END-TO-END RMS ENVELOPE MODEL OF THE ISAC-I LINAC

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ABSTRACT

A full end-to-end simulation of the ISAC-I linear accelerator has been built in the first order envelope code TRANSOPTR. This enables the fast tracking of rms sizes and correlations for a 6-dimensional hyperellipsoidal beam distribution defined around a Frenet-Serret reference particle frame, for which the equations guiding envelope evolution are numerically solved through a model of the machine's electromagnetic potentials. Further, the adopted formalism enables the direct integration of energy gain via time-dependent accelerating potentials, without resorting to transit-time factors.

TRANSOPTR and RMS envelope tracking

The Courant-Snyder Hamiltonian for a relativistic, charged particle is used [1]:

$$H_s = -qA_s - \sqrt{\left(\frac{E-q\Phi}{c}\right)^2 - m^2c^2 - (P_x - qA_x)^2 - (P_y - qA_y)^2}$$

Beams of charged particles are treated using the first and second moments of the distribution [2]:

$$\langle \mathbf{X} \rangle = \frac{1}{N}\sum_{n=1}^{N} \mathbf{X_n} \qquad \text{(centroids)}$$

$$\boldsymbol{\sigma} \equiv \frac{1}{N}\sum_{n=1}^{N} \mathbf{X}\mathbf{X}^T, \qquad \text{(beam matrix)}$$

The evolution of the ensemble forming the beam is:

$$\frac{d\mathbf{X}}{ds} = \mathbf{F}(s)\mathbf{X}$$

the **infinitesimal transfer matrix** is related to the Hamiltonian (and the EM fields):

$$\mathbf{F}(s) = \begin{pmatrix}
\frac{\partial^2 H}{\partial P_x \partial x} & \frac{\partial^2 H}{\partial P_x^2} & \frac{\partial^2 H}{\partial P_x \partial y} & \frac{\partial^2 H}{\partial P_x \partial P_y} & \frac{\partial^2 H}{\partial P_x \partial z} & \frac{\partial^2 H}{\partial P_x \partial P_z} \\
-\frac{\partial^2 H}{\partial x^2} & -\frac{\partial^2 H}{\partial x \partial P_x} & -\frac{\partial^2 H}{\partial x \partial y} & -\frac{\partial^2 H}{\partial x \partial P_y} & -\frac{\partial^2 H}{\partial x \partial z} & -\frac{\partial^2 H}{\partial x \partial P_z} \\
\frac{\partial^2 H}{\partial P_y \partial x} & \frac{\partial^2 H}{\partial P_y \partial P_x} & \frac{\partial^2 H}{\partial P_y \partial y} & \frac{\partial^2 H}{\partial P_y^2} & \frac{\partial^2 H}{\partial P_y \partial z} & \frac{\partial^2 H}{\partial P_y \partial P_z} \\
-\frac{\partial^2 H}{\partial y \partial x} & -\frac{\partial^2 H}{\partial y \partial P_x} & -\frac{\partial^2 H}{\partial y^2} & -\frac{\partial^2 H}{\partial y \partial P_y} & -\frac{\partial^2 H}{\partial y \partial z} & -\frac{\partial^2 H}{\partial y \partial P_z} \\
\frac{\partial^2 H}{\partial P_z \partial x} & \frac{\partial^2 H}{\partial P_z \partial P_x} & \frac{\partial^2 H}{\partial P_z \partial y} & \frac{\partial^2 H}{\partial P_z \partial P_y} & \frac{\partial^2 H}{\partial P_z \partial z} & \frac{\partial^2 H}{\partial P_z^2} \\
-\frac{\partial^2 H}{\partial z \partial x} & -\frac{\partial^2 H}{\partial z \partial P_x} & -\frac{\partial^2 H}{\partial z \partial y} & -\frac{\partial^2 H}{\partial z \partial P_y} & -\frac{\partial^2 H}{\partial z^2} & -\frac{\partial^2 H}{\partial z \partial P_z}
\end{pmatrix}$$

A point to point transformation for an infinitesimal step ds is then:

$$\mathcal{M}_{ds} = \mathbf{I} - \mathbf{F}ds. \quad \text{(I = identity matrix)}$$

TRANSOPTR computes the evolution of the beam envelope by numerically solving the **envelope equation [3]**:

$$\frac{d\boldsymbol{\sigma}}{ds} = \mathbf{F}(s)\boldsymbol{\sigma} + \boldsymbol{\sigma}\mathbf{F}(s)^T \qquad \text{(T = transposition)}$$

Which produces the s-evolution of the beam matrix.

