Design of double- and multi-bend achromat lattices with large dynamic aperture and approximate invariants

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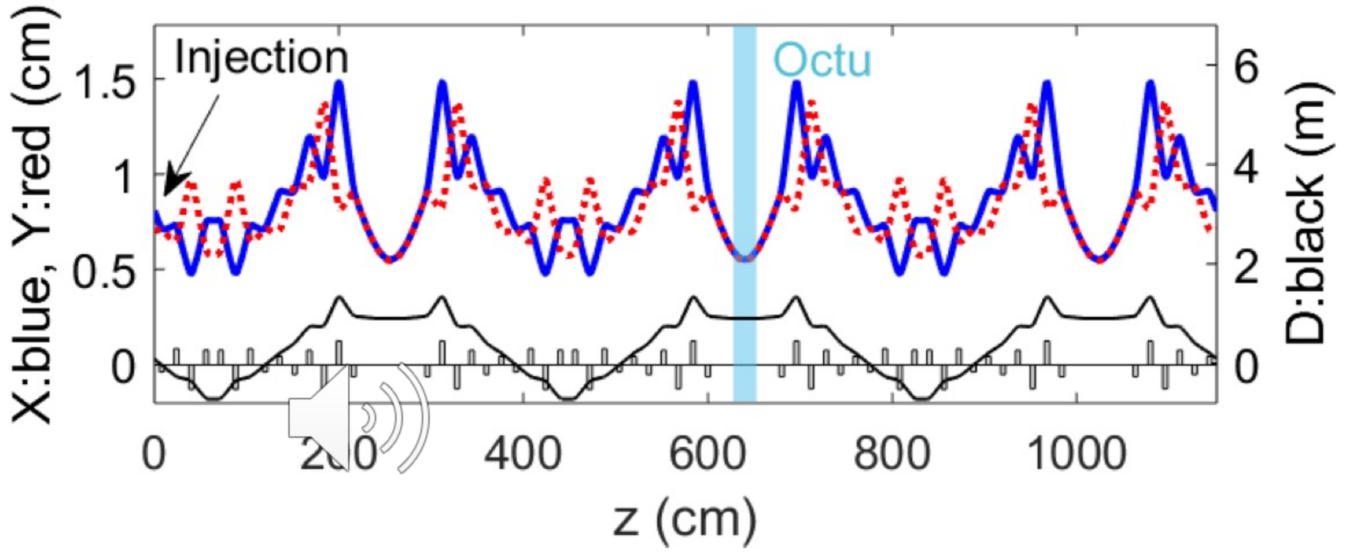
IOTA and UMER

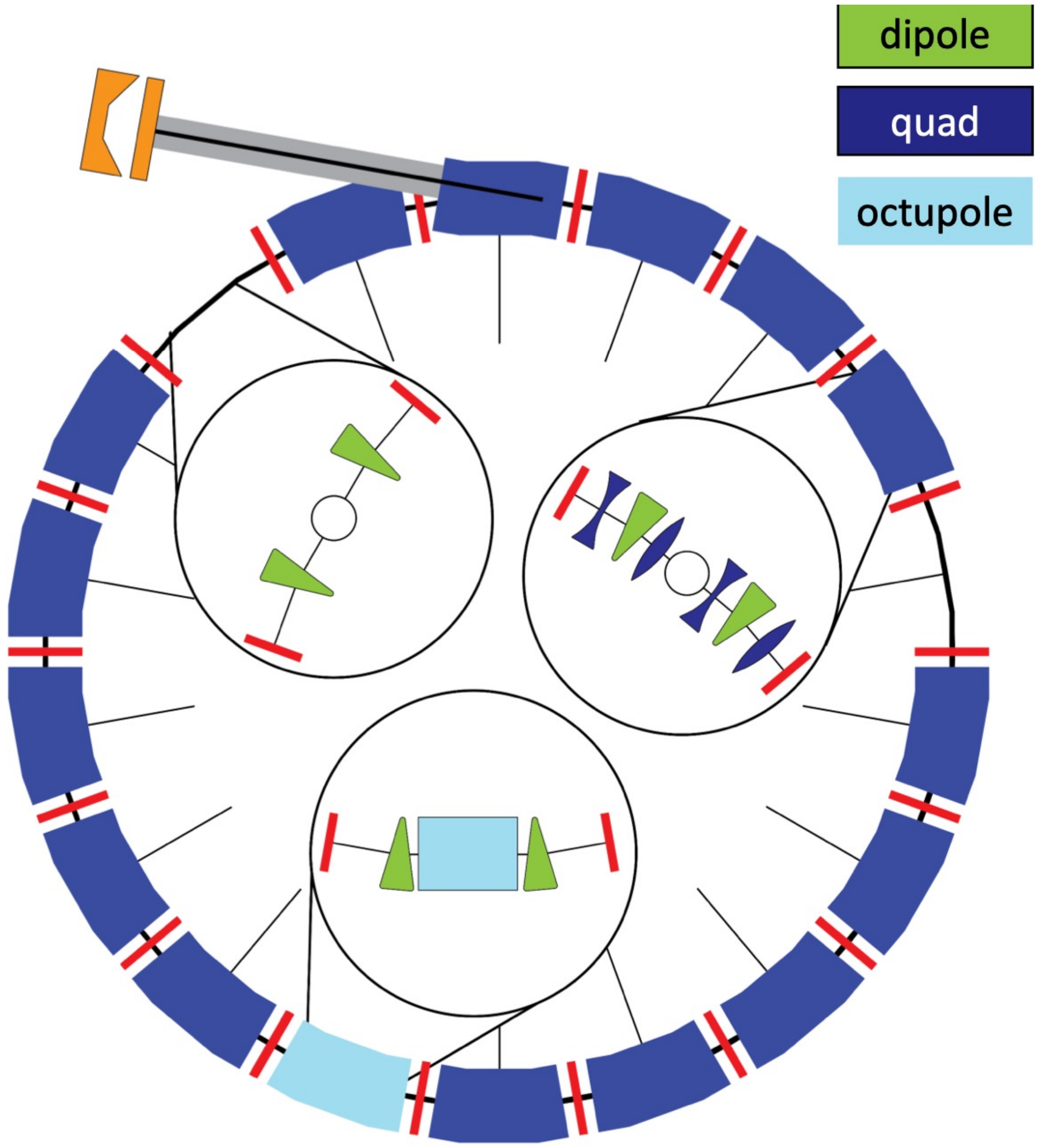
Nonlinear lattice with one or two invariants

Design of quadrupole focusing lattice

Linear lattice tune	Full ring tune
$\nu_x = 2.998$	$\nu_x = 3.124$
$\nu_x = 3.002$	$\nu_y = 3.128$

Solution assumes $\epsilon = 100\ \mu m$ and $I_{beam} = 60\ \mu A$

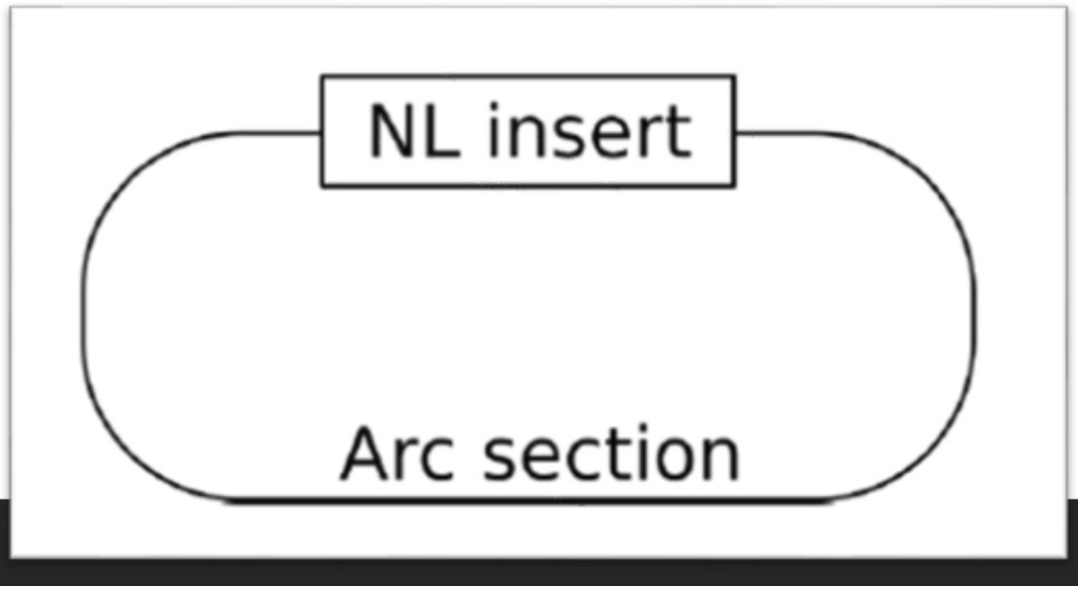




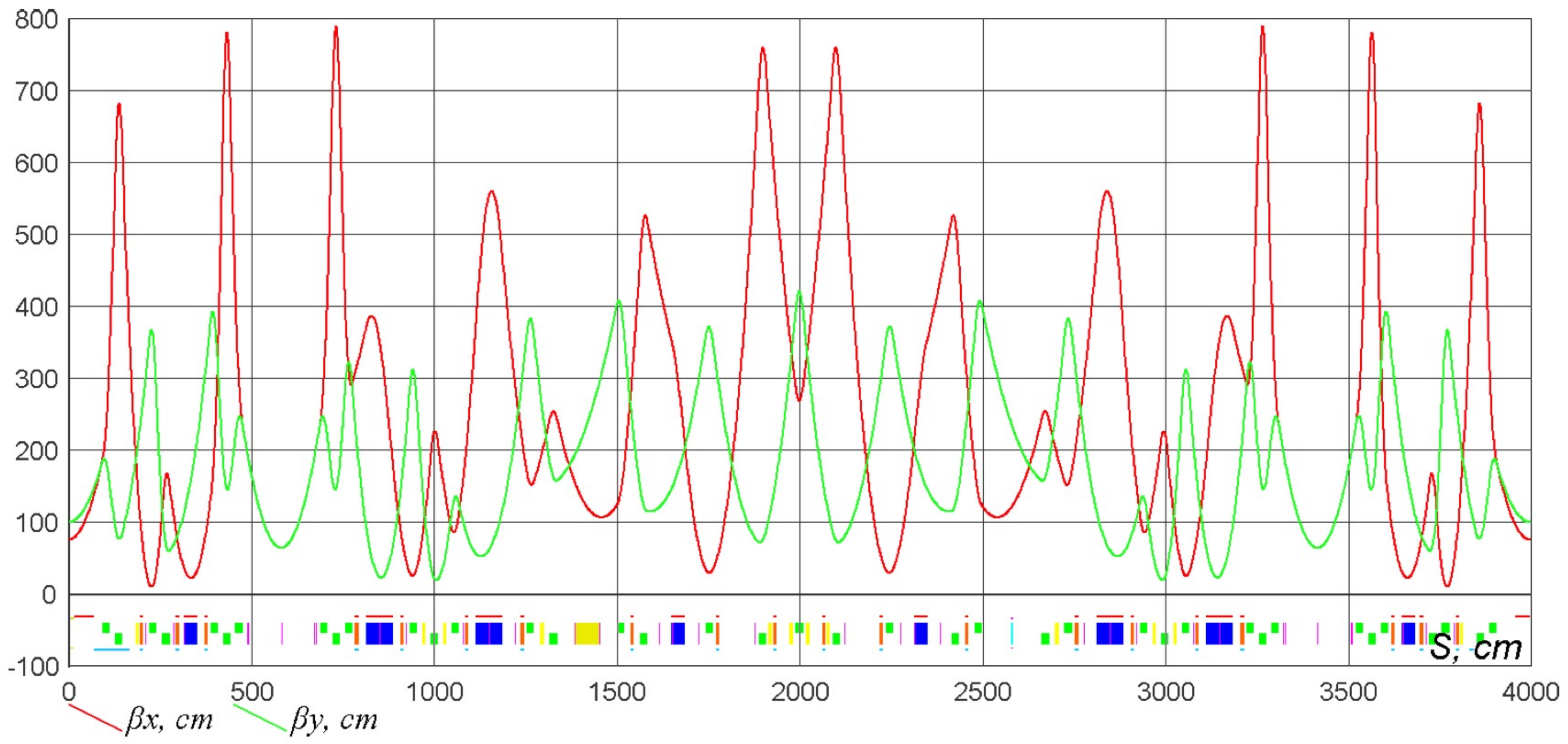
IOTA Toy model

IOTA lattice can be modeled by a nonlinear insert and a matrix map
- small nonlinearities in arc section, lattice errors and space charge can be modeled by introducing small tune shift μ in the matrix map

