IPAC21

DIFFRACTION AT THE OPEN-ENDED DIELECTRIC-LOADED CIRCULAR WAVEGUIDE

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Motivation

DWFA

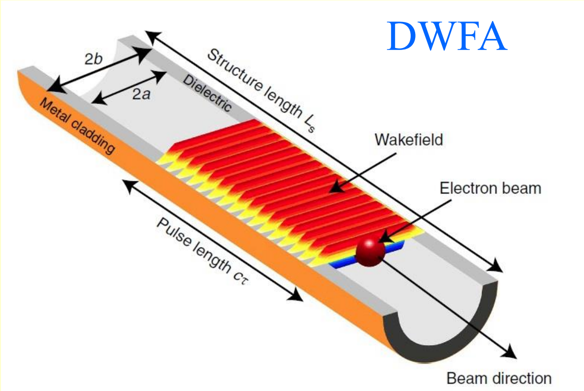
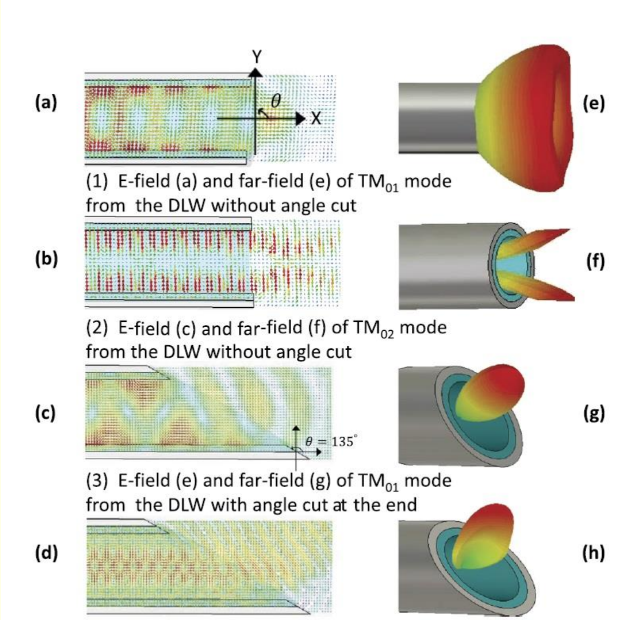


Figure 1 | Graphical representation of dielectric wakefield accelerator.

THz generation

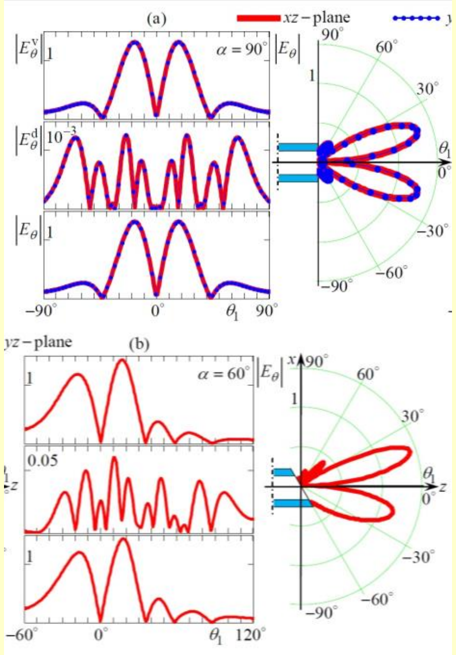


(1) E-field (a) and far-field (e) of TM_{01} mode from the DLW without angle cut

(2) E-field (c) and far-field (f) of TM_{02} mode from the DLW without angle cut

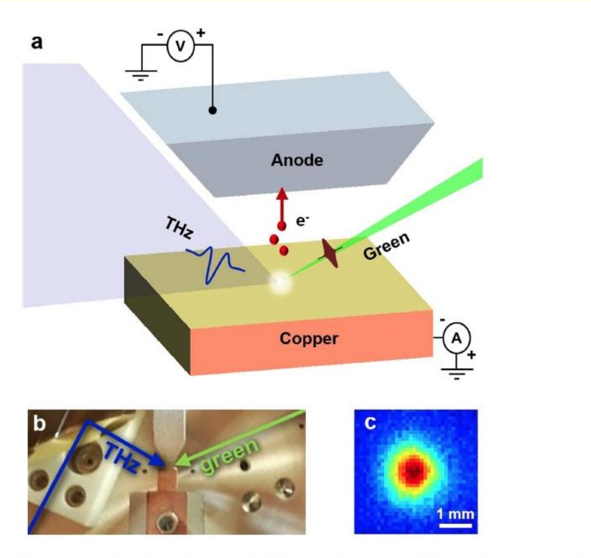
(3) E-field (e) and far-field (g) of TM_{01} mode from the DLW with angle cut at the end

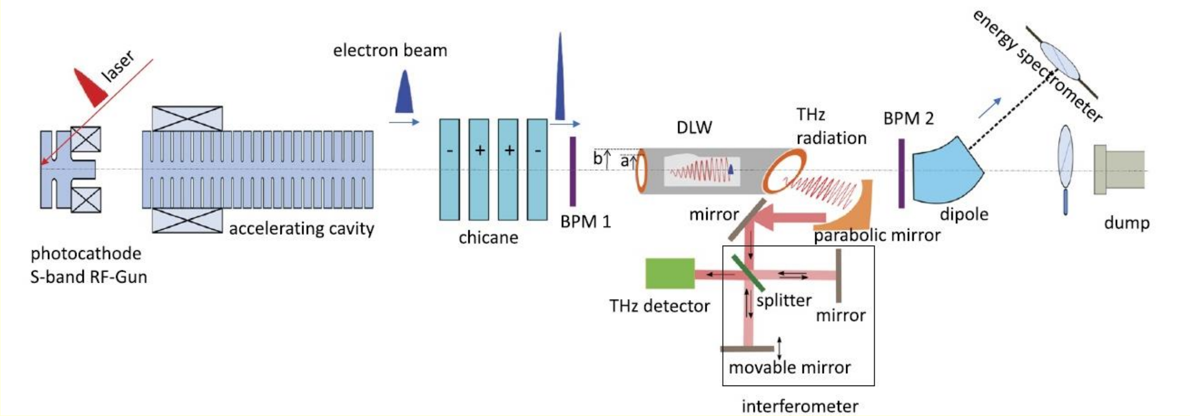
(4) E-field (g) and far-field (h) of TM_{02} mode from the DLW with angle cut at the end