2. Radiofrequency Quadrupole Linac

$$\mathbf{F} = \begin{pmatrix}
0 & \frac{1}{P} & 0 & 0 & 0 & 0 \\
-\mathcal{A}_+ & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{P} & 0 & 0 \\
0 & 0 & -\mathcal{A}_- & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mathcal{B} & \frac{1}{\gamma^2 P} \\
0 & 0 & 0 & 0 & -\mathcal{C} & -\mathcal{B}
\end{pmatrix}$$

$$\mathcal{A}_\pm = \frac{qV_0 \sin(\omega t_0 + \phi)\left(k^2 A_{10}\cos\psi \pm 4A_{01}\right)}{4\beta c},$$

$$\mathcal{B} = \frac{qV_0 A_{10}(k \sin\psi \sin(\omega t_0+\phi) + (\omega/(\beta c))\cos\psi\cos(\omega t_0+\phi))}{2\beta^2\gamma^3 mc^2},$$

$$\mathcal{C} = \frac{qV_0(\omega/(\beta c))^2 A_{10}\cos\psi (qV_0A_{10})/(\beta^2\gamma^3 mc^2)}{4\beta c} \times \frac{(\cos\psi\cos^2(\omega t_0+\phi) - 2\sin(\omega t_0+\phi))}{4\beta c}.$$

Input: RFQ parameters (a,m,k) [4]

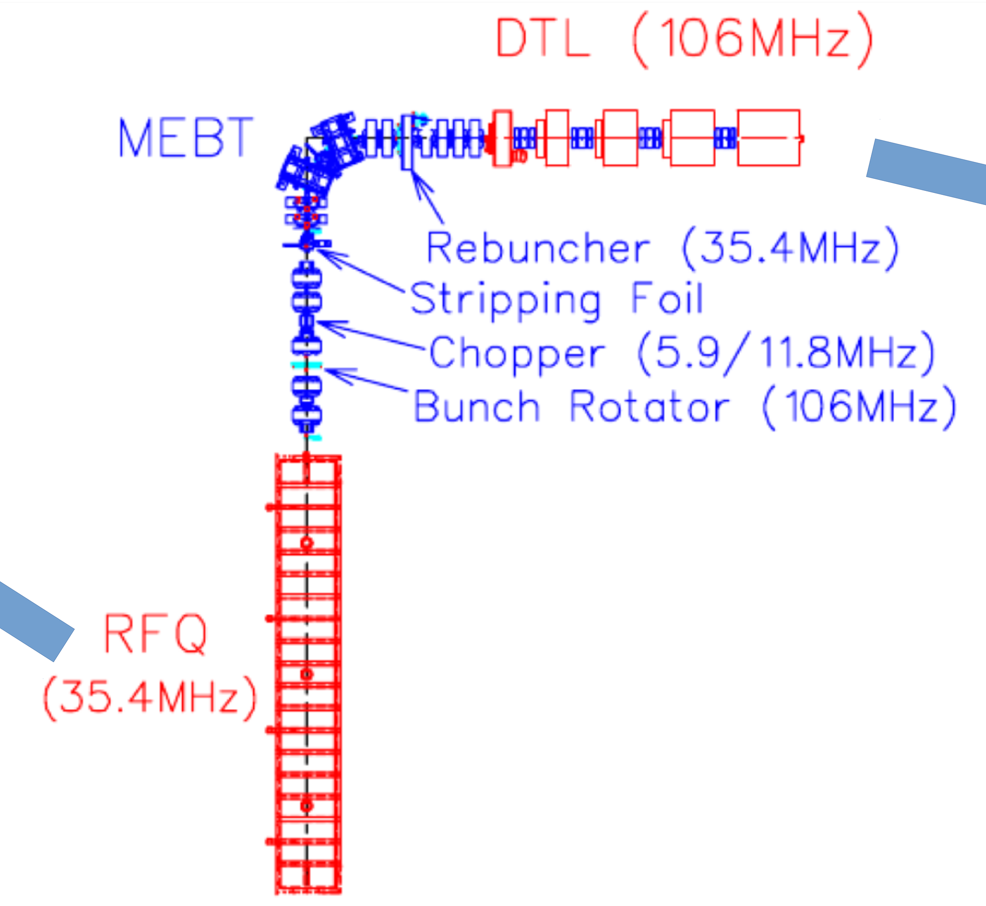

Figure 1: Overview of the ISAC-I linac

TRANSOPTR is fast: subsecond execution time.
All components of the ISAC-I linac are now in the code.