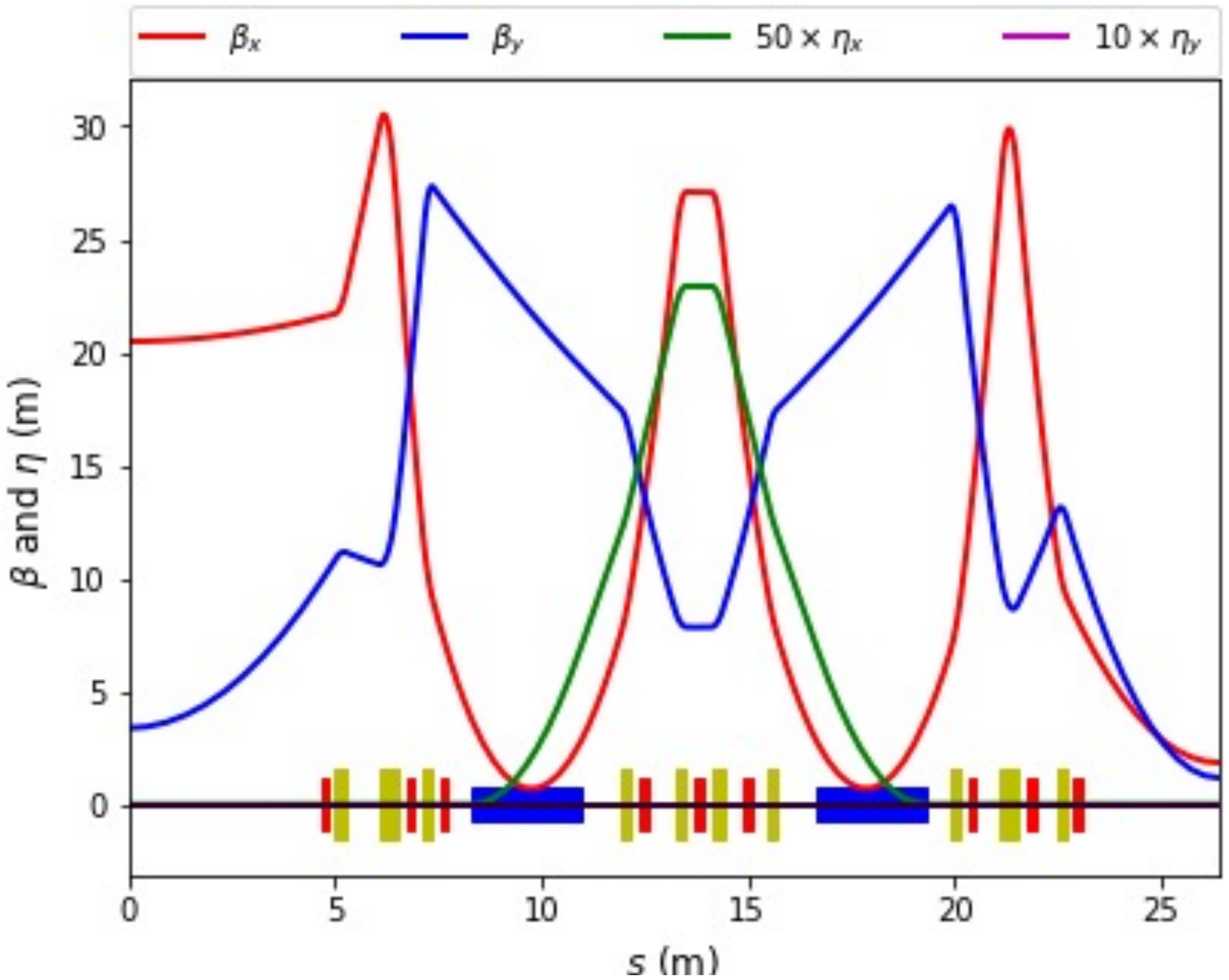
Throught this presentation, we use μ=0.02



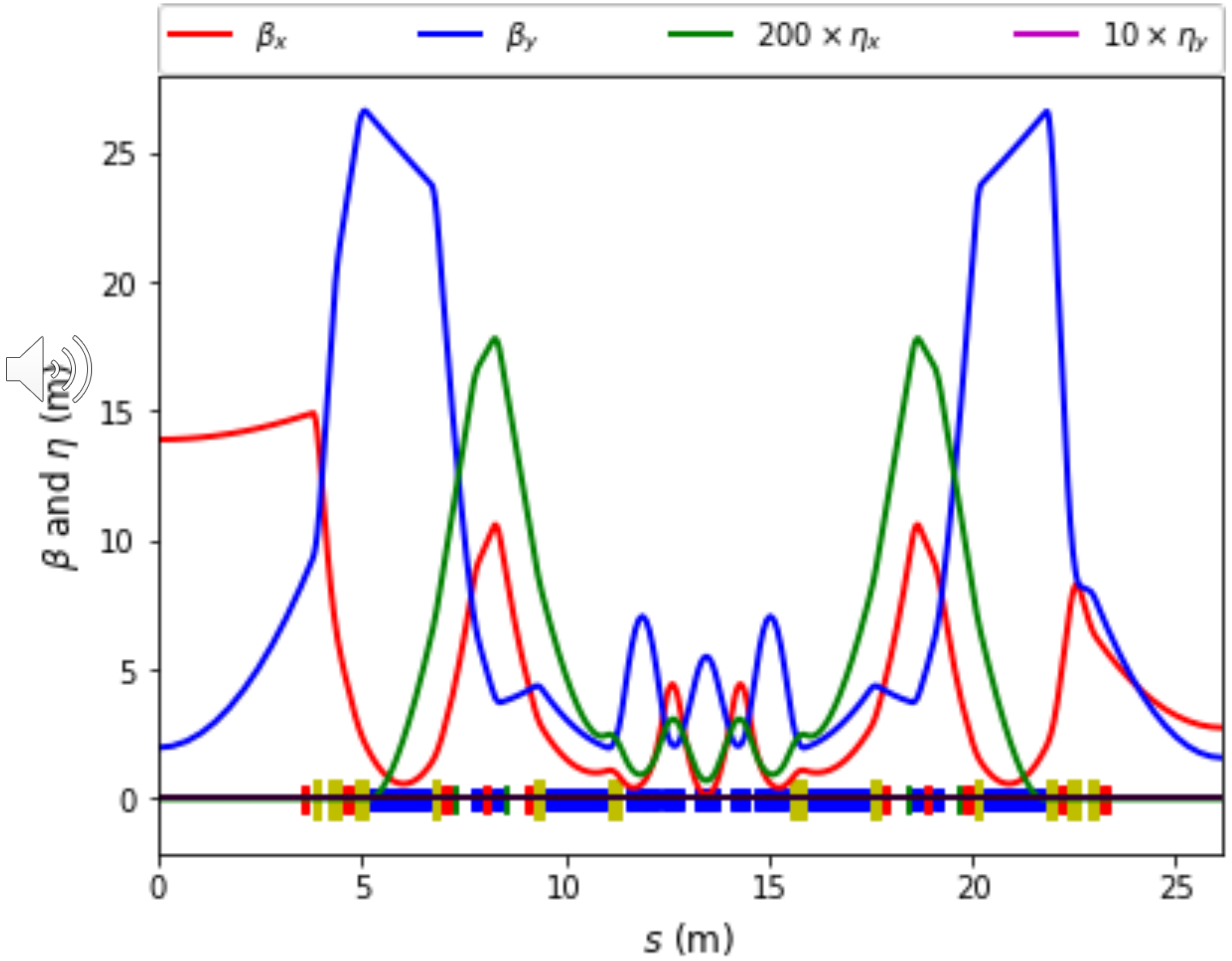
$$\mathcal{M} = \mathcal{N}\mathcal{R}$$
$$\mathcal{N} = e^{-2\pi\nu H_N}$$
$$\mathcal{R} = e^{-2\pi\mu H_R}$$



DBA or MBA lattice for light source?