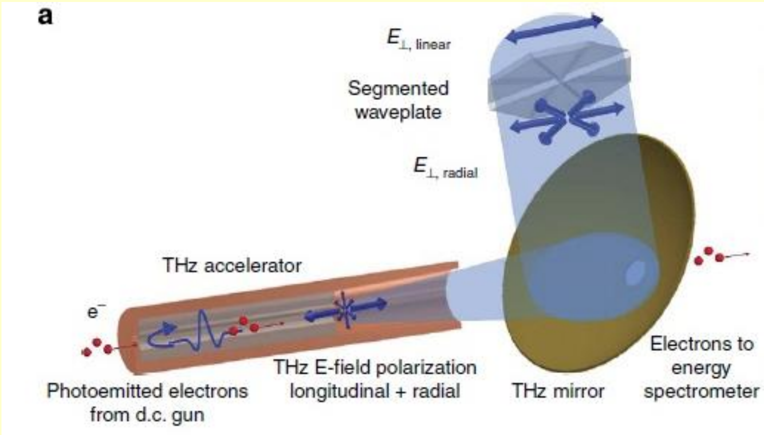
S.N. Galyamin *et al.*, Opt. Express, 22(8)8902 (2014).

THz-driven electron gun

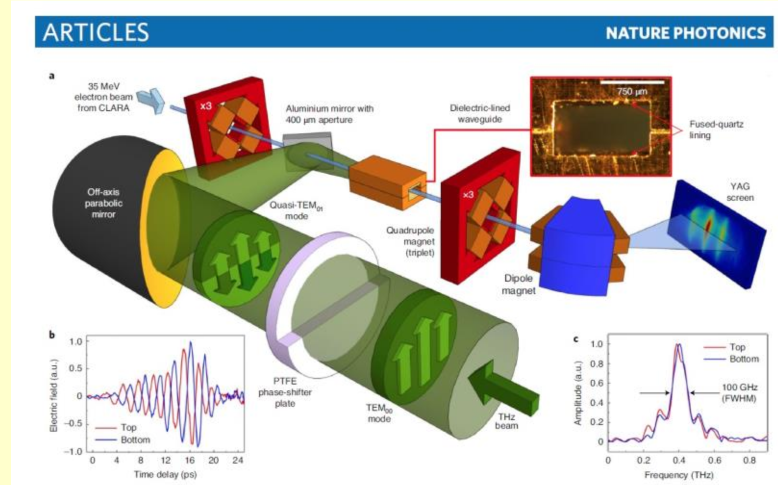




Motivation



THz-driven linear electron accelerator



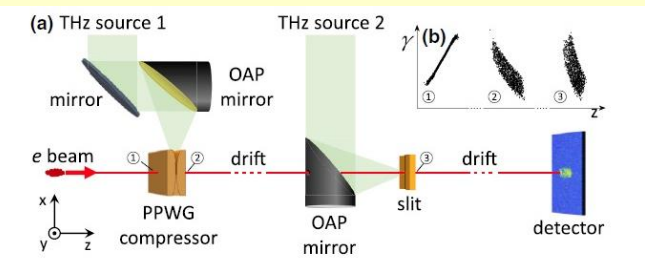
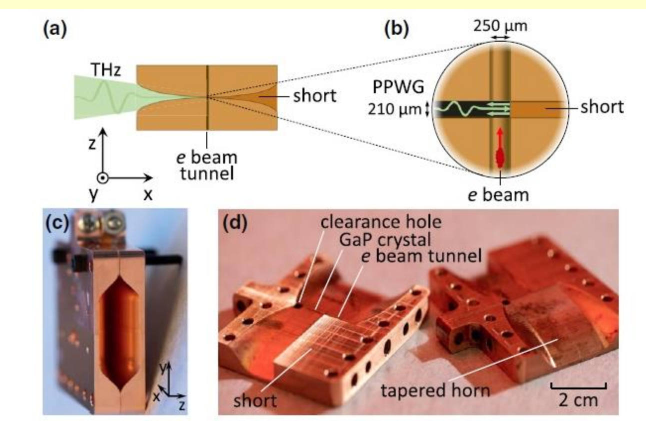
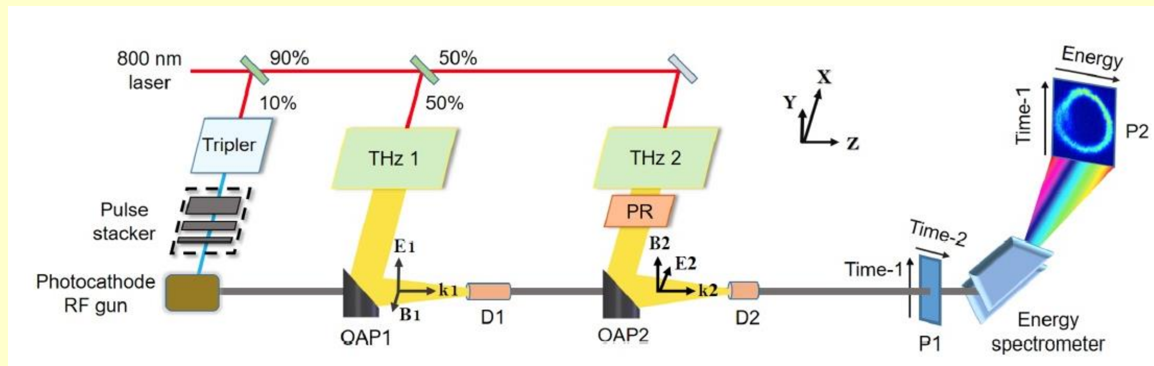


FIG. 1. (a) Schematic of the in-vacuum components for the THz driven compression and streaking setup. Within the PPWG



THz driven bunch compression & streaking



[1] B.D. O'Shea et al., Nature Commun., vol. 7, no. 1, p. 12763, Sep. 2016.
[2] W. R. Huang *et al.*, *Sci. Rep.*, vol. 5, no. 1, p. 14899, Dec. 2015.
[3] L. Zhao *et al.*, *Phys. Rev. Lett.*, vol. 124, no. 5, Feb. 2020, 054802.
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[5] E.A. Nanni *et al.*, *Nature Commun.*, vol. 6, no. 1, p. 8486, Dec. 2015.
[6] M.T. Hibberd *et al.*, *Nature Photon.*, vol. 14, no. 12, pp. 755–759, Dec. 2020.
[7] D. Wang *et al.*, *Rev. Sci. Instrum.*, vol. 89, no. 9, Sep. 2018, 093301.

