Motivation

• Danilov distribution: self-consistent assuming linear fields and coasting beam

• We think we can inject (paint) Danilov-like distribution in the SNS ring*

• How to quantify level of success of painting scheme?
  • Ideally: $\varepsilon_{4D} = \varepsilon_1 \varepsilon_2 = \sqrt{|\Sigma|} = 0$

$$
\varepsilon_1 = \frac{1}{2} \sqrt{-tr[(\Sigma U)^2] + \sqrt{tr[(\Sigma U)^2]} - 16|\Sigma|}
$$

$$
\varepsilon_2 = \frac{1}{2} \sqrt{-tr[(\Sigma U)^2] - \sqrt{tr[(\Sigma U)^2]} - 16|\Sigma|}
$$

$$
\Sigma = 
\begin{bmatrix}
\langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\
\langle xx' \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle \\
\langle xy \rangle & \langle x'y \rangle & \langle y^2 \rangle & \langle yy' \rangle \\
\langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'^2 \rangle 
\end{bmatrix}
$$

Available resources

- 5 wire-scanners available in Ring-Target-Beam-Transport (RTBT) line
Quadrupole scan technique

- Well-established method*

- Measure $\langle x^2 \rangle$, $\langle y^2 \rangle$, $\langle xy \rangle$ with wire-scanner at $B$

- Repeat $N$ times with different optics between $A$ and $B$ to give $3N$ equations

- Fit 10 moments at $A$ assuming linear transport

- Possible to use multiple wire-scanners

- 180 degree coverage in phase advances is optimal

\[
\Sigma_B = M \Sigma_A M^T
\]

Reconstruction point \[\xrightarrow{s} \]
Wire-scanner

\[
\begin{align*}
\langle x^2 \rangle_B &= M_{11}^2 \langle x^2 \rangle_A + M_{12}^2 \langle x^2 \rangle_A + 2M_{11}M_{12} \langle xx' \rangle_A \\
\langle y^2 \rangle_B &= M_{33}^2 \langle y^2 \rangle_A + M_{34}^2 \langle y^2 \rangle_A + 2M_{33}M_{34} \langle yy' \rangle_A \\
\langle xy \rangle_B &= M_{13}M_{33} \langle xy \rangle_A + M_{14}M_{33} \langle x'y \rangle_A + M_{11}M_{34} \langle xy' \rangle_A + M_{12}M_{34} \langle x'y' \rangle_A
\end{align*}
\]

Example of simulated scan in PyORBIT

- Launch beam from start of RTBT (perfect Danilov distribution matched to design optics)
  - Scan $x$ and $y$ phases simultaneously at WS24
    - Constrain beta function along transport line to $< 40$ m/rad
    - Enforce production values of beta functions on the target

![Graphs showing phase and position data](image)
Error analysis

- Add random errors and track envelope: repeat 200 trials
- Most errors are small; largest is from mismatched beam at reconstruction point
- Rough estimates need to be refined
- Other sources of error to consider (energy spread, fringe fields, wire-scanner resolution, etc.)

\[
\begin{align*}
\epsilon_1 &= 0.57 \pm 0.63 \\
\epsilon_2 &= 40.66 \pm 1.04 \\
\epsilon_x &= 21.04 \pm 0.75 \\
\epsilon_y &= 20.31 \pm 0.55 \\
\end{align*}
\]

Sim [mm mrad]  
- $\epsilon_1 = 0.0$  
- $\epsilon_2 = 40.0$  
- $\epsilon_x = 20.0$  
- $\epsilon_y = 20.0$
Error analysis

- Plot obtained by tracking envelope with space charge — no other errors
- Near SNS intensities, the smaller intrinsic emittance stays near zero

Max SNS intensity is \(1.5 \times 10^{14}\)
Outlook

- Measurement seems feasible to perform in the SNS
  - Diagnostics and optics sufficient for reconstruction
  - Initial simulations including several sources of error lead to acceptable accuracy
- Work in progress
  - Refine error estimates
  - Make predictions for realistic painted beam and compare with production beam
  - Carry out measurement experimentally