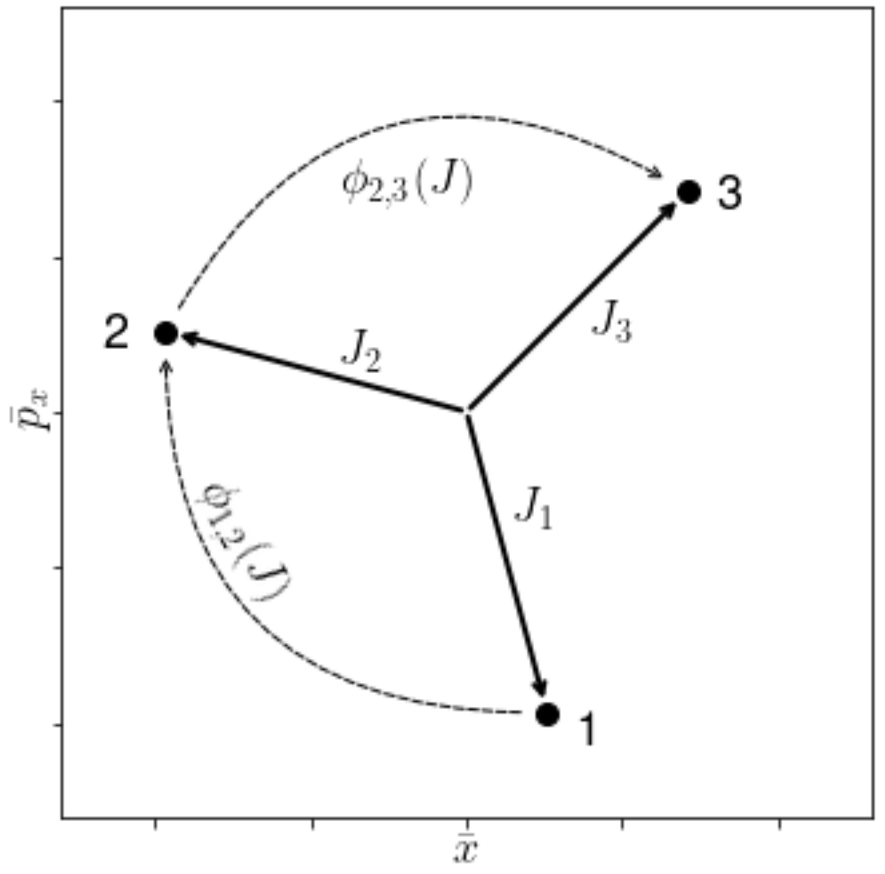


DBA lattice for 3rd light source



MBA for 4th light source

(Action-angle)-like quasi-invariants



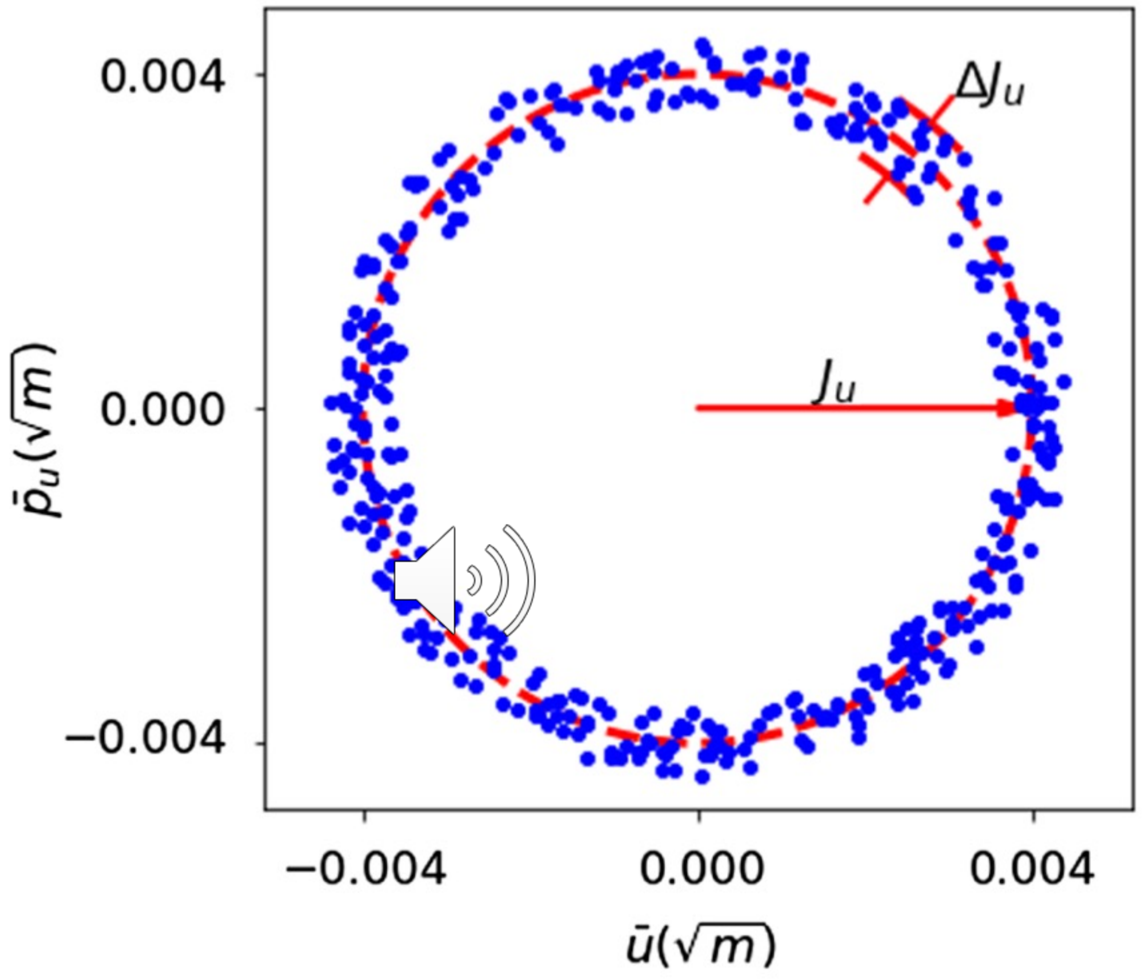
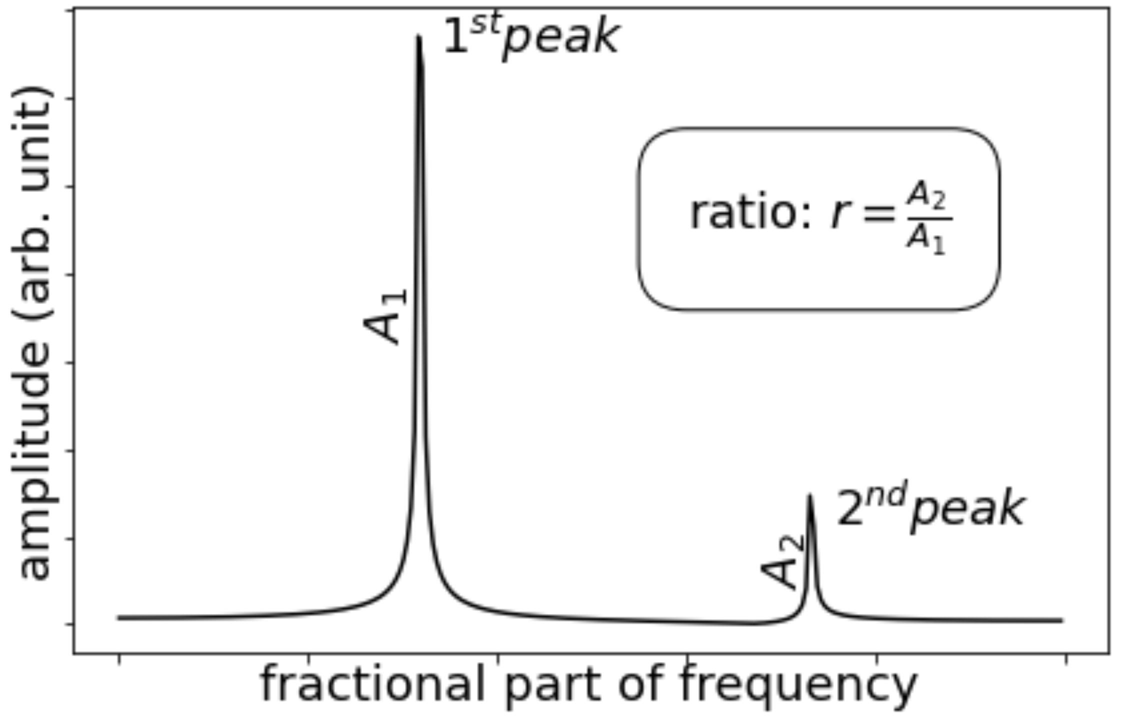


FIG. 2. The root means squared (rms) spread of action from a constant is used as an optimization objective. The dashed circle represents a constant linear action at different angles. The dots are the normalized turn-by-turn coordinates.



$$J_x = \frac{1}{2}(\bar{x}^2 + \bar{p}_x^2) = \frac{1}{2}\left(\gamma_x x^2 + 2\alpha_x x p_x + \beta_x p_x^2\right)$$

$$\begin{bmatrix} \bar{x} \\ \bar{p}_x \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{\beta_x}} & 0 \\ \frac{\alpha_x}{\sqrt{\beta_x}} & \sqrt{\beta_x} \end{bmatrix} \begin{bmatrix} x \\ p_x \end{bmatrix}.$$

$$\begin{aligned} \Delta\phi_x &= \phi_{x,i+1} - \phi_{x,i} \\ &= \arctan\left(\frac{\bar{p}_{x,i+1}}{\bar{x}_{i+1}}\right) - \arctan\left(\frac{\bar{p}_{x,i}}{\bar{x}_i}\right) + k \cdot 2\pi, \end{aligned}$$

$$\phi \neq \int \frac{1}{\sqrt{\beta}} \mathrm{d}s$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}n} = 0$$

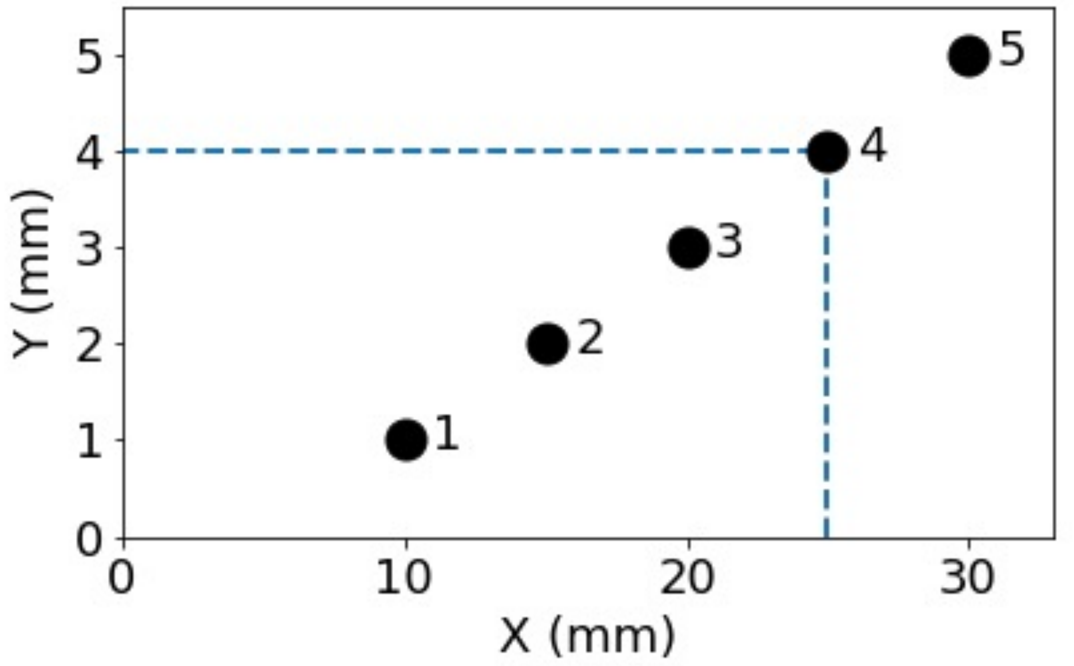
Numerical construction via tracking

Multi-objective genetic algorithm for optimization

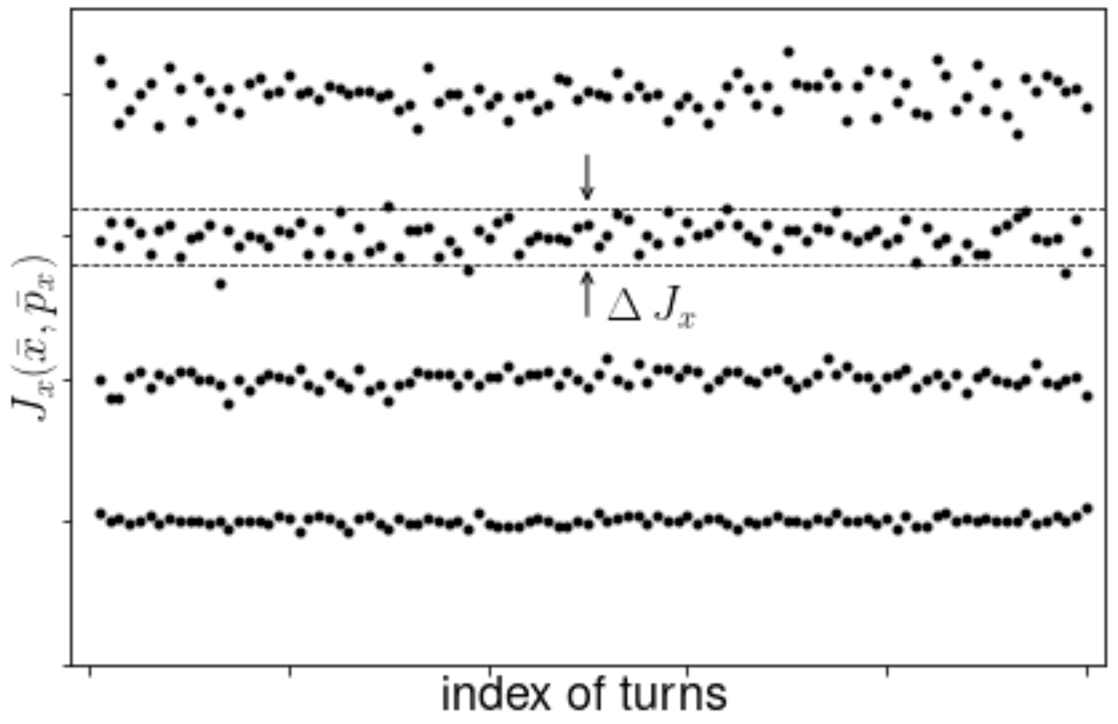
$$\begin{cases} \min\limits_{K_i} \left| [J_x(x,p_x), H(K_i,x,p_x,y,p_y)] \right| \to 0 \\ \min\limits_{K_i} \left| [\phi_x(x,p_x), H(K_i,x,p_x,y,p_y)] \right| \to 0 \end{cases}$$

