Simulation of 4D emittance measurement at the Spallation Neutron Source

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*J. A. Holmes, T. Gorlov, N. J. Evans, M. Plum, and S. Cousineau, *Injection of a self-consistent beam with linear space charge force into a ring*, Phys. Rev. Accel. Beams 21, 124403 (2018).

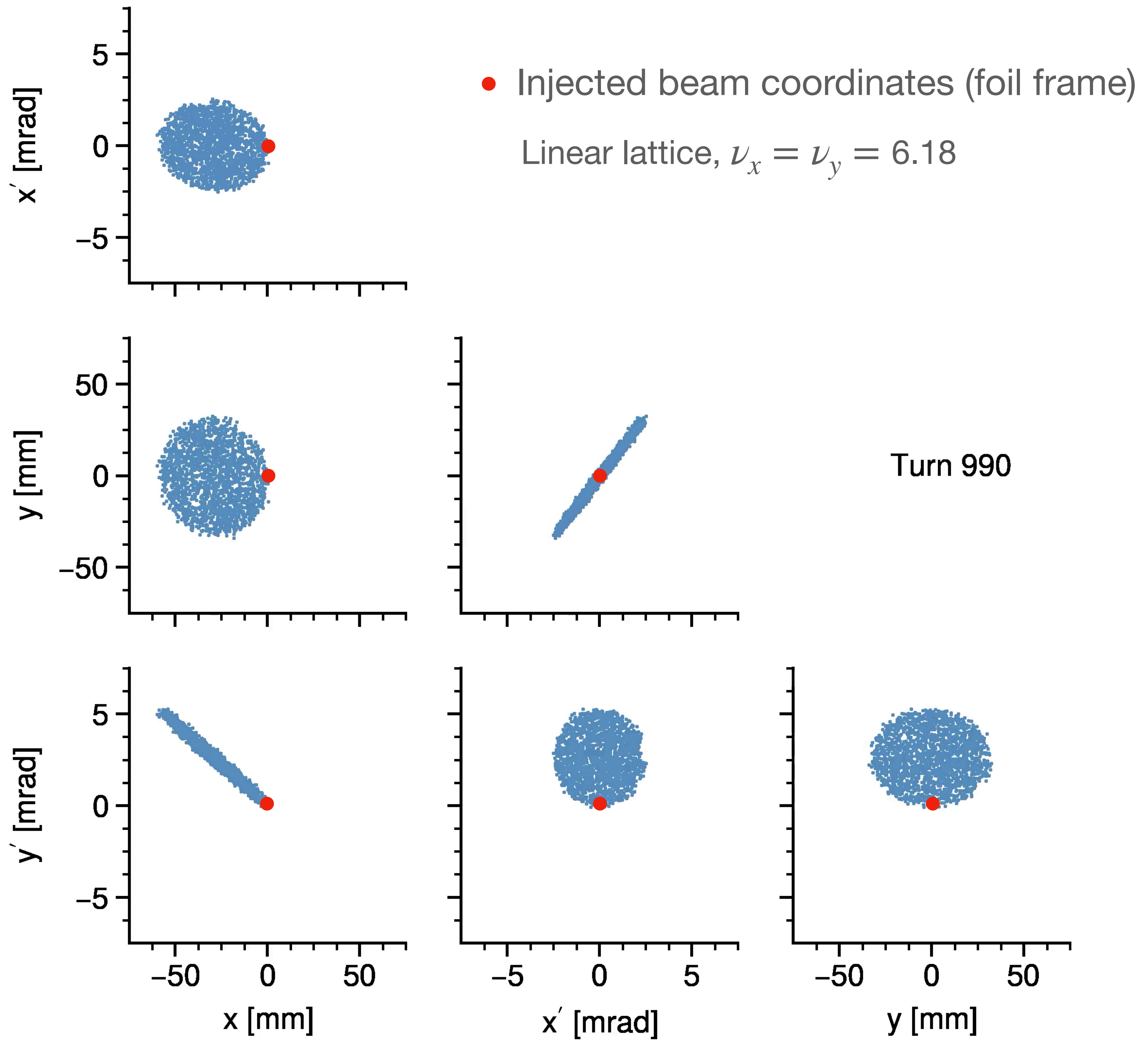
Motivation

- Danilov distribution: self-consistent assuming linear fields and coasting beam
- We think we can inject (paint) Danilov-like distribution in the SNS ring*
- How to quantify level of success of painting scheme?
  - Ideally: $\varepsilon_{4D} = \varepsilon_1 \varepsilon_2 = \sqrt{|\Sigma|} = 0$

$$\varepsilon_1 = \frac{1}{2}\sqrt{-tr\left[(\Sigma U)^2\right] + \sqrt{tr\left[(\Sigma U)^2\right] - 16|\Sigma|}}$$