3. IH-DTL

$$\mathbf{F}(s) = \begin{pmatrix}
0 & \frac{1}{P_0} & 0 & 0 & 0 & 0 \\
\mathcal{A}(s) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{P_0} & 0 & 0 \\
0 & 0 & \mathcal{A}(s) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\beta'}{\beta} & \frac{1}{\gamma^2 P_0} \\
0 & 0 & 0 & 0 & \mathcal{B}(s) & -\frac{\beta'}{\beta}
\end{pmatrix} \qquad (13)$$

with:

$$\mathcal{A}(s) = -\frac{q}{2\beta c}\left(\mathcal{E}'(s)C - \mathcal{E}(s)S\frac{\omega\beta}{c}\right)$$

$$\mathcal{B}(s) = \frac{q\mathcal{E}(s)\omega S}{\beta^2 c^2}.$$

Input: $\mathcal{E}(s)$, on-axis E-field intensity (bead pull or simulation) [5]

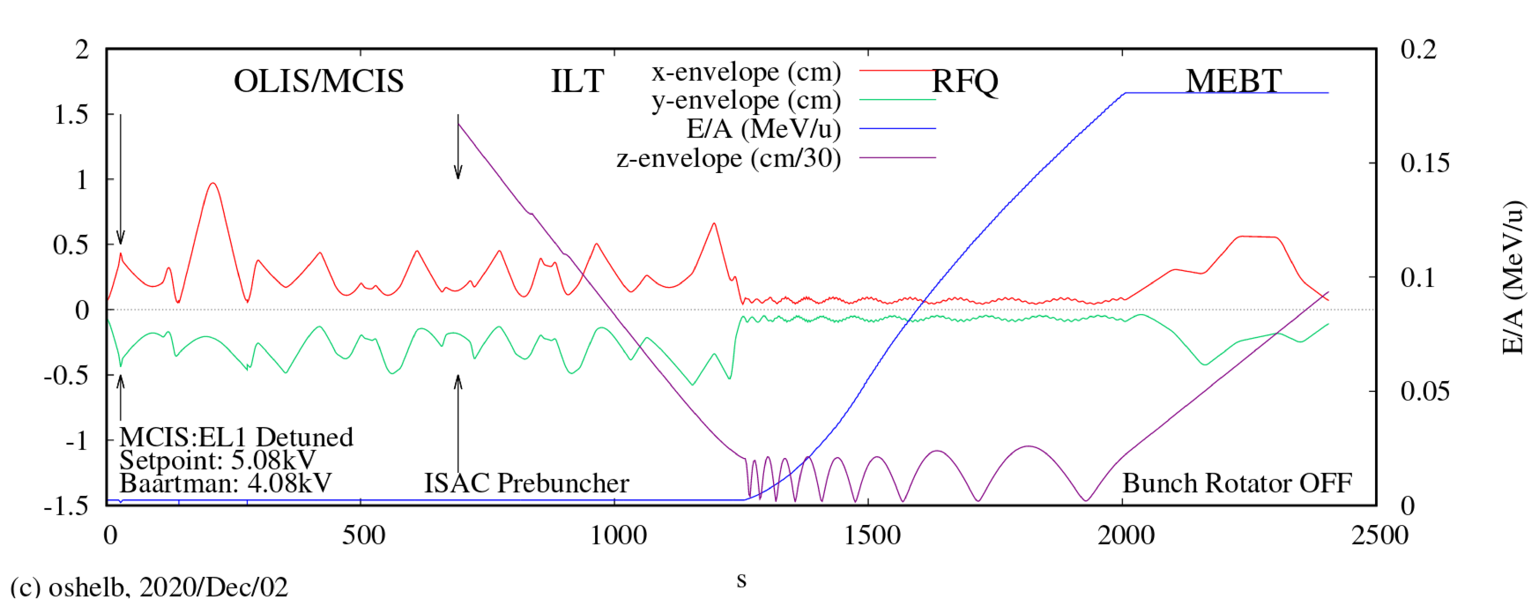

Figure 2: 2rms envelopes through the ISAC-RFQ

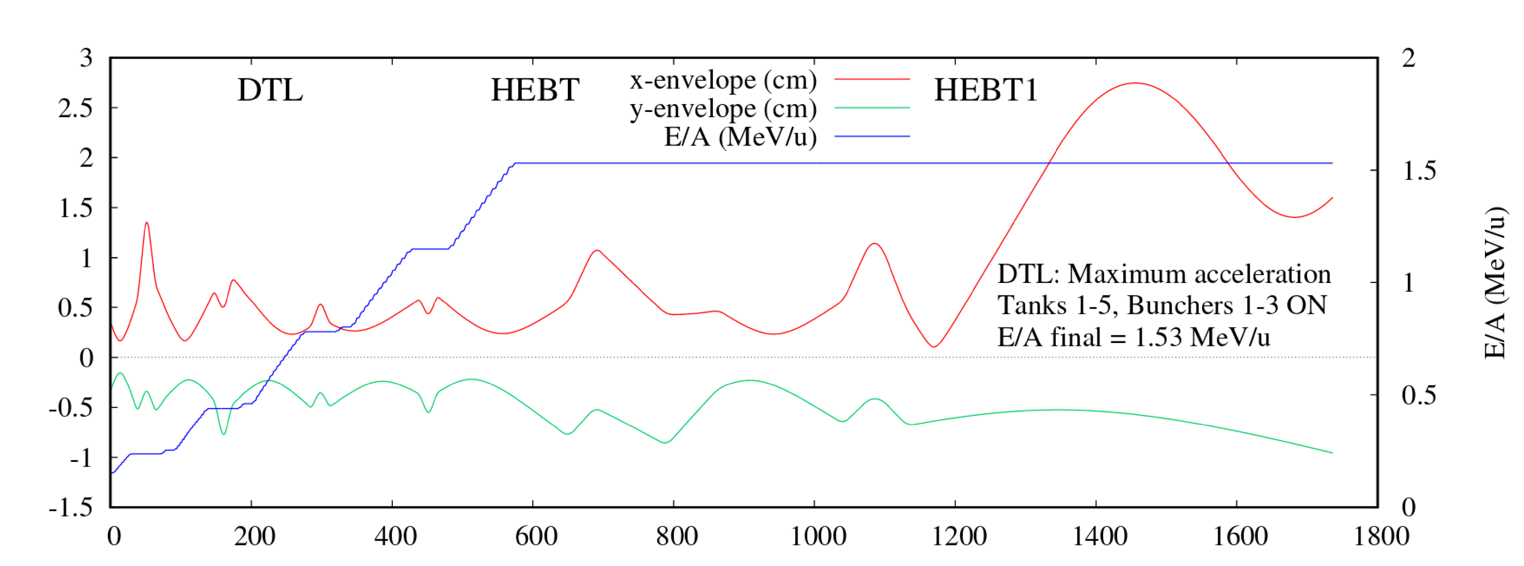

Figure 3: 2rms envelopes through the ISAC-DTL

4. No TTF

To first order in TRANSOPTR, the potential directly modifies the canonical energy [3,4,5]:

$$c^2 P \Delta P = (E - q\Phi)\Delta E$$

Energy gain is directly integrated from the field:

$$E(s) = E_0 + qV_s \int_0^l \mathcal{E}(s)\cos(\omega t(s) + \phi_0)ds,$$

This allows for a straightforward integration of the reference particle energy, without resorting to transit time factor approximations.

5. Conclusion

The envelope code TRANSOPTR has now been extended to represent the entire ISAC linear accelerator. This notably includes a novel RFQ simulation capability, only requiring the vane modulation parameters as input. In addition, the code possesses an axially symmetric linac feature.

The model is now being used at TRIUMF-ISAC for investigations and studies of the linac's tune and performance.

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