Rigorous approach

Planar waveguide

РАДИОТЕХНИКА И ЭЛЕКТРОНИКА

1976 № 12

КРАТКИЕ СООБЩЕНИЯ

УДК 621.372.81.09

ИЗЛУЧЕНИЕ ИЗ ОТКРЫТОГО КОНЦА ПЛОСКОГО ВОЛНОВОДА С ДИЭЛЕКТРИЧЕСКИМ ЗАПОЛНЕНИЕМ

Г. В. Воскресенский, С. М. Журав

Хотя волноводные излучающие системы, заполненные диэлектриком, используются на практике, теоретически они исследованы недостаточно. Основной интерес представляет выяснение влияния диэлектрического заполнения на частотные зависимости коэффициента отражения основной волны и на диаграмму излучения. В настоящей работе рассматривается задача об излучении из открытого конца плоского волновода и плоского волновода с фланцем, заполненных однородным диэлектриком. Способ решения, основанный на сшивании полей в плоскости раскрыва волновода и сведении функциональных уравнений типа Винера – Хопфа к системе линейных уравнений, аналогичен использованному в[1] при рассмотрении задачи об излучении из пустого волновода с фланцем.

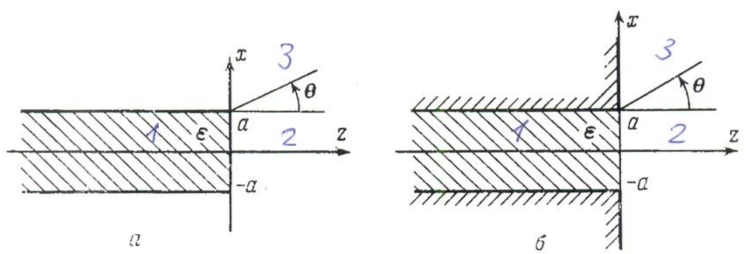


Рис. 1

РАДИОТЕХНИКА И ЭЛЕКТРОНИКА

1978 № 4

УДК 621.372.826:621.315.61

ИЗЛУЧЕНИЕ ИЗ ОТКРЫТОГО КОНЦА ПЛОСКОГО ВОЛНОВОДА, ЧАСТИЧНО ЗАПОЛНЕННОГО ДИЭЛЕКТРИКОМ

Г. В. Воскресенский, С. М. Журав

Рассматривается задача об излучении электрических волн из полубесконечного плоского волновода с исчезающе тонкими стенками, нагруженного полубесконечной диэлектрической пластиной, высота которой меньше высоты волновода. Решение основано на методе Винера – Хопфа. Приведены результаты вычислений.

Для изменения характеристик излучения и отражающих свойств открытого конца волновода [1] можно нагружать волновод диэлектриком. В работе [2] получено решение задачи о полубесконечном волноводе, полностью заполненном диэлектриком. При этом показано, что заполнение диэлектриком не влияет на характеристики излучения, а изменяет лишь коэффициенты возбуждения собственных волн. Ниже будет рассмотрен случай, когда высота нагружающей диэлектрической пластины меньше поперечного размера волновода.

Пусть в полубесконечный плоский волновод с идеально проводящими тонкими стенками ($x=\pm b$, $z<0$) симметрично относительно средней плоскости $x=0$ вставлена полубесконечная пластина ($|x|\leqslant a$, $z<0$) из диэлектрика с проницаемостью ε (рис. 1). Структура возбуждается собственной электрической волной E_{0l}, набега-

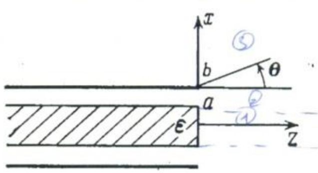
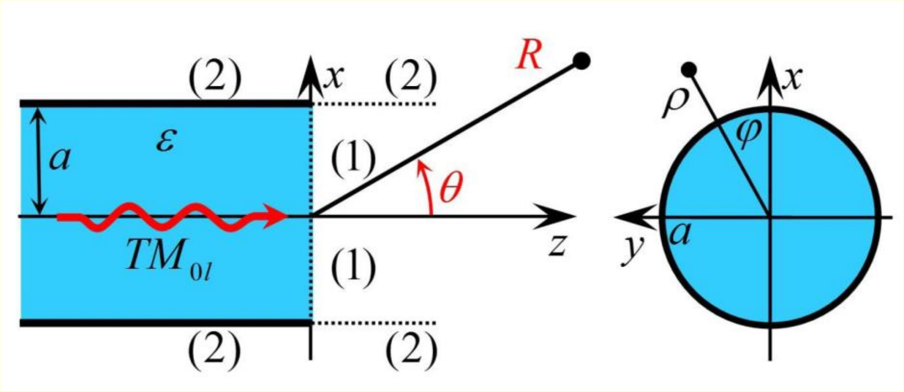


Рис. 1

Purpose: generalization for cylindrical waveguides

Cylindrical waveguide with uniform filling



Incident field: symmetric TM_{0l} mode

$$\left\{E_\rho,\, E_z,\, H_\varphi\right\} = \int_{-\infty}^{+\infty} d\omega \left\{E_{\omega\rho},\, E_{\omega z},\, H_{\omega\varphi}\right\} \exp(-i\omega t)$$

$$H_{\omega\varphi}^{(i)} = M^{(i)} J_1\left(\rho \frac{j_{0l}}{a}\right) e^{ik_{zl}z}, \qquad k_{zl} = \sqrt{k_0^2\varepsilon - \left(\frac{j_{0l}}{a}\right)^2}, \qquad k_0 = \omega / c + i\delta, \qquad J_0(j_{0l}) = 0$$

$$E_{\omega\rho} = \frac{1}{ik_0\varepsilon}\frac{\partial H_{\omega\varphi}}{\partial z}, \qquad E_{\omega z} = \frac{i}{k_0\varepsilon}\left(\frac{H_{\omega\varphi}}{\rho} + \frac{\partial H_{\omega\varphi}}{\partial \rho}\right)$$

Reflected field

$$H_{\omega\varphi}^{(r)} = \sum_{m=1}^{\infty} M_m J_1\left(\rho \frac{j_{0m}}{a}\right) e^{-ik_{zm}z} \qquad \left\{M_m\right\} = ?$$