Tuning sexts/octs to minimize the fluctuations in multi-turn tracking

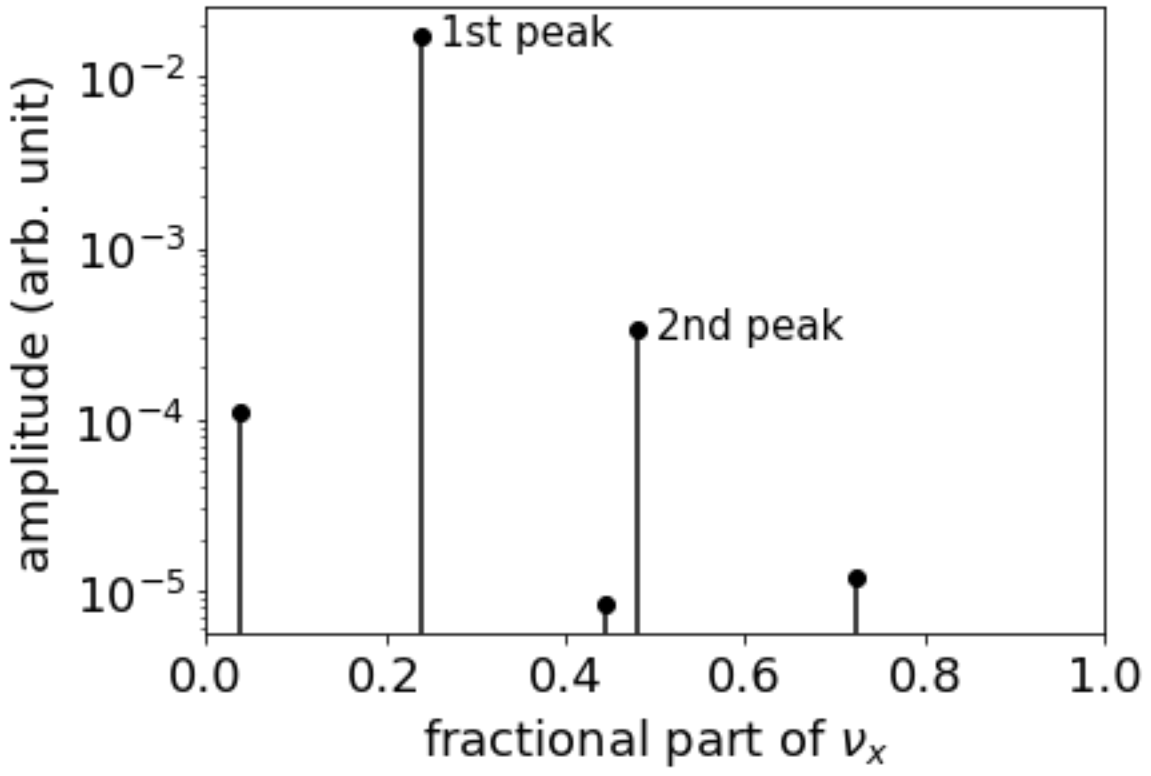
Multiple initial conditions



Symplectic tracking



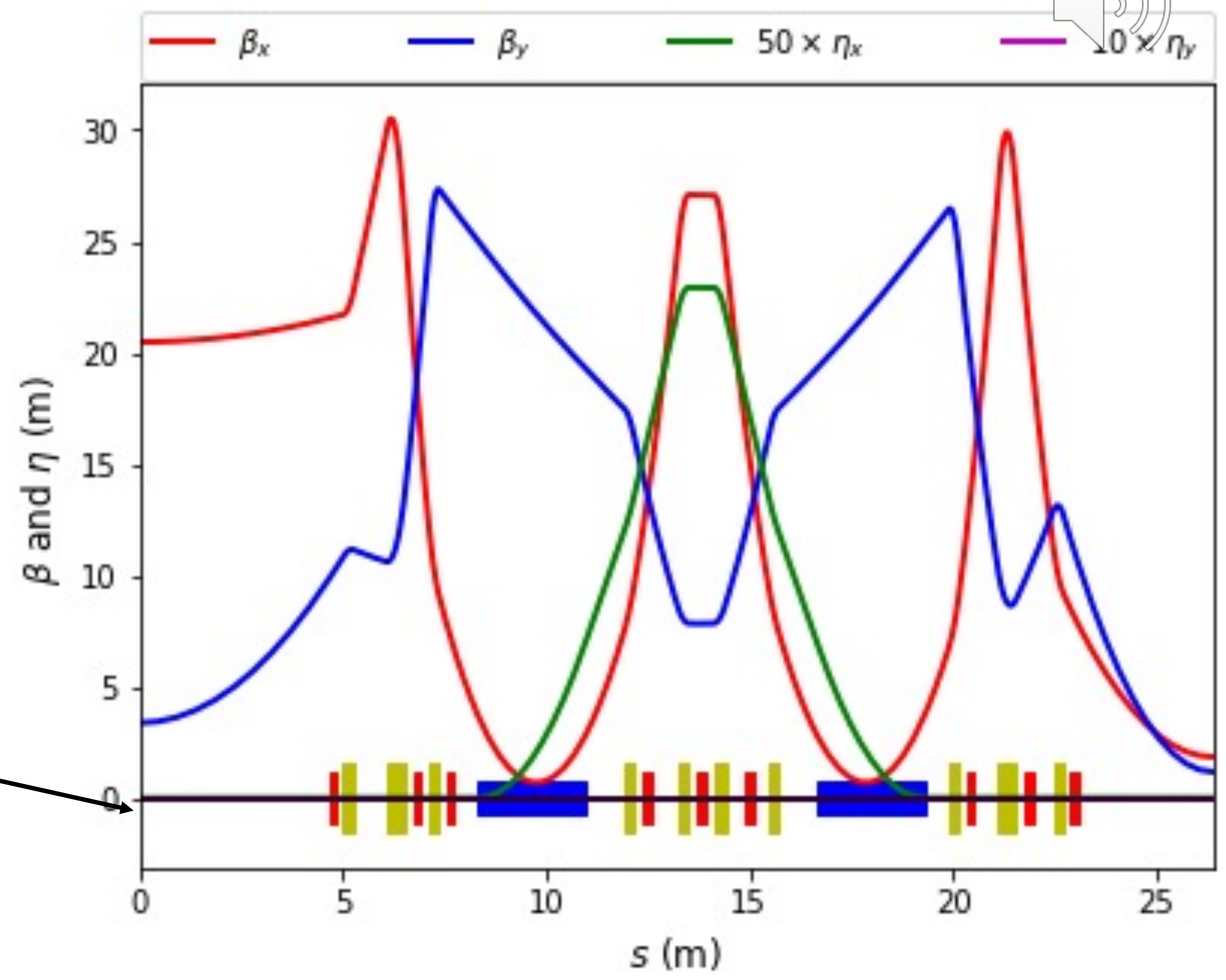
NAFF



Applied to NSLS-II DBA

- 3 chromatic sexts for chromaticity correction
- 6 harmonic sexts to construct 4 quasi-invariants

Observation point



Sext knobs: red blocks

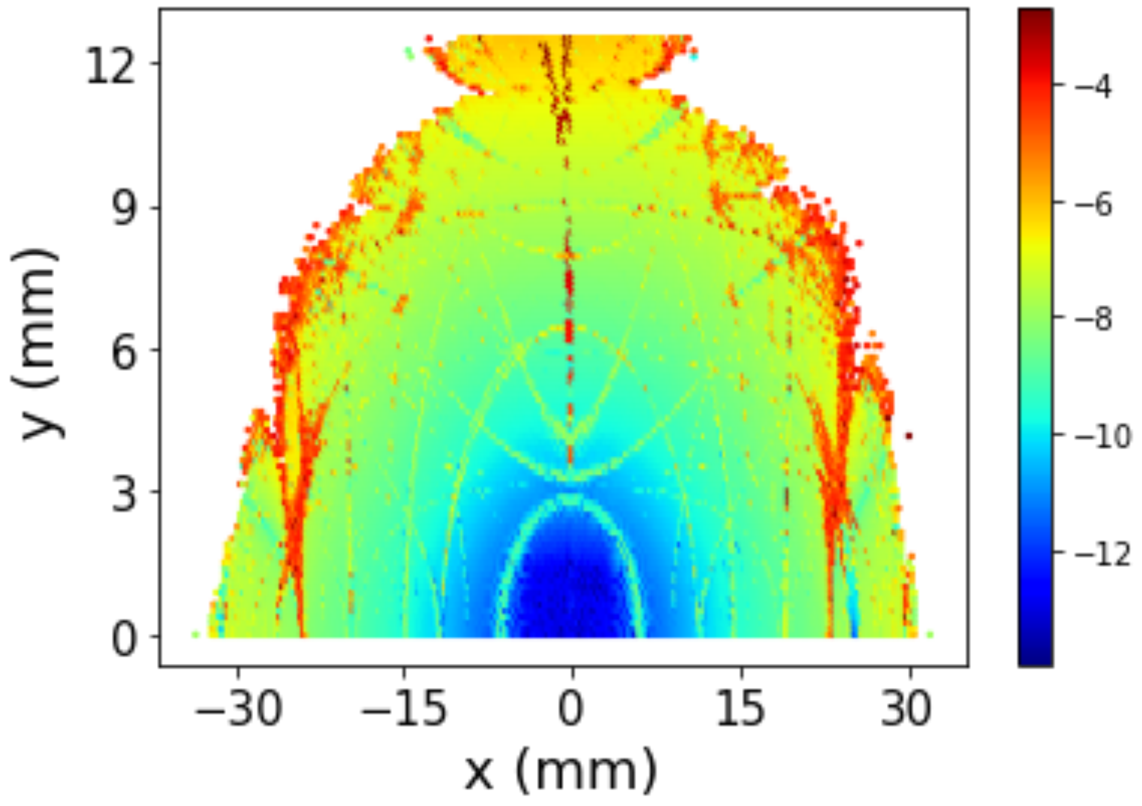
Table I. Main parameters of NSLS-II storage ring

Parameters	Values
Hor. emit. (nm)	2.1
Natural chrom. (x/y)	-101/-40
Tune (x/y)	33.22/16.26
Energy spread	5.1×10^{-4}
Damp. partition (x/y/s)	1.0/1.0/2.0