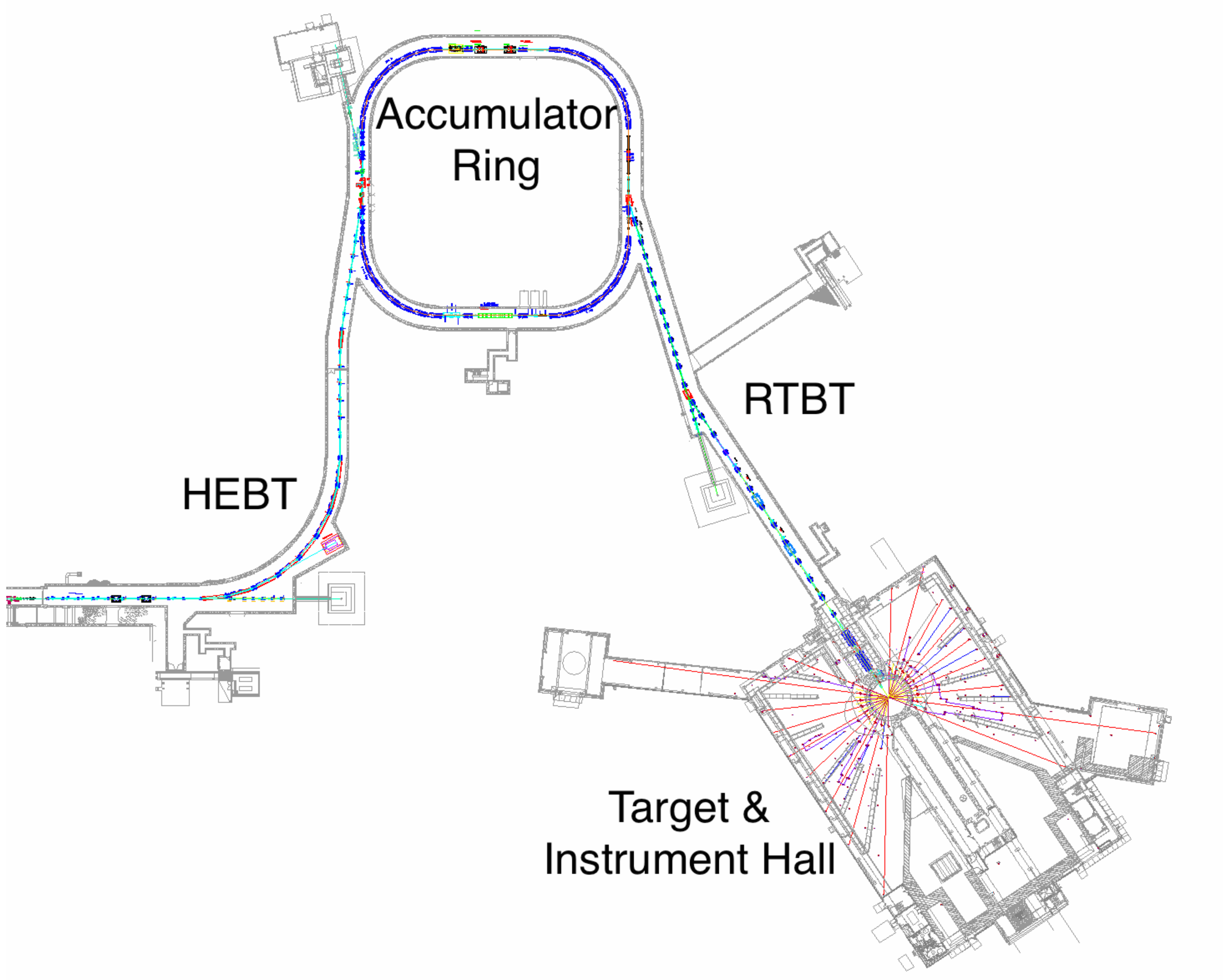
$$\varepsilon_2 = \frac{1}{2}\sqrt{-tr\left[(\Sigma U)^2\right] - \sqrt{tr\left[(\Sigma U)^2\right] - 16|\Sigma|}}$$

