finite answer.

Introduction-1

From Ref.[1]: average energy loss and energy spread due to CSR radiation in 0.5 m long bend.



What happens with $\langle \Delta E \rangle$ and $\langle (\Delta E - \langle \Delta E \rangle)^2 \rangle^{1/2}$ when the distance after the bend goes to infinity? The answer: they go to infinity.

[1] G. Stupakov and P. Emma, "CSR wake for a short magnet in ultrarelativistic limit," in Proceedings of 8th European Particle Accelerator Conference, Paris, France, 2002, p. 1479.

Introduction-2

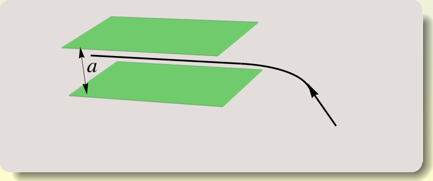
We compare the formation length, ℓ_f, of the edge radiation in free space for a bunch with a finite value of γ (σ_z if the rms bunch length),

$$\ell_f \sim \gamma^2 \sigma_z, \tag{1}$$

with the formation length in a vacuum chamber with transverse dimensions a,

$$\ell_f \sim a^2/\sigma_z. \tag{2}$$

Here, we assume that the length given by Eq. (2) is much shorter than that in Eq. (1); in this case the effect of the shielding dominates and we can keep the assumption $\gamma = \infty$ in our calculations. We will also adopt a model where the vacuum chamber is treated as two parallel metal plates separated by distance a, with the beam orbit situated in the middle plane between the plates.



Beam trajectory passing through a bending magnet. In the straight part of the orbit after the exit there are two parallel metal plates (shown by green color).

CSR wake in drift without shielding

If we neglect the shielding (which is formally valid in the limit $a \to \infty$), the wake (per unit path length) generated by a bunch in a drift after the exit from the magnet is given by

$$W(z,s) = \frac{4}{\rho} \int_{-\infty}^{z} \frac{1}{\psi(z',z,s) + 2s/\rho} \frac{d\lambda(z')}{dz'} dz'. \tag{3}$$

In this formula, s is the position of the bunch in the drift measured from the exit of the magnet, $\lambda(z)$ is the one-dimensional bunch distribution normalized by unity, and the function $\psi(z',z,s)$ is defined by the equation

$$z - z' = \frac{\rho\psi^3}{24} \frac{\psi + 4s/\rho}{\psi + s/\rho}. \tag{4}$$

In the limit $s \to \infty$, the integral in (3) can be simplified,

$$W(z,s) \approx \frac{2}{s}\lambda(z). \tag{5}$$

This wake decays as $1/s$, and if integrated to $s = \infty$ gives an infinite energy loss of the beam.

CSR impedance in drift with shielding

The CSR wake for a bend of finite length with the shielding represented by two parallel metal plates was studied in[2]. A general expression was derived in the ultra-relativistic limit, $v = c$, for the longitudinal impedance, $Z(k)$, related to the wake of a point charge, $w(z)$, by the equation

$$Z(k) = \frac{1}{c}\int_{-\infty}^{\infty} w(z) e^{-ikz} dz, \tag{6}$$

The CSR wake in the drift after a bend magnet was calculated in Appendix B.4 of that paper and (here we use the notation Z for that impedance). A small bending angle was assumed, which means that $L \ll \rho$, where L is the bend length. It was also assumed that $ka \gg 1$ and $k\rho \gg 1$ (which means that the bunch length σ_z is much smaller than the gap between the plates, as well as the bending radius).

[2] G. Stupakov and D. Zhou, "Analytical theory of coherent synchrotron radiation wakefield of short bunches shielded by conducting parallel plates," Phys. Rev. Accel. Beams, vol. 19, p. 044402, 2016.

CSR impedance in drift with shielding

The expression for Z is given by

$$Z(k, L_d) \approx \frac{Z_0}{4\pi}(i-1)\frac{\sqrt{\pi k}}{a\rho^2}\exp\left(\frac{1}{3}ik\theta_0^2 L\right)\sum_{p=0}^{\infty}\int_{-\infty}^{L}(L-s')^2 ds' \times \tag{7}$$

$$\int_{L}^{L+L_d}\frac{ds}{\sqrt{\zeta}}\exp\left[i\left(\frac{ks'^3}{6\rho^2}-\frac{1}{2}k\theta_0^2 s+\frac{k\theta_0^2}{2\zeta}\left(s-\frac{1}{2}L-\frac{s'^2}{2L}\right)^2-\zeta\frac{(2p+1)^2\pi^2}{2ka^2}\right)\right]$$

where $Z_0 = 377$ Ohm, L_d is the length of the drift, $\theta_0 = L/\rho$ and $\zeta = s - s'$, with the entrance to the bend corresponding to $s' = s = 0$[3]. The integration over s in the second integral extends from the exit from the bend ($s = L$) to the end of the drift ($s = L + L_d$). The lower zero limit in the integral over s' corresponds to the entrance to the bend; if the length of the bend much longer then the formation length, $L \gg (24\rho^2\sigma_z)^{1/3}$, which we assume here, this limit can be replaced by $-\infty$. This means that the transient effects at the entrance to the bend do not interfere with the wake after the exit (as was also assumed for Eq. (3)).