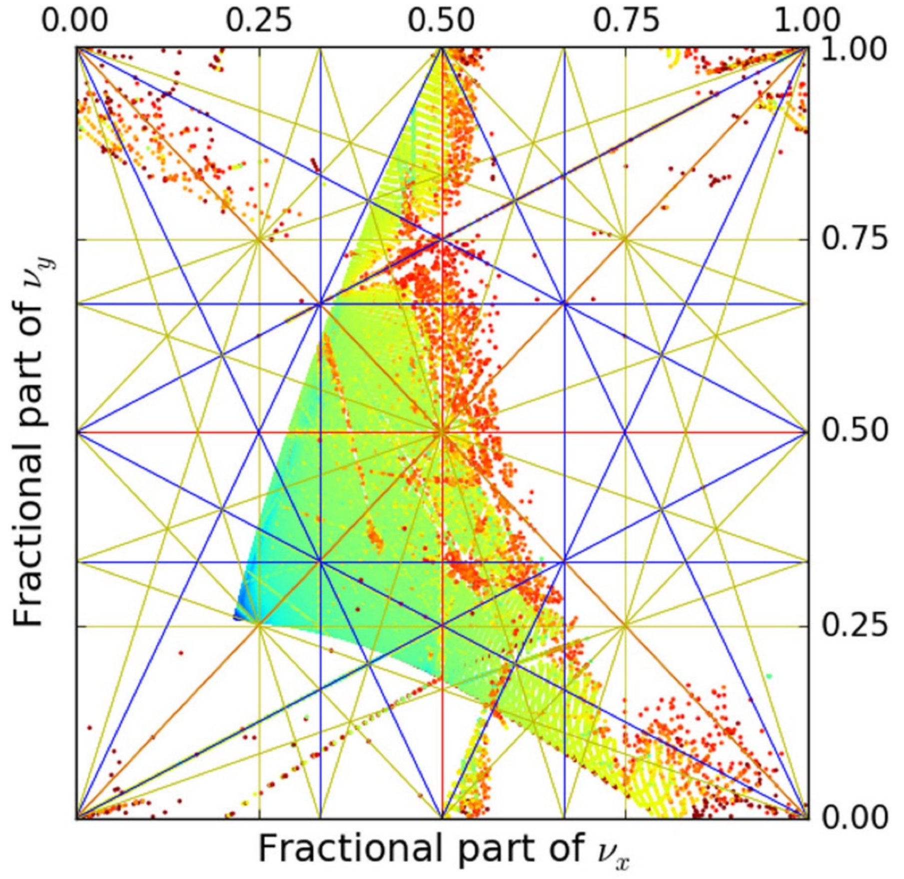
Properties of DBA

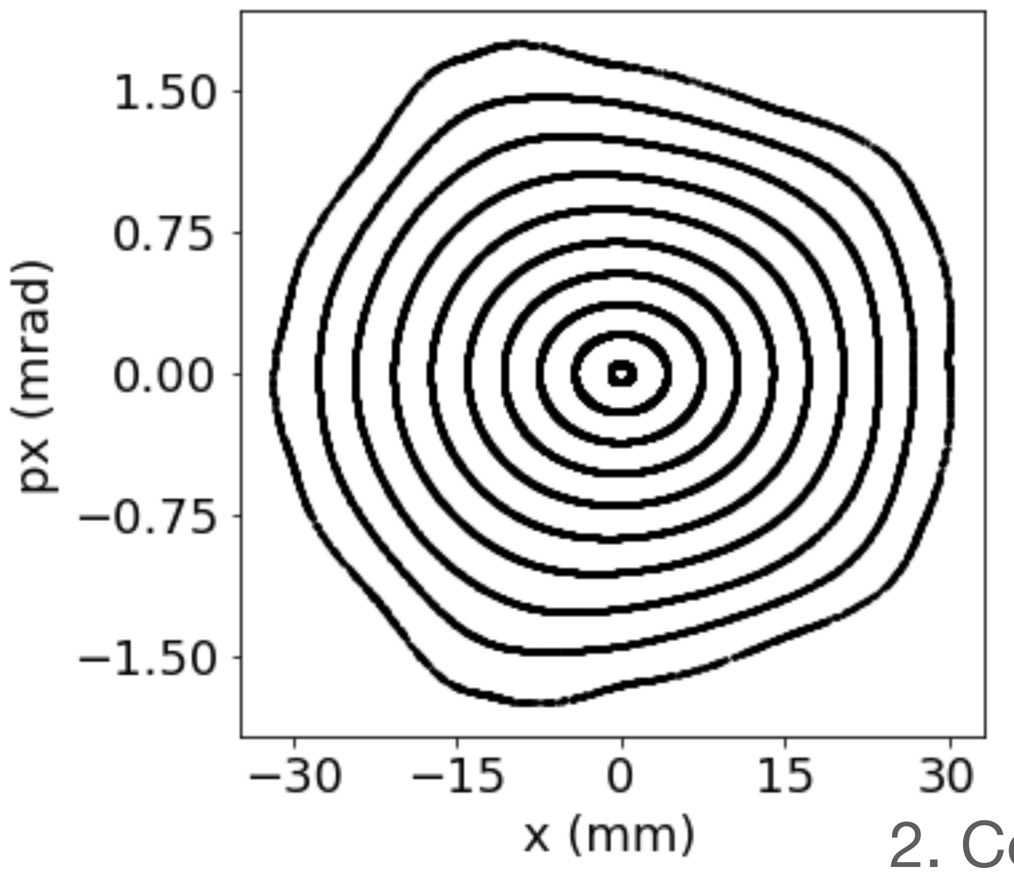
1. Large DA
2. Closed tori with quasi-invariants
3. Clean spectrum
4. Large tune spread