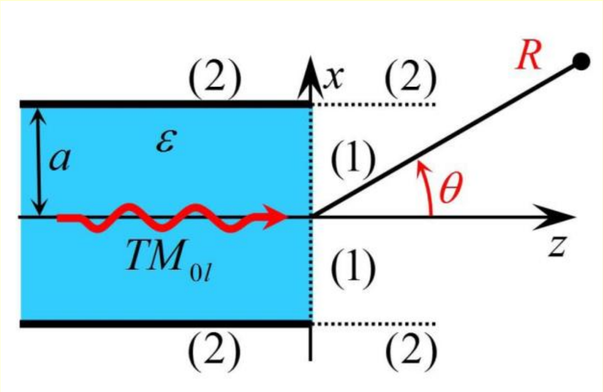
Cylindrical waveguide with uniform filling

$$\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial}{\partial \rho}-\frac{1}{\rho^2}+k_0^2\right)H_{\omega\varphi}^{(1,2)}=0$$

Integral representation for Hφ:

$$\Psi_{+}^{(1,2)}(\rho,\alpha)=\frac{1}{2\pi}\int_0^{\infty} dz H_{\omega\varphi}^{(1,2)}(\rho,z)e^{i\alpha z}$$

$$\Psi_{-}^{(2)}(\rho,\alpha)=\frac{1}{2\pi}\int_{-\infty}^{0} dz H_{\omega\varphi}^{(2)}(\rho,z)e^{i\alpha z}$$



$$\left(\frac{\partial^2}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial}{\partial \rho}-\frac{1}{\rho^2}+\kappa^2\right)\begin{Bmatrix}\Psi_{+}^{(1)}\\ \Psi_{-}^{(2)}+\Psi_{+}^{(2)}\end{Bmatrix}=\begin{Bmatrix}F^{(1)}\\ 0\end{Bmatrix},\qquad \kappa=\sqrt{k_0^2-\alpha^2}$$

$$F^{(1)}=(2\pi)^{-1}\,\partial H_{\omega\varphi}^{(1)}\big/\partial z\Big|_{z=+0}-(2\pi)^{-1}\,i\alpha H_{\omega\varphi}^{(1)}\Big|_{z=+0}$$

Continuity at z = 0: $\partial H_{\omega\varphi}/\partial z\sim E_{\omega\rho},\ \ H_{\omega\varphi}\quad\Rightarrow\quad F^{(1)}$

$$\Psi_{+}^{(1)}(\rho,\alpha)=C_1 J_1(\rho\kappa)+\Psi_{p}^{(1)}(\rho,\alpha),\qquad \Psi_{-}^{(2)}(\rho,\alpha)+\Psi_{+}^{(2)}(\rho,\alpha)=C_2 H_1^{(1)}(\rho\kappa),$$

$$\Psi_{p}^{(1)}(\rho,\alpha)=\frac{i}{2\pi}\left[M^{(i)}\frac{\frac{k_{zl}}{\varepsilon}-\alpha}{\alpha_l^2-\alpha^2}J_1\left(\frac{\rho j_{0l}}{a}\right)-\sum_{m=1}^{\infty}M_m\frac{\frac{k_{zm}}{\varepsilon}+\alpha}{\alpha_m^2-\alpha^2}J_1\left(\frac{\rho j_{0m}}{a}\right)\right],\qquad \alpha_m=\sqrt{k_0^2-\left(\frac{j_{0m}}{a}\right)^2}$$

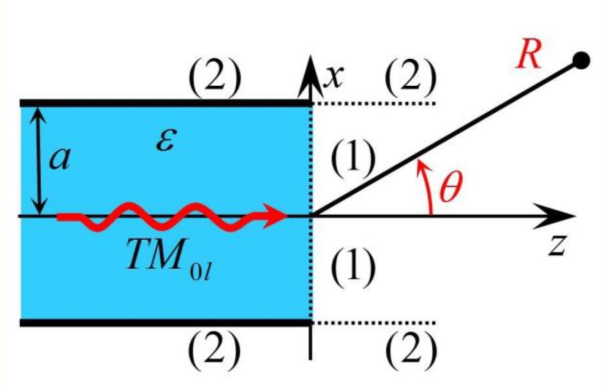
Cylindrical waveguide with uniform filling

Integral representation for Ez:

$$\Phi_+^{(1,2)}(\rho,\alpha) = \frac{1}{2\pi}\int_0^\infty dz E_{\omega z}^{(1,2)}(\rho,z)e^{i\alpha z}\frac{k_0}{i}$$

$$\Phi_-^{(2)}(\rho,\alpha) = \frac{1}{2\pi}\int_{-\infty}^0 dz E_{\omega z}^{(2)}(\rho,z)e^{i\alpha z}\frac{k_0}{i}$$

$$E_{\omega z} = \frac{i}{k_0\varepsilon}\left(\frac{H_{\omega\varphi}}{\rho} + \frac{\partial H_{\omega\varphi}}{\partial\rho}\right)$$



$$\Phi_+^{(1)}(\rho,\alpha) = C_1\kappa J_0(\rho\kappa) + \Phi_p^{(1)}(\rho,\alpha),$$

$$\Phi_-^{(2)}(\rho,\alpha) + \Phi_+^{(2)}(\rho,\alpha) = C_2\kappa H_0^{(1)}(\rho\kappa),$$

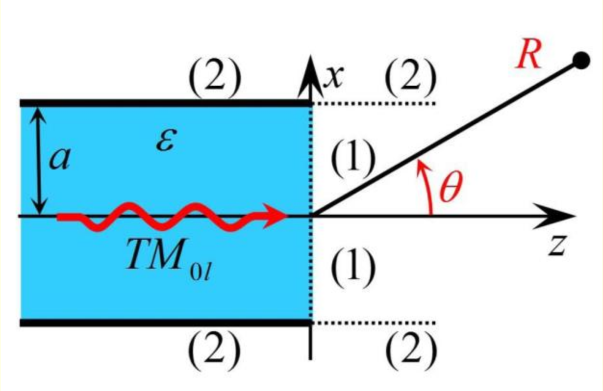
$$\Phi_p^{(1)}(\rho,\alpha) = \frac{i}{2\pi}\left[M^{(i)}\frac{\frac{k_{zl}}{\varepsilon}-\alpha}{\alpha_l^2-\alpha^2}\frac{j_{0l}}{a}J_0\left(\frac{\rho j_{0l}}{a}\right) - \sum_{m=1}^{\infty} M_m \frac{\frac{k_{zm}}{\varepsilon}+\alpha}{\alpha_m^2-\alpha^2}\frac{j_{0m}}{a}J_0\left(\frac{\rho j_{0m}}{a}\right)\right]$$