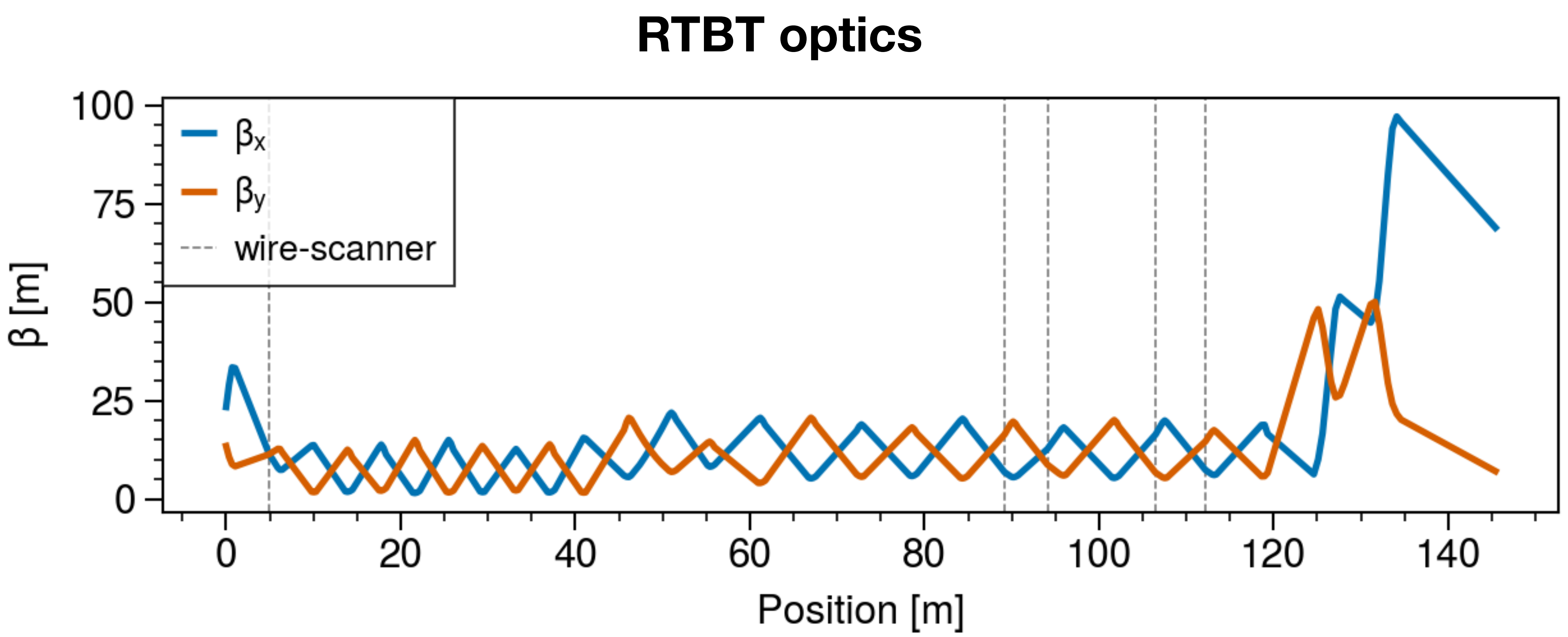
$$\Sigma = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle \\ \langle xy \rangle & \langle x'y \rangle & \langle y^2 \rangle & \langle yy' \rangle \\ \langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'^2 \rangle \end{bmatrix}$$



Available resources

- 5 wire-scanners available in Ring-Target-Beam-Transport (RTBT) line



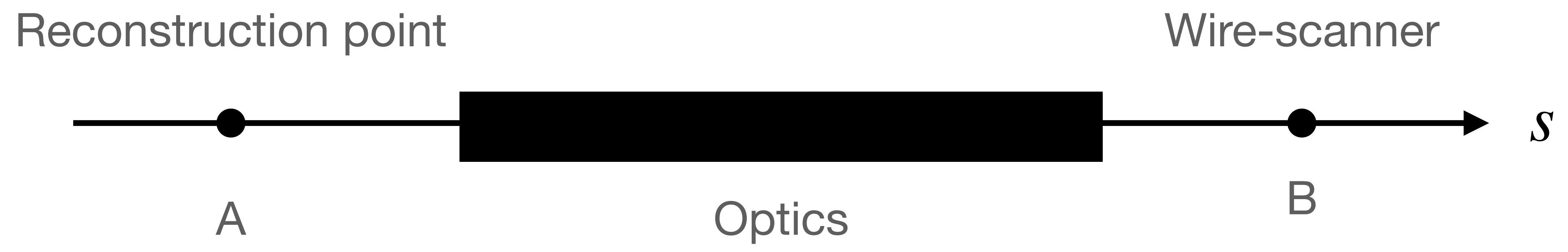


*E. Prat and M. Aiba, *Four-dimensional transverse beam matrix measurement using the multiple- quadrupole scan technique*, Phys. Rev. ST Accel. Beams 17, 052801 (2014).