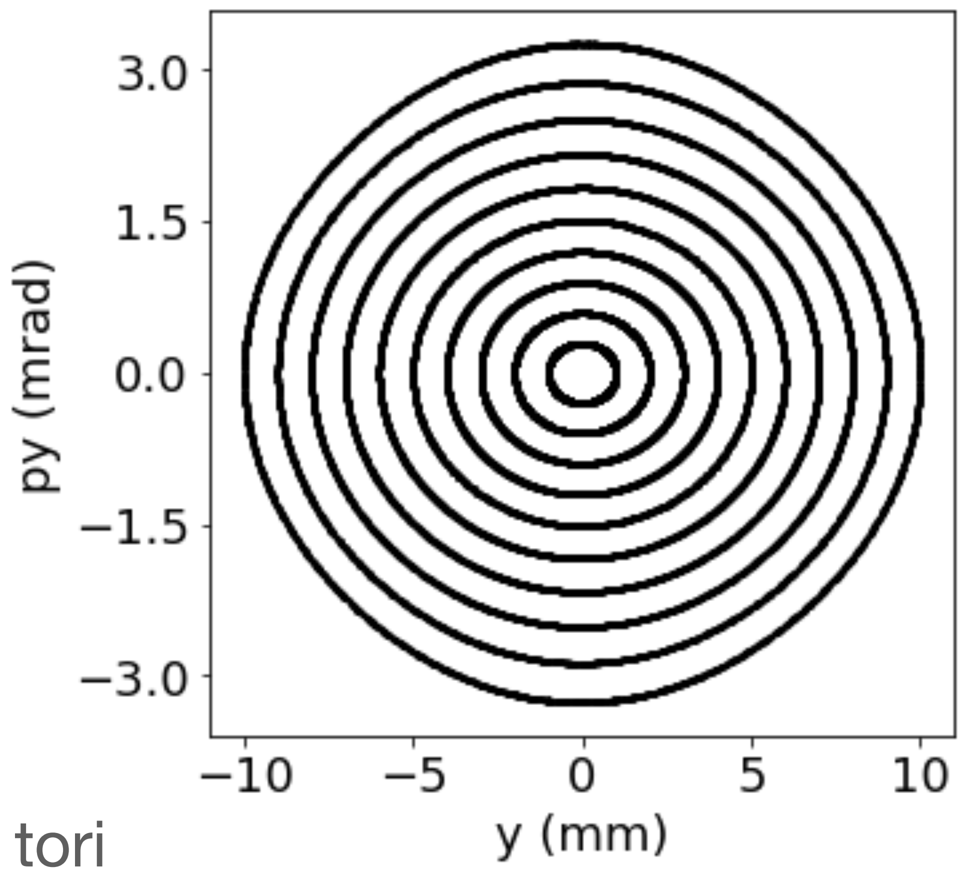
1. DA



4. FMA

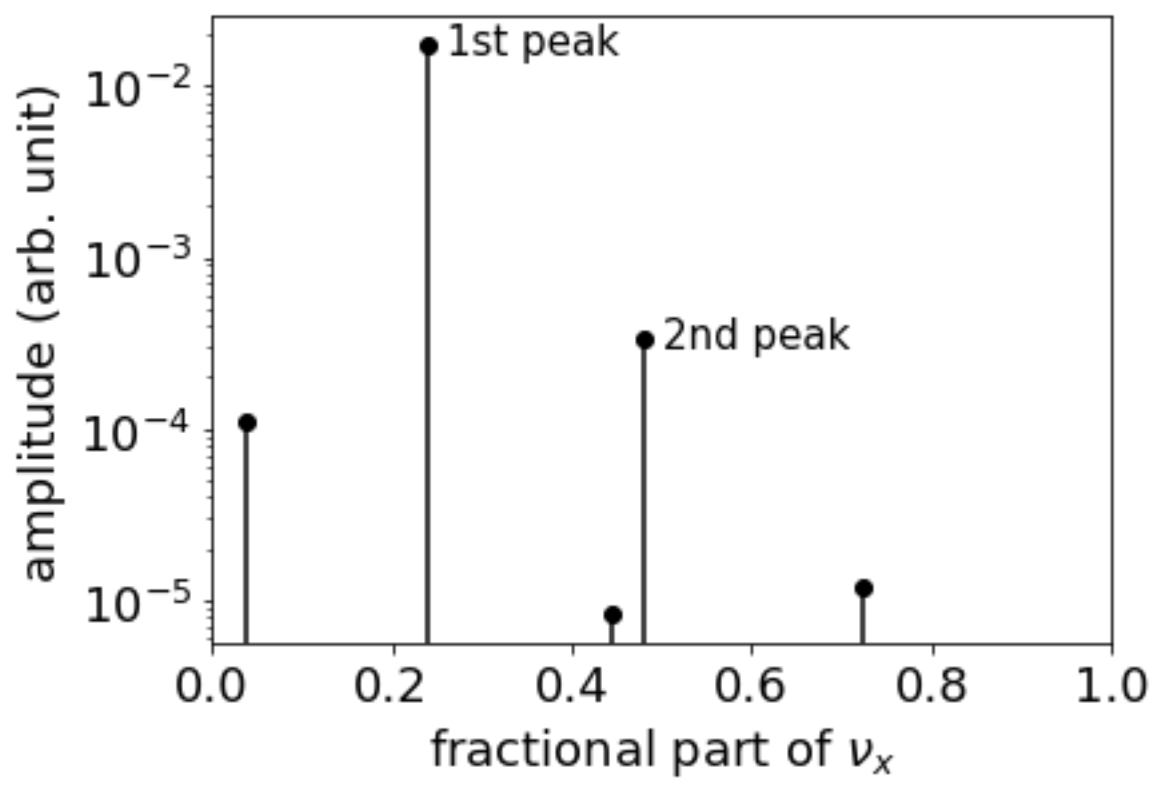


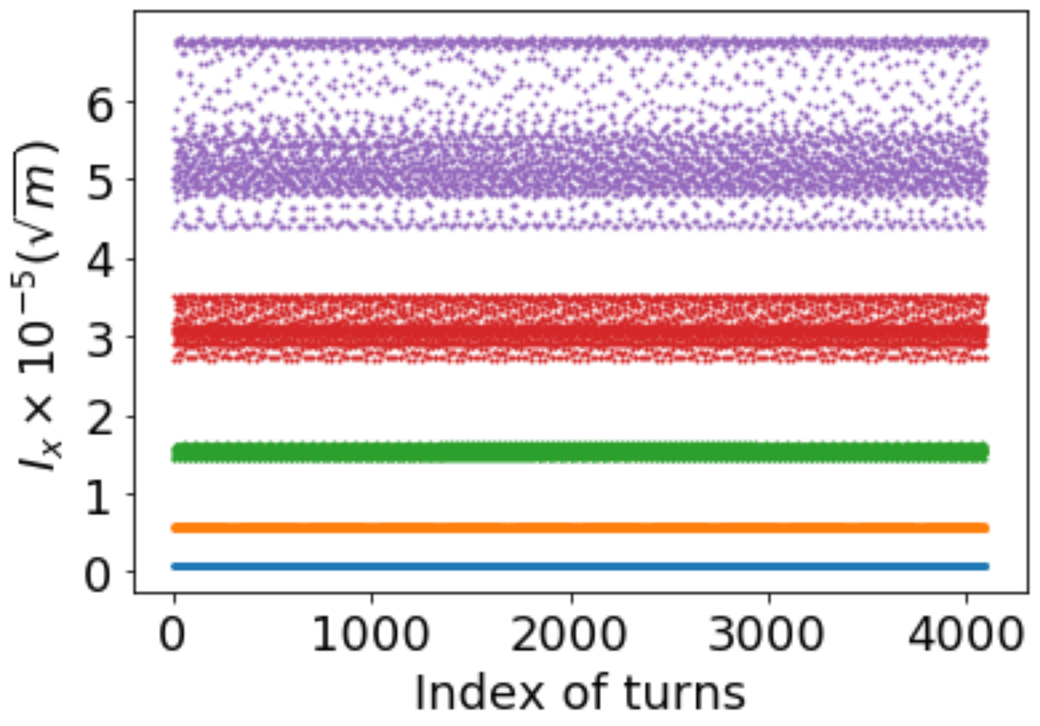


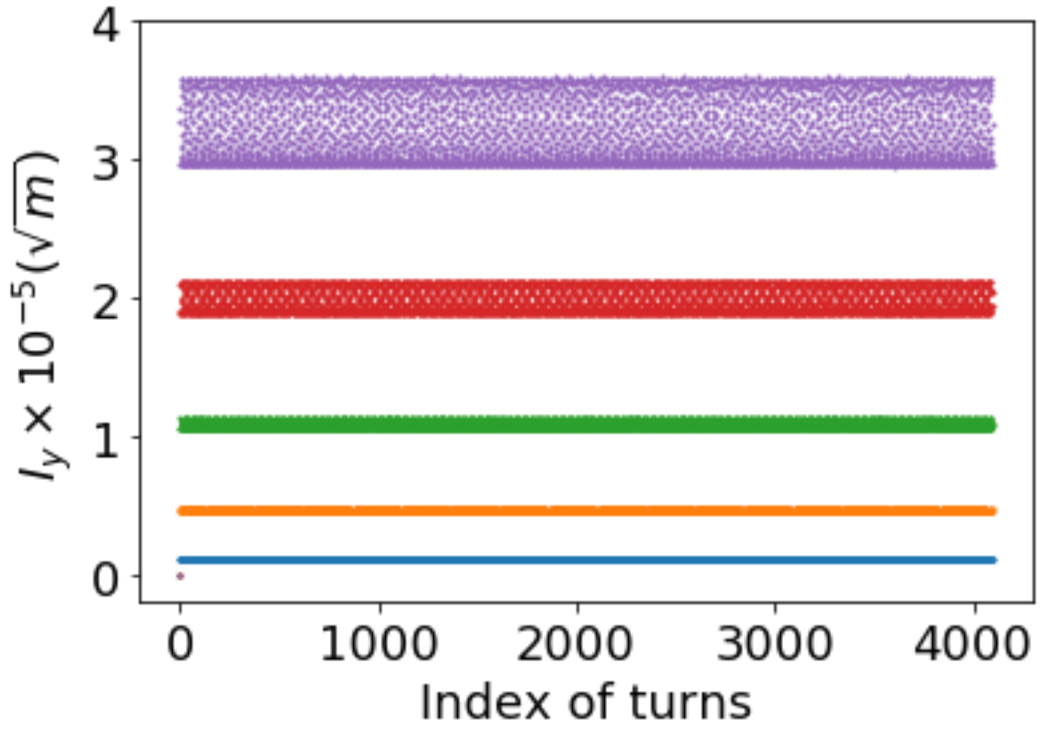


2. Confined tori

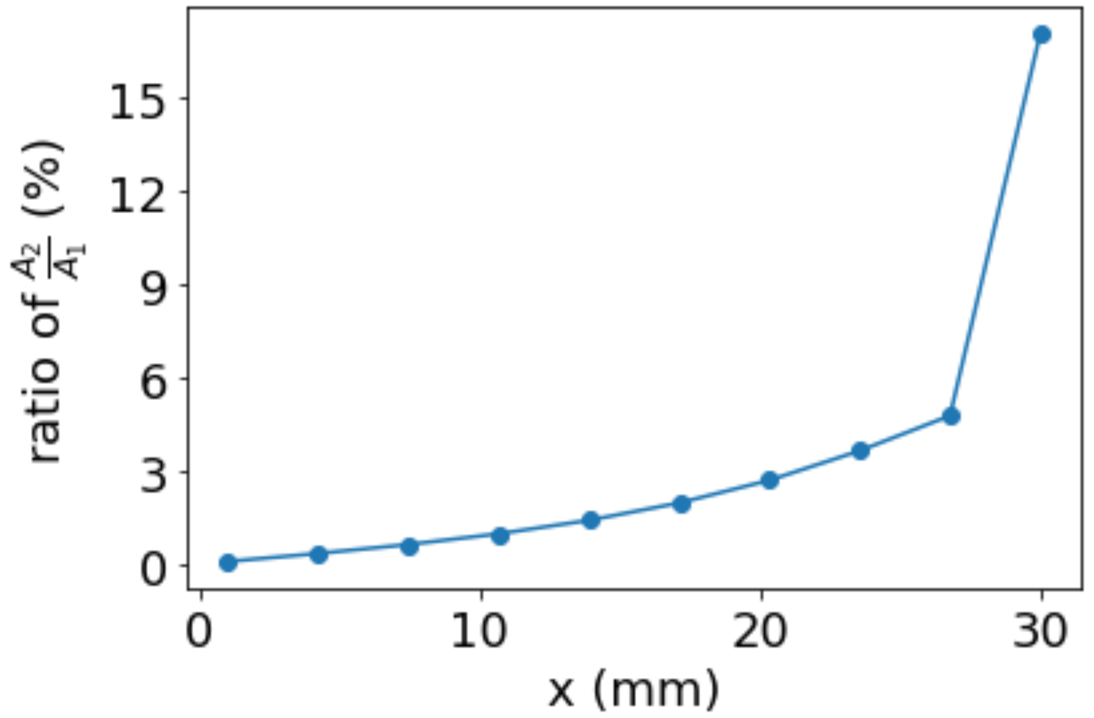
3. NAFF

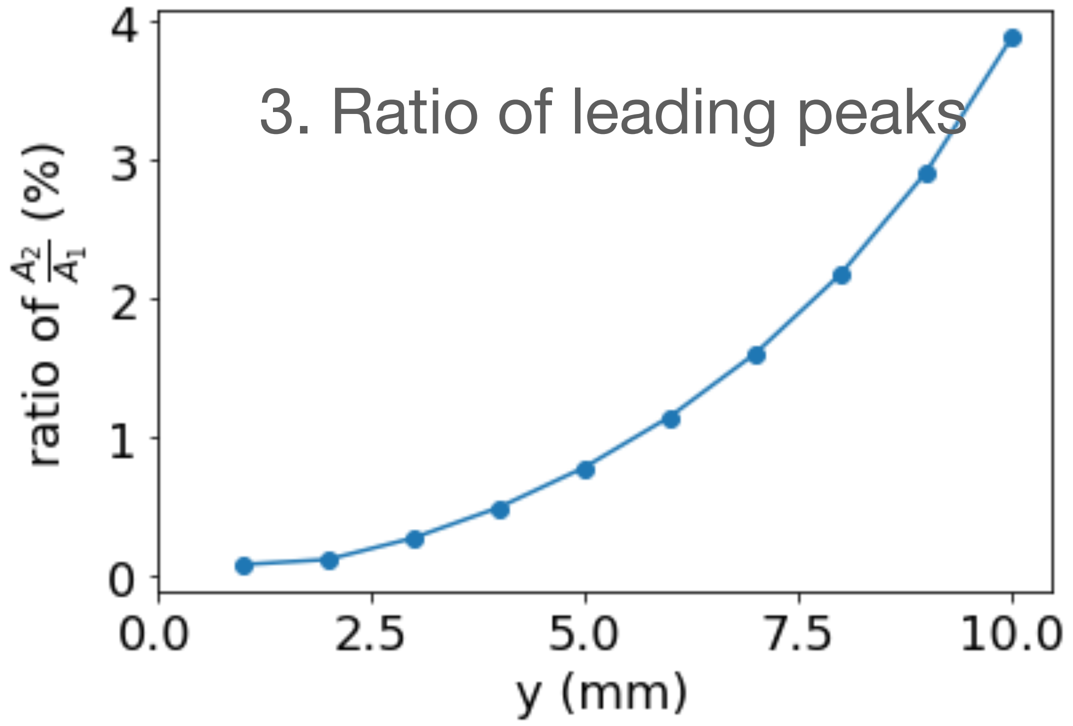






Fluctuations of actions



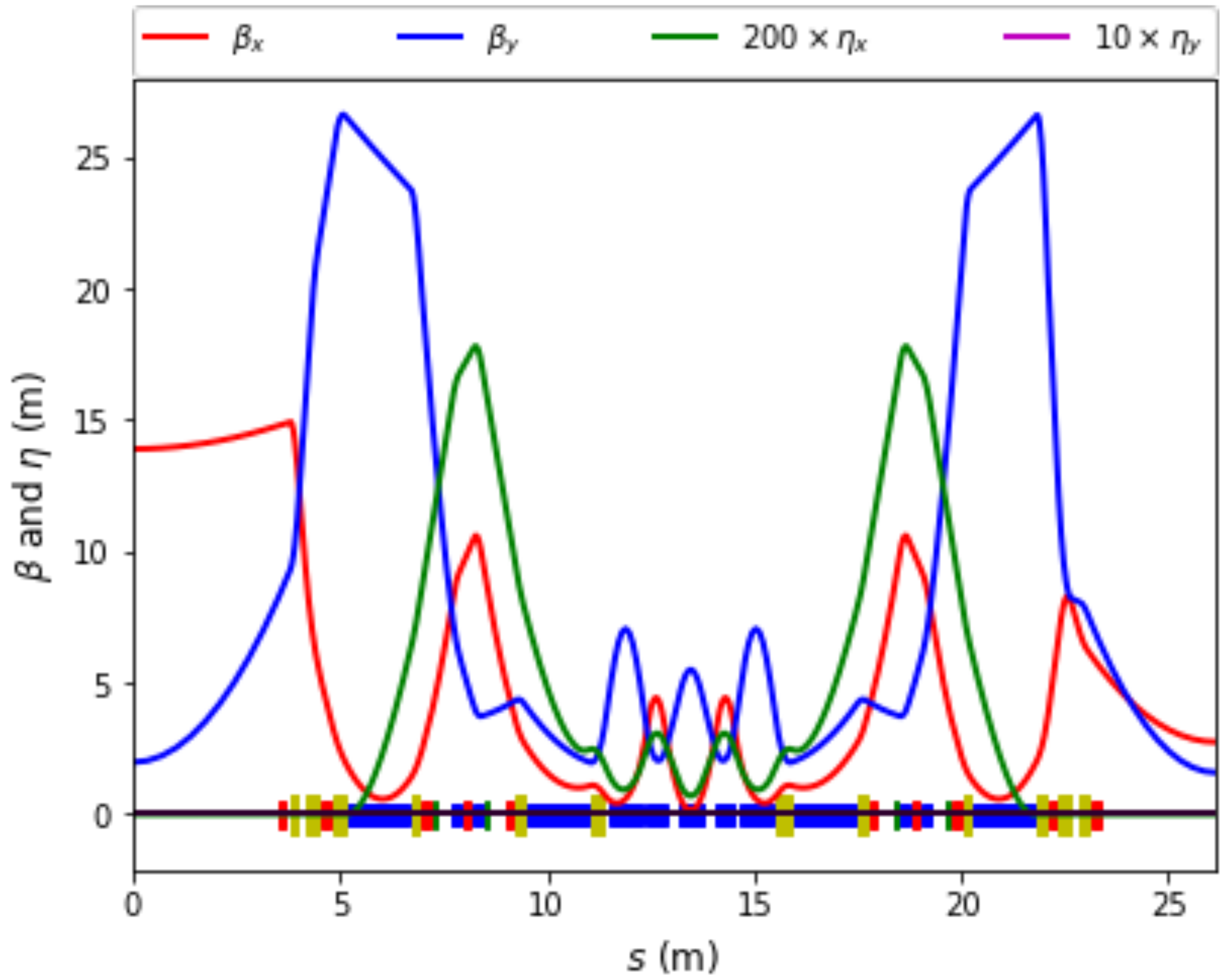


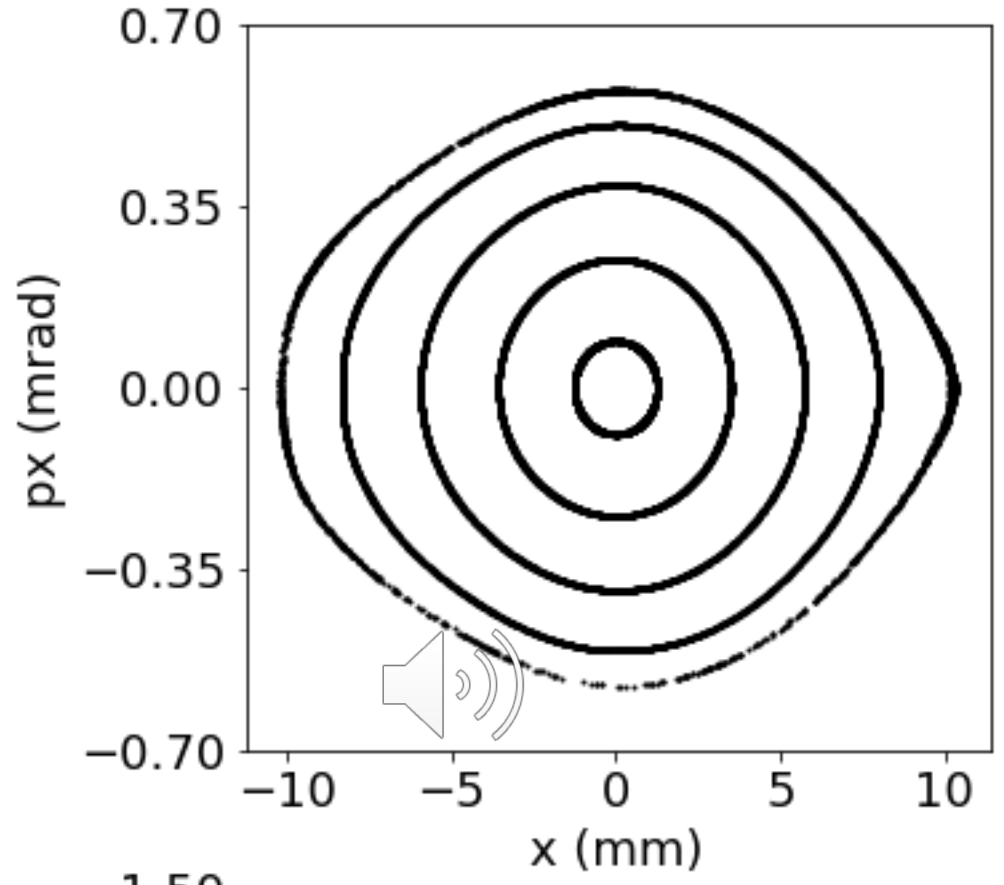
Ratios of harmonics amplitude to the fundamental one

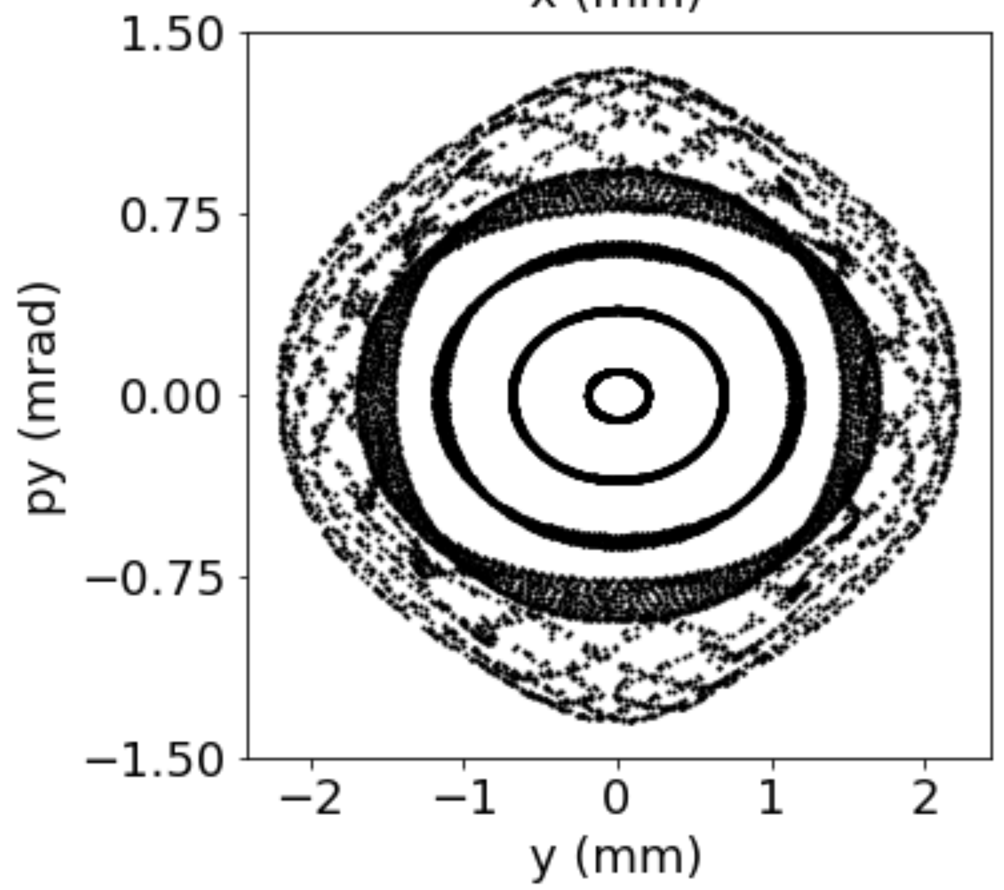