PEC for $\rho = a$, $z < 0$: $\Phi_-^{(2)}(\rho,\alpha) = 0 \quad \Rightarrow \quad C_2 = \dfrac{\Phi_+^{(2)}(a,\alpha)}{\kappa H_0^{(1)}(\kappa a)}$

Cylindrical waveguide with uniform filling

Continuity at $\rho = a$, $z > 0$: $H_{\omega\varphi}$, $E_{\omega z}$

$$\begin{cases} \Phi_+^{(1)}(a,\alpha) = \Phi_+^{(2)}(a,\alpha) \\ \Psi_+^{(1)}(a,\alpha) = \Psi_+^{(2)}(a,\alpha) \end{cases} \Rightarrow \textbf{ exclude } C_1 \Rightarrow$$



$$\Psi_+^{(2)}(a,\alpha) = J_1(\kappa a)\frac{\Phi_+^{(2)}(a,\alpha)}{\kappa J_0(\kappa a)} +$$

$$+\frac{i}{2\pi}\left[M^{(i)} \frac{\frac{k_{zl}}{\varepsilon} - \alpha}{\alpha_l^2 - \alpha^2} J_1(j_{0l}) - \sum_{m=1}^{\infty} M_m \frac{\frac{k_{zm}}{\varepsilon} + \alpha}{\alpha_m^2 - \alpha^2} J_1(j_{0m}) \right]$$

Right-hand side has poles in the upper half-plane for $\alpha = \alpha_p$:

$$\alpha_m = \sqrt{k_0^2 - \left(\frac{j_{0m}}{a}\right)^2}, \qquad \kappa(\alpha_m) = \frac{j_{0m}}{a} \qquad \Rightarrow \qquad J_0(a\kappa(\alpha_m)) = J_0\left(a\frac{j_{0m}}{a}\right) = 0$$

"Regularity" condition

$$\Phi_+^{(2)}(a,\alpha_p) = \frac{ia}{4\pi} J_1(j_{0p}) \left[M^{(i)} \delta_{lp} \left(\frac{k_{zp}}{\varepsilon} - \alpha_p\right) - M_p \left(\frac{k_{zp}}{\varepsilon} + \alpha_p\right) \right], \qquad p = 1, 2, \ldots$$

Cylindrical waveguide with uniform filling

Wiener-Hopf equation

$$\frac{2i}{\kappa}\frac{\Phi_+^{(2)}(a,\alpha)}{G(\alpha)}+\Psi_-^{(2)}(a,\alpha)+\frac{i}{2\pi}\left[M^{(i)}\frac{\frac{k_{zl}}{\varepsilon}-\alpha}{\alpha_l^2-\alpha^2}J_1\left(j_{0l}\right)-\sum_{m=1}^{\infty}M_m\frac{\frac{k_{zm}}{\varepsilon}+\alpha}{\alpha_m^2-\alpha^2}J_1\left(j_{0m}\right)\right]=0$$

Factorization and decomposition

$$G(\alpha)=\pi a\kappa J_0(a\kappa)H_0^{(1)}(a\kappa)=G_+(\alpha)G_-(\alpha),\qquad \kappa(\alpha)=\kappa_+(\alpha)\kappa_-(\alpha),\qquad \kappa_\pm(\alpha)=\sqrt{k_0\pm\alpha},$$

$$\eta_l(\alpha)=\kappa_-(\alpha)G_-(\alpha)\frac{\frac{k_{zl}}{\varepsilon}-\alpha}{\alpha_l^2-\alpha^2}=\eta_{l+}(\alpha)+\eta_{l-}(\alpha),\qquad \eta_{l+}(\alpha)=\frac{\kappa_+(\alpha_l)G_+(\alpha_l)\left(\frac{k_{zl}}{\varepsilon}+\alpha_l\right)}{2\alpha_l\left(\alpha+\alpha_l\right)}$$

$$\zeta_m(\alpha)=\kappa_-(\alpha)G_-(\alpha)\frac{\frac{k_{zm}}{\varepsilon}+\alpha}{\alpha_m^2-\alpha^2}=\zeta_{m+}(\alpha)+\zeta_{m-}(\alpha),\qquad \zeta_{m+}(\alpha)=\frac{\kappa_+(\alpha_m)G_+(\alpha_m)\left(\frac{k_{zm}}{\varepsilon}-\alpha_m\right)}{2\alpha_m\left(\alpha+\alpha_m\right)}$$

Formal solution of W.-H. equation

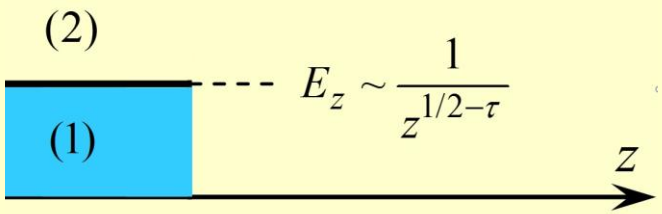
$$\frac{2i\,\Phi_+^{(2)}(a,\alpha)}{\kappa_+G_+}+\frac{i}{2\pi}\left[M^{(i)}J_1\left(j_{0l}\right)\eta_{l+}(\alpha)-\sum_{m=1}^{\infty}M_mJ_1\left(j_{0m}\right)\zeta_{m+}(\alpha)\right]=P(\alpha)$$

- some polynomial

Cylindrical waveguide with uniform filling

Meixner edge condition

$(|\alpha| \to \infty, |\operatorname{Im}\alpha| < \delta)$



$$\tau = \frac{1}{\pi}\operatorname{asin}\left[\frac{\varepsilon - 1}{2\varepsilon + 2}\right]$$

$M_m \sim m^{-1-\tau}$ $\quad \Phi_+^{(2)}(a,\alpha) \sim \alpha^{-1/2-\tau}$ $\quad \Psi_-^{(2)}(a,\alpha) \sim \alpha^{-3/2}$ $\quad P(\alpha) \to 0 \quad \Rightarrow \quad P(\alpha) = 0$

Solution of W.-H. equation

$$\Phi_+^{(2)}(a,\alpha) = -\frac{\kappa_+(\alpha)G_+(\alpha)}{4\pi}\left[M^{(i)}J_1(j_{0l})\eta_{l+}(\alpha) - \sum_{m=1}^{\infty} M_m J_1(j_{0m})\zeta_{m+}(\alpha)\right]$$