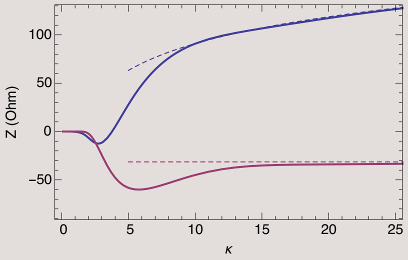
[3] Note that here the coordinate s is measured from the entrance to the magnet, while in Eq. (3) it is measured from the exit.

Infinitely long drift $L_d = \infty$

We numerically computed the impedance (7) for an infinitely long drift, $L_d = \infty$.

Plot of the real (blue) and imaginary (magenta) parts of the impedance Z (in Ohms) as a function of the dimensionless wavenumber $\varkappa = ka^{3/2}/\rho^{1/2}$. The dashed lines show the high-frequency approximation:

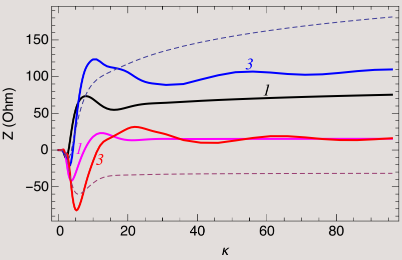
$$Z = \frac{Z_0}{4\pi}\left[1.33\ln\left(0.97\varkappa\right) - 1.05i\right]$$



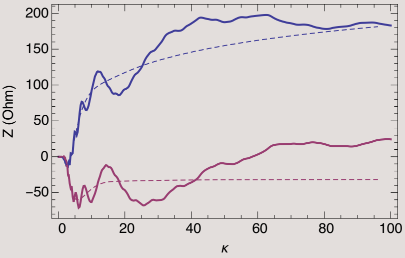
Note that the real part of Z is negative in the region $\varkappa < 4$ which seems to contradict to the requirement that the real part of any impedance be always positive. One has to remember, however, that here we only calculate the contribution to the impedance from a part of the beam trajectory—the impedance for the full trajectory that also includes the circular part of the orbit inside the bend will have a positive real part for all frequencies.

Finite length drift $L_d < \infty$

Plots of the impedance ($\mathrm{Re}\, Z$ - blue, $\mathrm{Im}\, Z$ - red) for a finite length L_d of the drift for two values of the parameter $\ell = L_d/\sqrt{a\rho}$ (the values of this parameter are indicated by the numbers near the curves). The dashed lines show the real and imaginary parts of Z corresponding to $L_d = \infty$.



Finite length drift $L_d < \infty$



Plot of the real (blue) and imaginary (magenta) parts of the impedance Z for $\ell = 20$. The dashed lines show the real and imaginary parts of Z corresponding to $L_d = \infty$.

Note that a non-smooth character of the solid lines in this figure is not an artifact of calculation errors—it reflects the fact that the impedance curves approach the limit $\ell = \infty$ only on average, executing rapid oscillations around the limiting values of the impedance. A similar behavior was observed in the past for the impedance of collimators and tapers in a perfectly conducting pipe[4].

[4] G. Stupakov and B. Podobedov, "High-frequency impedance of small-angle tapers and collimators," PRAB, **13**, p. 104401, 2010.

Numerical example

Using the impedance $Z(k)$ one can calculate the average energy loss per particle of the beam due to the CSR impedance in the drift:

$$\langle \Delta \mathcal{E} \rangle = \frac{Qe}{\pi} c \int_0^\infty dk \, \mathrm{Re}\, Z(k) |\hat{\lambda}(k)|^2, \tag{8}$$

where $\hat{\lambda}(k)$ is the Fourier transform of the distribution function $\hat{\lambda}(k) = \int_{-\infty}^{\infty} dz \lambda(z) e^{ikz}$. For a Gaussian function with the rms bunch length σ_z we have $\hat{\lambda}(k) = e^{-k^2 \sigma_z^2 / 2}$.

For a numerical example we take the same parameters of the beam and the bending magnet as in[5]: $Q = 1$ nC, $\rho = 1.5$ m, $\sigma_z = 50$ μm, and assume the gap between the plates $a = 2$ cm. For an infinitely long drift we find $\langle \Delta \mathcal{E} \rangle = 0.18$ MeV. Calculating the wake of the bunch, we can also find the rms energy spread induced by the wake in the beam, which turns out to be 0.10 MeV.

[5] G. Stupakov and P. Emma, "CSR wake for a short magnet in ultrarelativistic limit," in Proceedings of 8th European Particle Accelerator Conference, Paris, France, 2002, p. 1479.