Applied to diffraction-limited Hybrid-M(7)BA

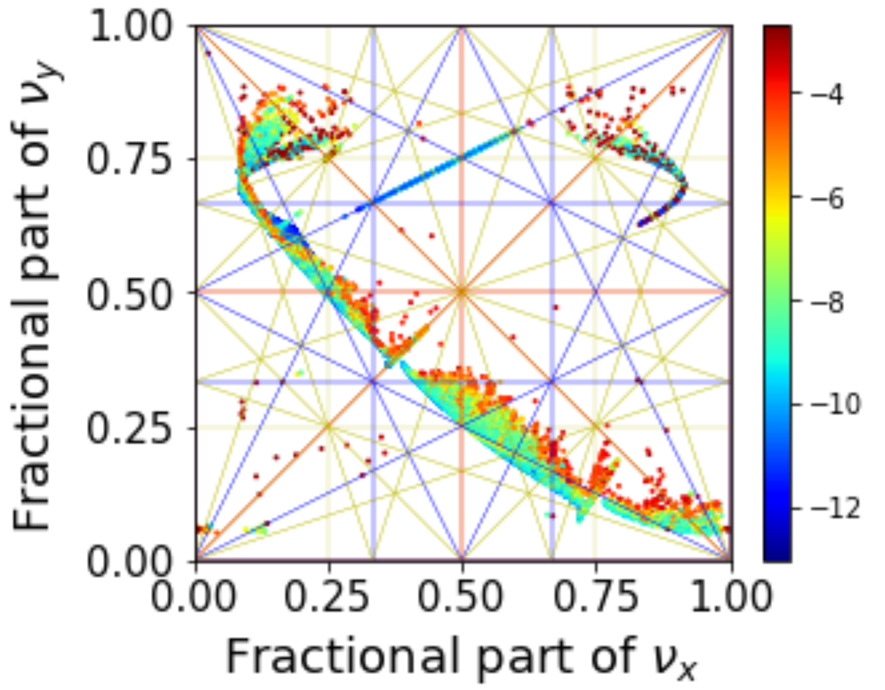
Table II. Main parameters of the test hybrid MBA ring

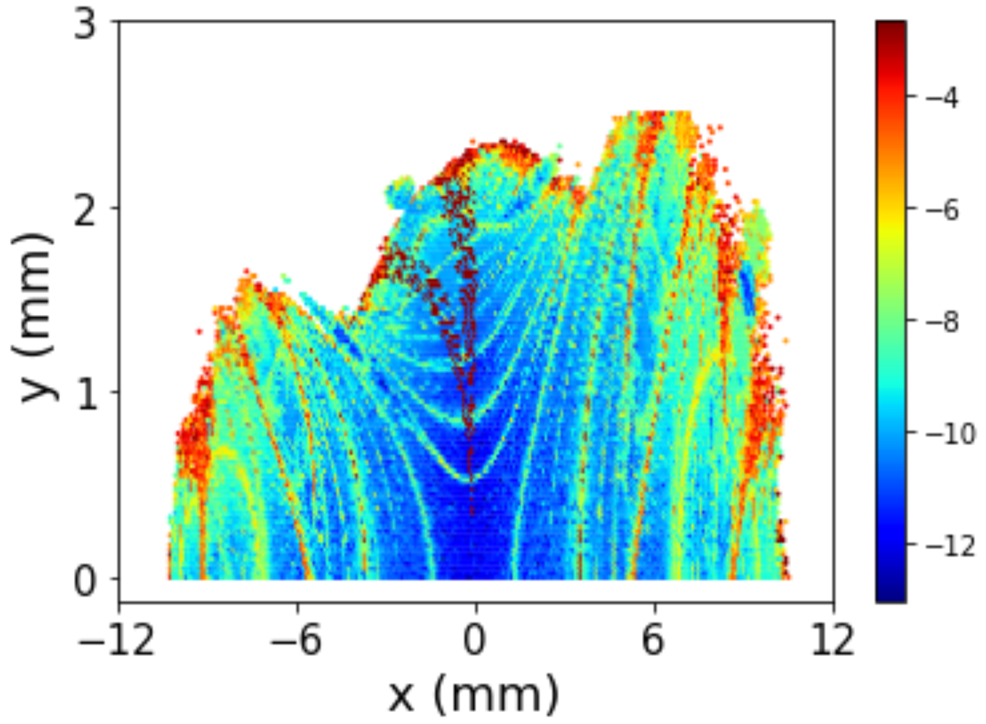
Parameters	Values
Hor. emit. (*pm*)	31
Natural chrom. (x/y)	-125/-108
Tune (x/y)	73.19/28.62
Energy spread	7.1×10^{-4}
Damp. partition (x/y/s)	2.0/1.0/1.0







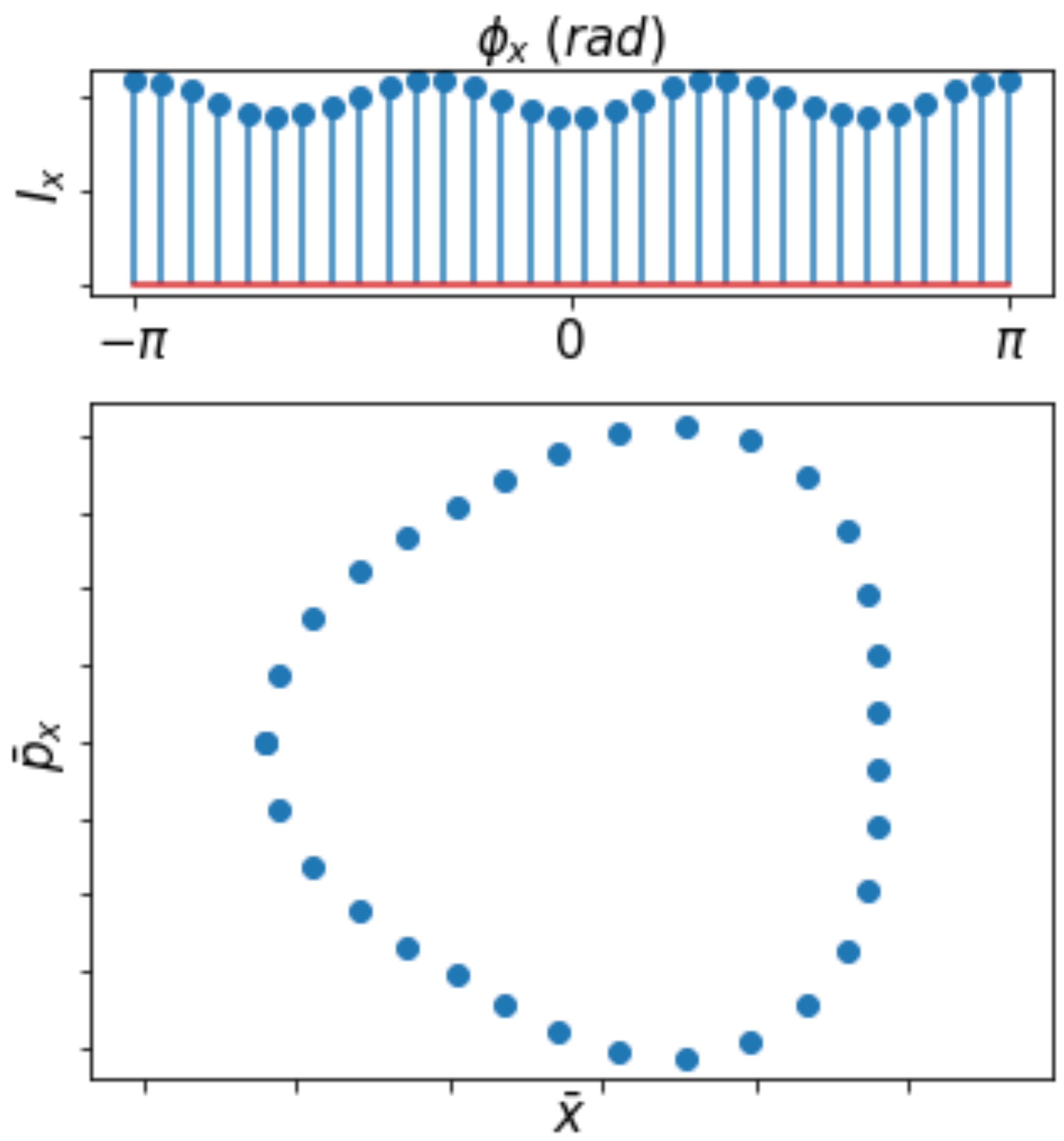




Two-stage optimization: sexts, then octs. Combined as an one-stage is not efficient as the two stage.