Regularity condition

$$\Phi_+^{(2)}(a,\alpha_p) = \frac{ia}{4\pi}J_1(j_{0p})\left[M^{(i)}\delta_{lp}\left(\frac{k_{zp}}{\varepsilon} - \alpha_p\right) - M_p\left(\frac{k_{zp}}{\varepsilon} + \alpha_p\right)\right], \qquad p = 1, 2, \ldots$$

Cylindrical waveguide with uniform filling

Infinite linear system for coefficients of reflected modes

$$\sum_{m=1}^{\infty} M_m W_{mp} = M^{(i)} w_p,$$

$$W_{mp} = \left[ia\delta_{mp}\left(\frac{k_{zm}}{\varepsilon}+\alpha_m\right) + G_+(\alpha_p)\kappa_+(\alpha_p)\frac{\kappa_+(\alpha_m)G_+(\alpha_m)}{2\alpha_m\left(\alpha_p+\alpha_m\right)}\left(\frac{k_{zm}}{\varepsilon}-\alpha_m\right)\right] J_1(j_{0m}) \qquad \Rightarrow \qquad \{M_m\}$$

$$w_p = \left[ia\delta_{lp}\left(\frac{k_{zl}}{\varepsilon}-\alpha_l\right) + G_+(\alpha_p)\kappa_+(\alpha_p)\frac{\kappa_+(\alpha_l)G_+(\alpha_l)}{2\alpha_l\left(\alpha_p+\alpha_l\right)}\left(\frac{k_{zl}}{\varepsilon}+\alpha_l\right)\right] J_1(j_{0l})$$

Far-field

$$\begin{aligned} H_{\omega\varphi}^{(2)}(\rho, z) &= -M^{(i)} J_1(j_{0l})\kappa_+(\alpha_l)G_+(\alpha_l)\frac{\frac{k_{zl}}{\varepsilon}+\alpha_l}{2\alpha_l}\frac{I_l^{(2)}}{4\pi} \\ &+ \sum_{m=1}^{\infty} M_m J_1(j_{0m})\kappa_+(\alpha_m)G_+(\alpha_m)\frac{\frac{k_{zm}}{\varepsilon}-\alpha_m}{2\alpha_m}\frac{I_m^{(2)}}{4\pi} \end{aligned}$$

$$I_m^{(2)}(\rho, z) \approx \pi a \frac{e^{ik_0R}}{R}\frac{\kappa_-(k_0\cos\theta)}{G_+(k_0\cos\theta)}\frac{2J_0(ak_0\sin\theta)}{k_0\cos\theta-\alpha_m}$$

Cylindrical waveguide with uniform filling

S-parameters

$$S_{ml} = \sqrt{\Sigma_m^{(r)} / \Sigma^{(i)}}$$

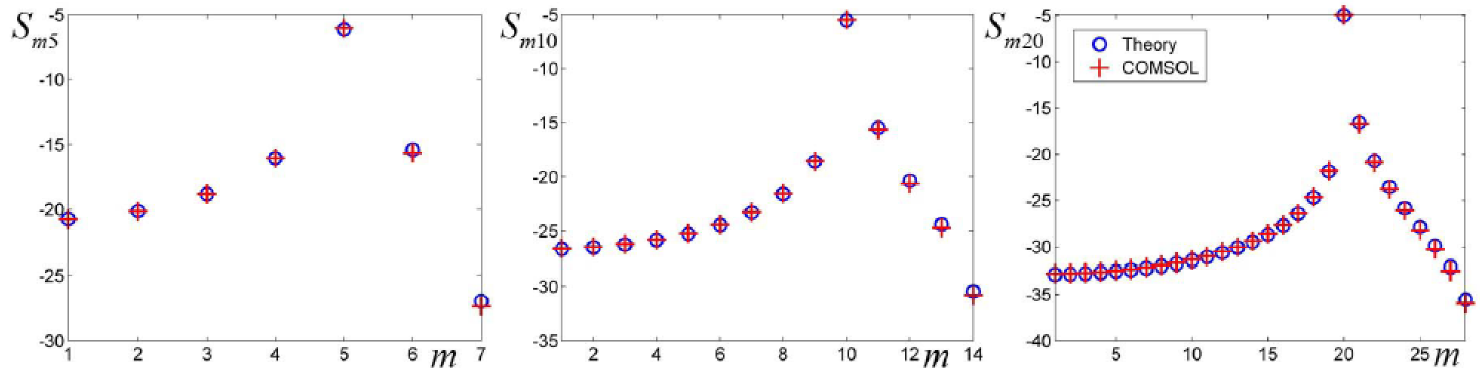
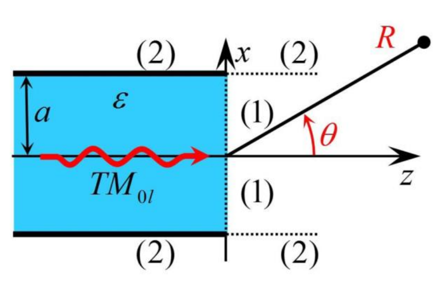


Fig. 3. Comparison between S-parameters (in dB) obtained via the presented analytical approach and via COMSOL simulations: S_{ml} corresponds to frequency f_l^{CR} (54) and incident mode with number l. We have seven propagating modes for $l=5$ ($f_5^{\mathrm{CR}} = 300$ GHz), 14 for $l=10$ ($f_{10}^{\mathrm{CR}} = 615$ GHz), and 28 for $l=20$ ($f_{20}^{\mathrm{CR}} = 1.247$ THz). Other parameters: $a = 0.24$ cm and $\varepsilon = 2$.

Far-field



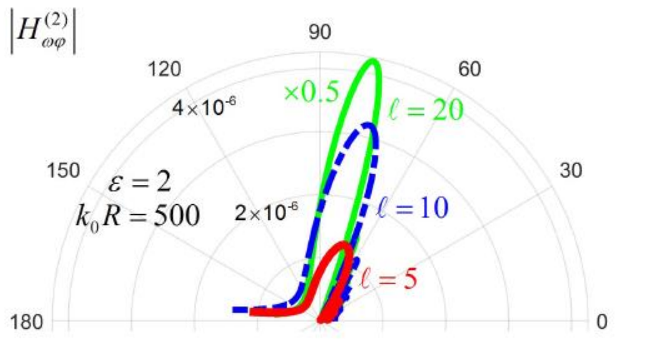


Fig. 4. Far-field distribution of the absolute value of $H_{\omega\varphi}^{(2)}$ in the region "2" calculated via (38) and (51). Observation distance $R = 500/k_0$, frequency, and other parameters correspond to those in Fig. 3, and a number indicated near each curve means the number of the exciting CR mode. $M^{(i)}$ is chosen so that $\Sigma^{(i)} = 1$ for each case. The curve $l = 20$ is multiplied by the factor 1/2.