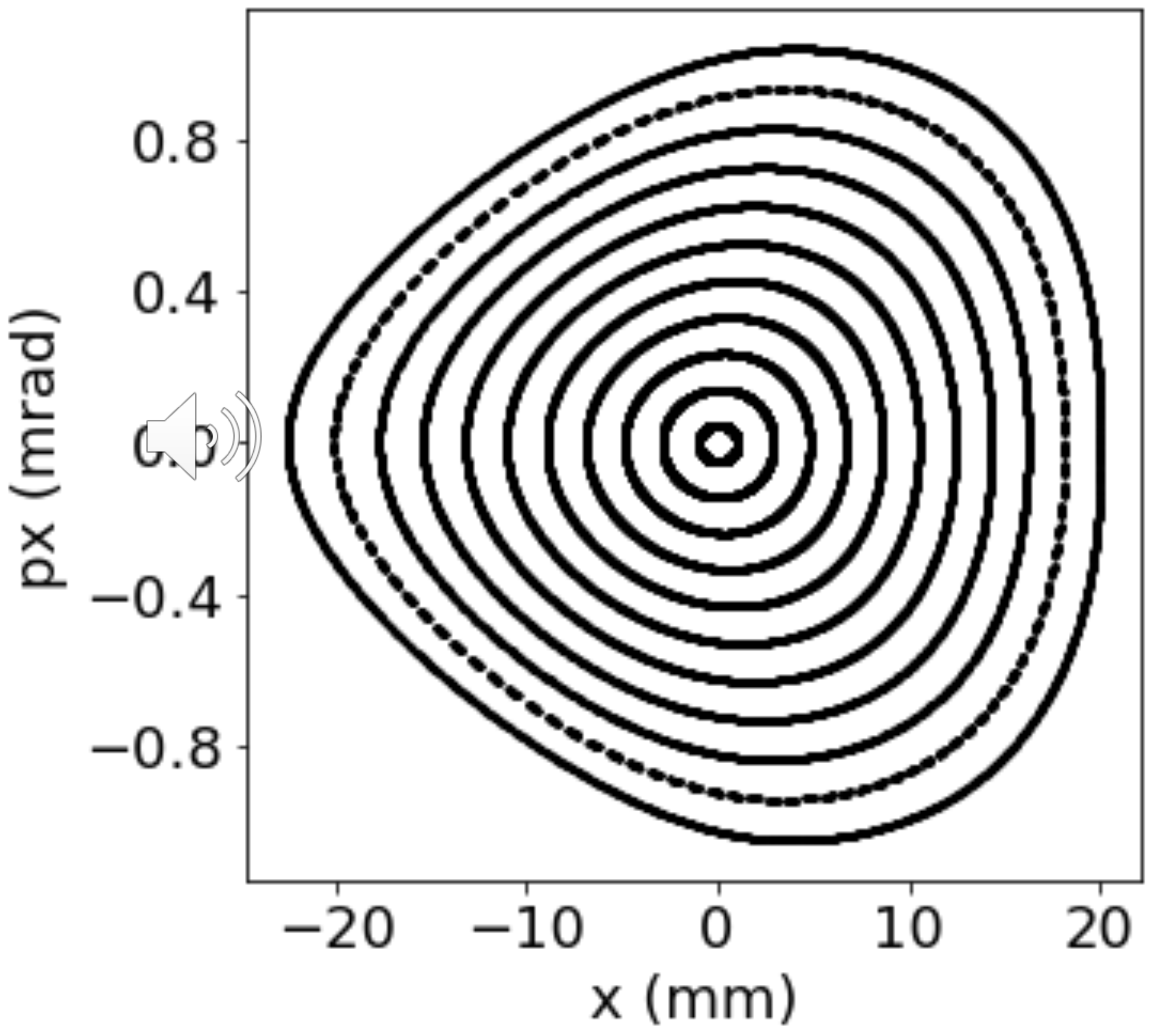
Modulation on action-like quasi-invariant

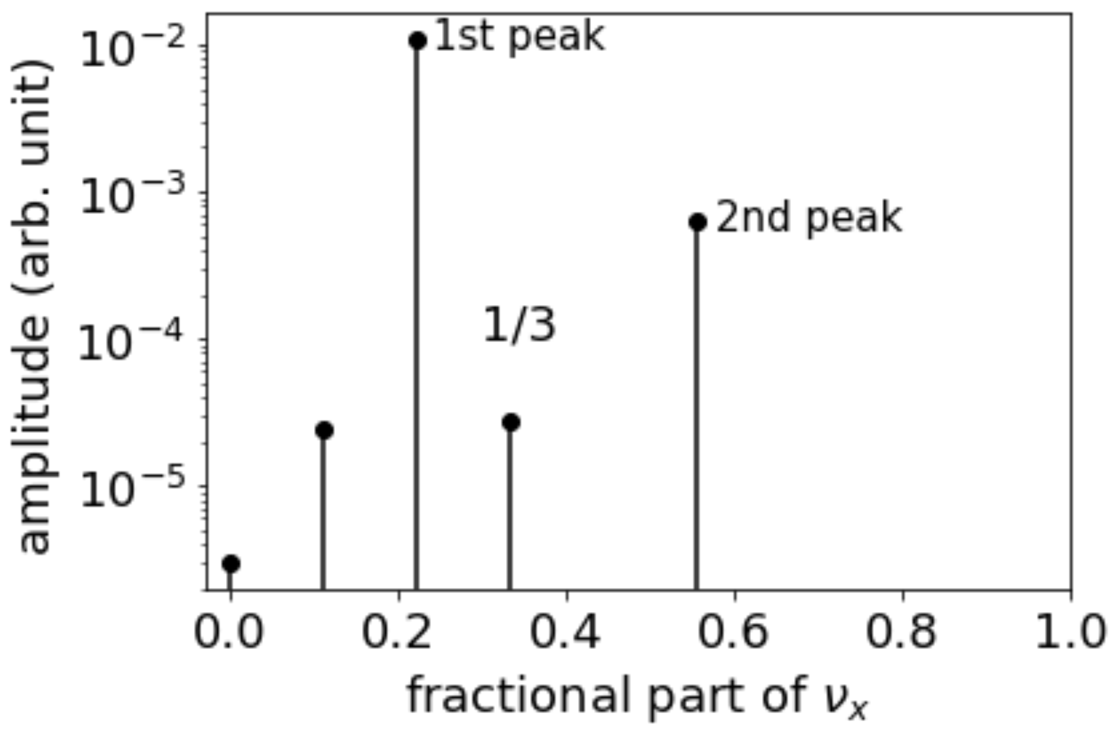
Desired torus



$$J(\phi_x) = J_0 + \Delta J_x(\phi_x)$$
$$= J_0 \left\{ 1 + \delta J_x \sin\left[n \left(\phi_x - \frac{\pi}{2n} \right) \right] \right\},$$

Obtained and Confirmed with simulation

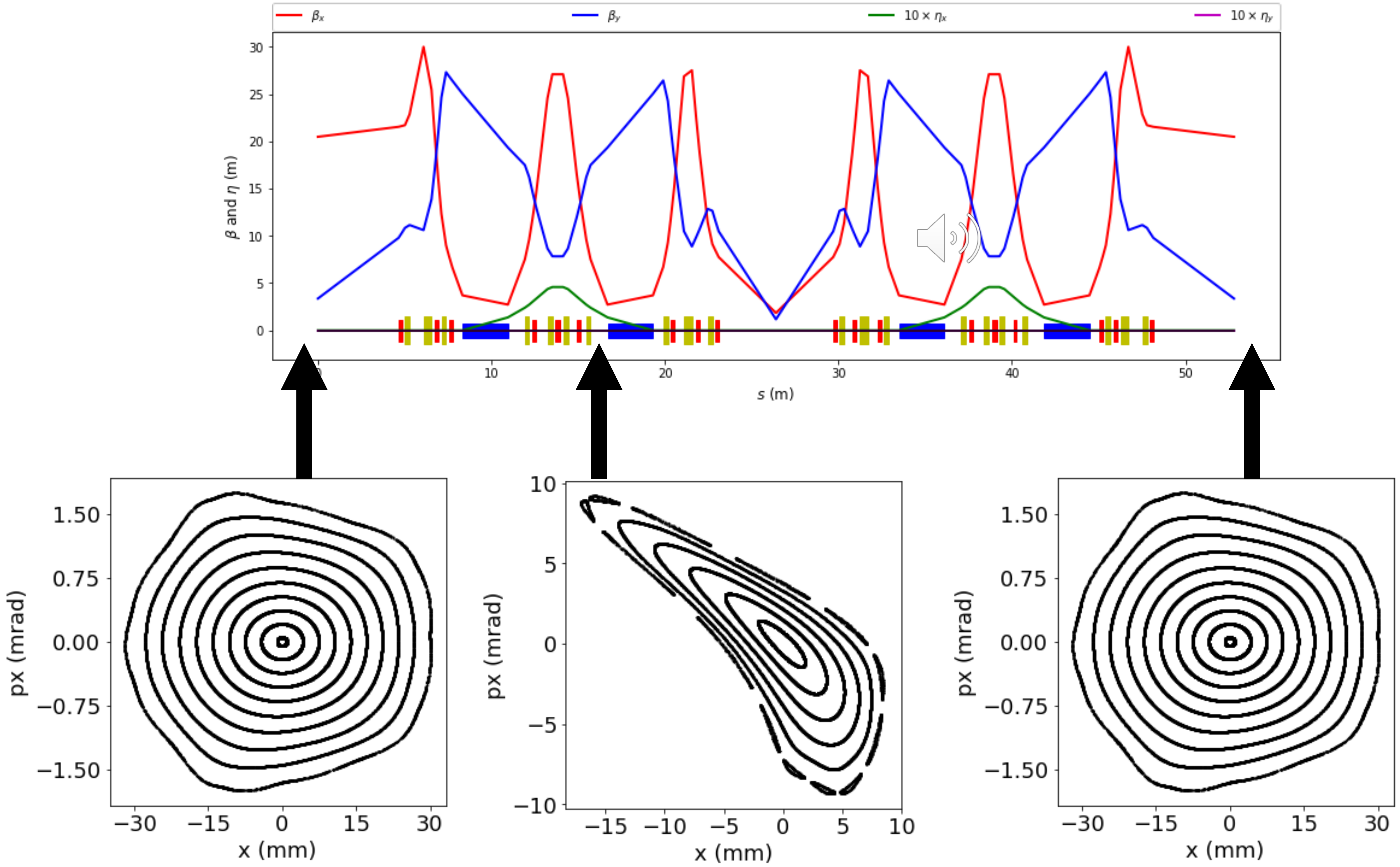




Implementation on DBA lattice

Location dependence of quasi-invariants

- Regular tori only at certain *s* location, but distortion and chaos (bounded) were found in-between



“anastigmat”