Cylindrical waveguide with uniform filling

Near field

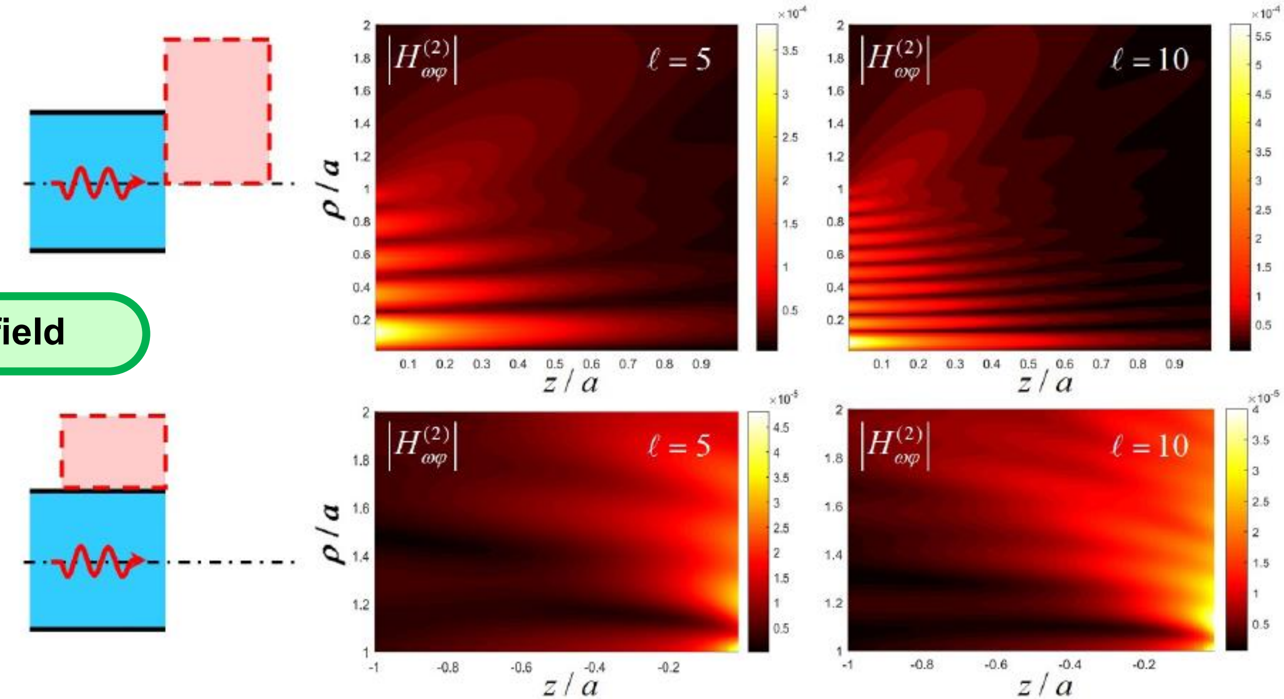


Fig. 6. Near-field distribution of the absolute value of $H^{(2)}_{\omega\varphi}$ in the regions outside the waveguide (these regions are shown at the first column) calculated via (46), (48), (49), (47), and (50). Calculation parameters correspond to the left and middle plots of Fig. 3. $M^{(i)}$ is chosen so that $\Sigma^{(i)} = 1$ for each case.

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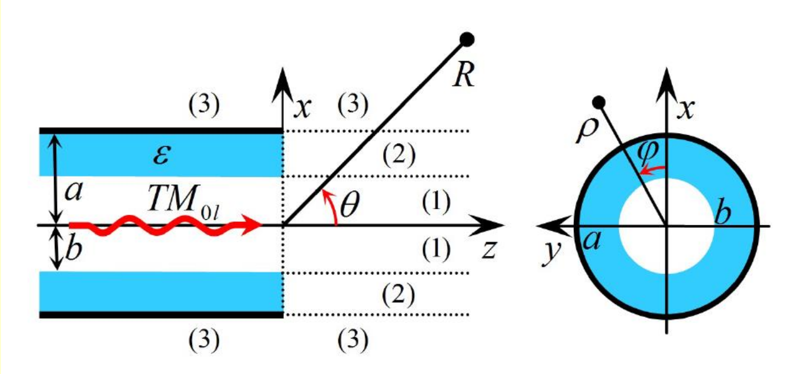
https://ieeexplore.ieee.org/document/9382413

Cylindrical waveguide with layered filling



https://arxiv.org/abs/2104.12375

S-parameters

$$S_{ml} = \sqrt{\Sigma_m^{(r)} / \Sigma^{(i)}}$$

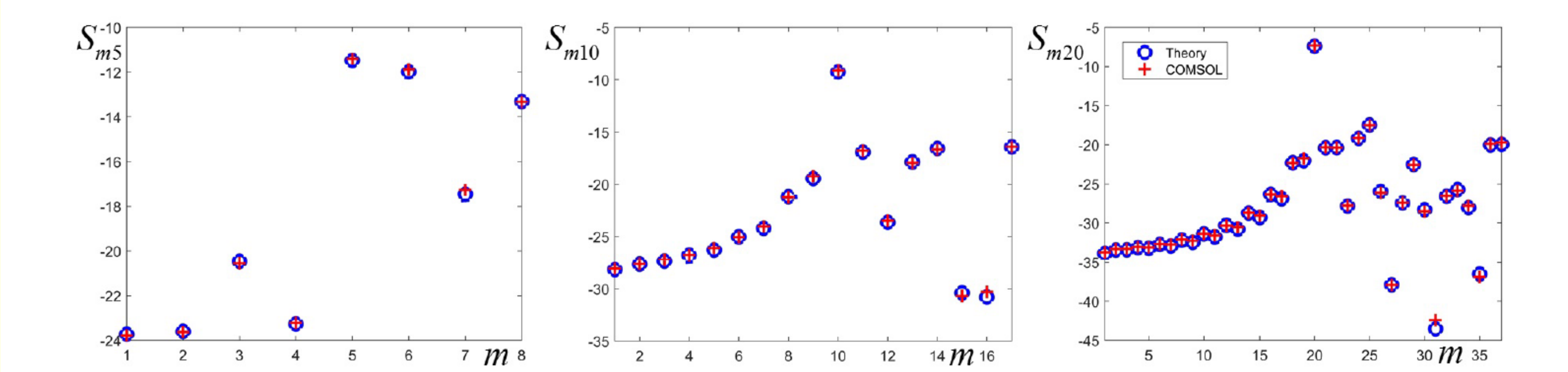
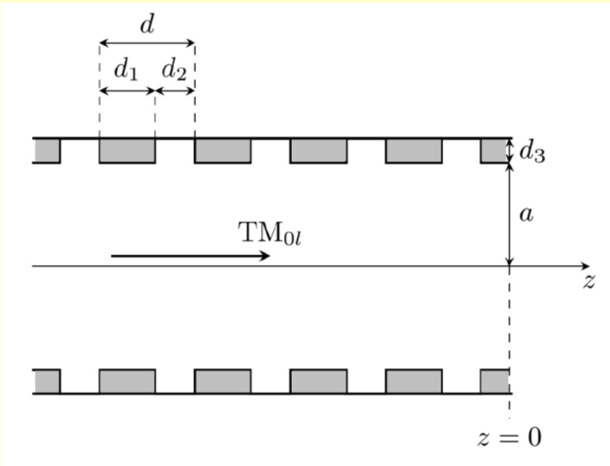


Figure 2. Comparison between S-parameters (in dB) obtained via the presented analytical approach and via COMSOL simulations: S_{ml} corresponds to the Cherenkov frequency $f_l^{\rm CR}$ and incident mode with number l. We have 8 propagating modes for $l = 5$ ($f_5^{\rm CR} = 397$ GHz), 17 for $l = 10$ ($f_{10}^{\rm CR} = 864$ GHz) and 37 for $l = 20$ ($f_{20}^{\rm CR} = 1.81$ THz). Other parameters: $a = 0.24$ cm, $b = a/3$, $\varepsilon = 2$.

Cylindrical waveguide with shallow corrugated wall

https://arxiv.org/abs/2005.05020



$$d \ll a,\ d_3 \ll a,\ d \ll \lambda, d_3 \ll \lambda,$$

S-parameters

Far-field

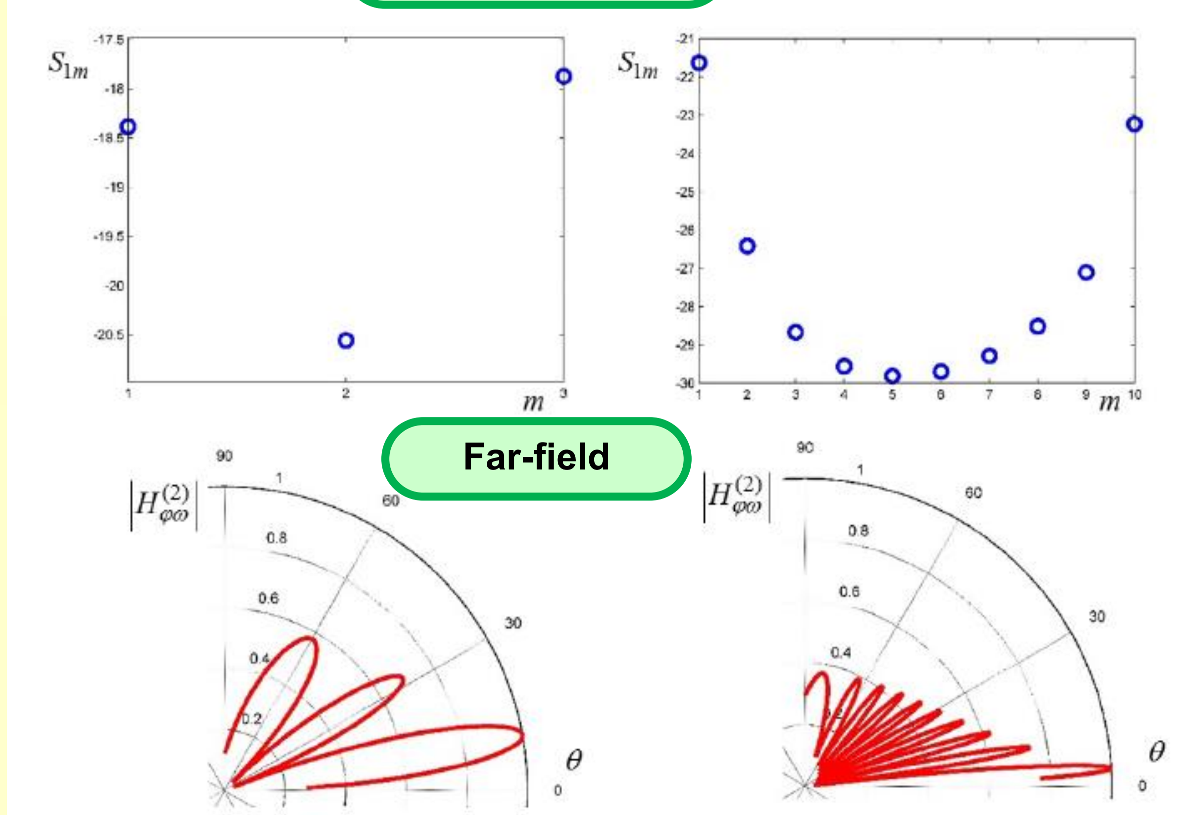


Figure 3. S-parameters (top row) and far-field patterns (bottom row) for the case of excitation by the first mode (this is only slow mode in this case) of the corrugated vacuum vaveguide. In all cases $a = 0.24$cm, other parameters: $d_1 = 0.005$cm, $d_2 = d_3 = 0.01$cm, $\omega = 2\pi \cdot 200$GHz, waveguide supports 3 propagating modes, $\beta = k_0/k_{z1} = 0.9996$ (left); $d_1 = d_2 = d_3 = 0.005$cm, $\omega = 2\pi \cdot 615$GHz, waveguide supports 10 propagating modes, $\beta = k_0/k_{z1} = 0.9982$ (right). Far-field paterns are normalized by the first lobe, θ is counted from positive z direction, $k_0 R = 1000$.

Thank you for your attention!