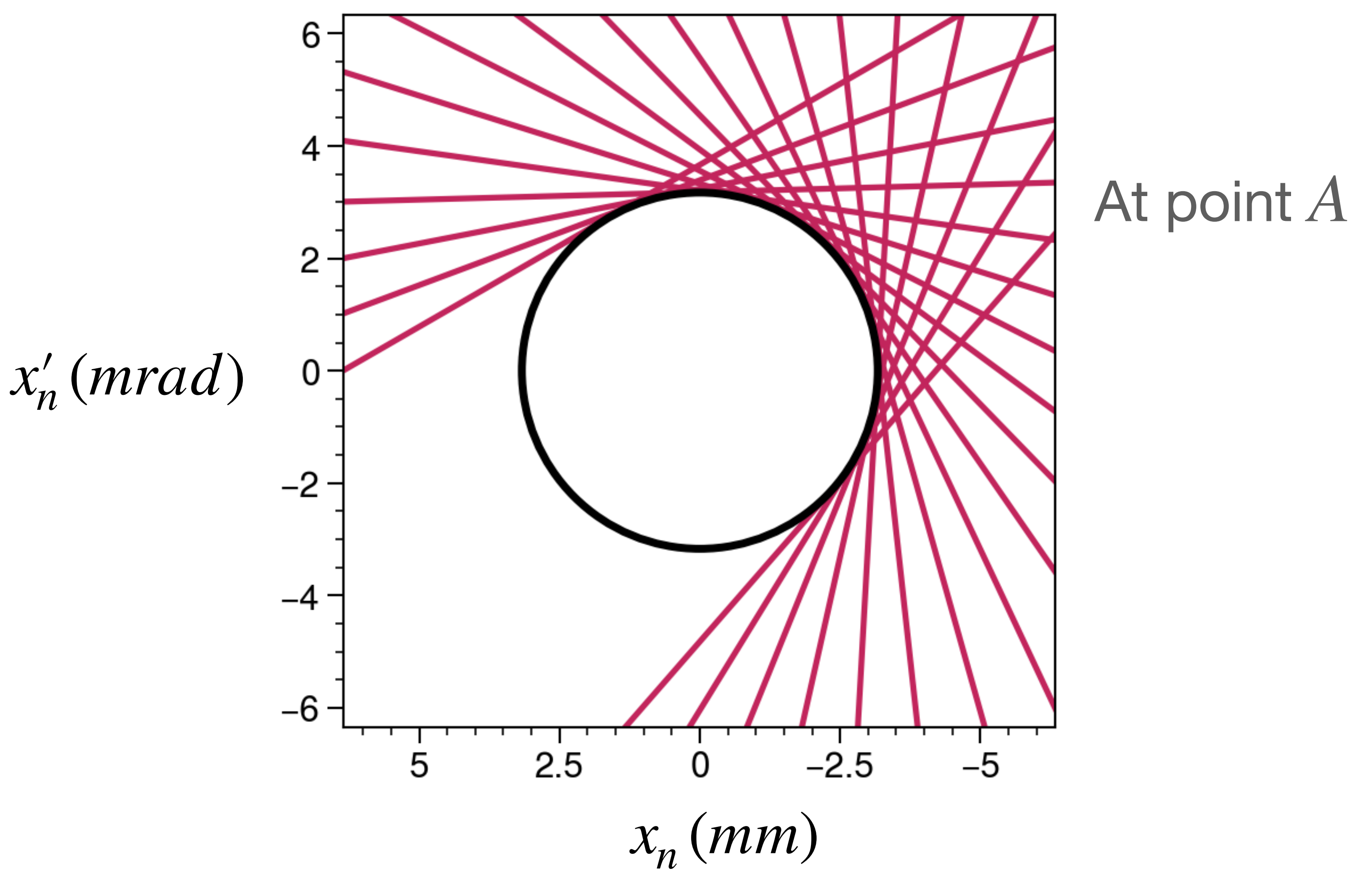
Quadrupole scan technique

- Well-established method*
- Measure $\langle x^2 \rangle$, $\langle y^2 \rangle$, $\langle xy \rangle$ with wire-scanner at B
- Repeat N times with different optics between A and B to give $3N$ equations
- Fit 10 moments at A assuming linear transport
- Possible to use multiple wire-scanners
- 180 degree coverage in phase advances is optimal

$$\Sigma_B = M \, \Sigma_A \, M^T$$

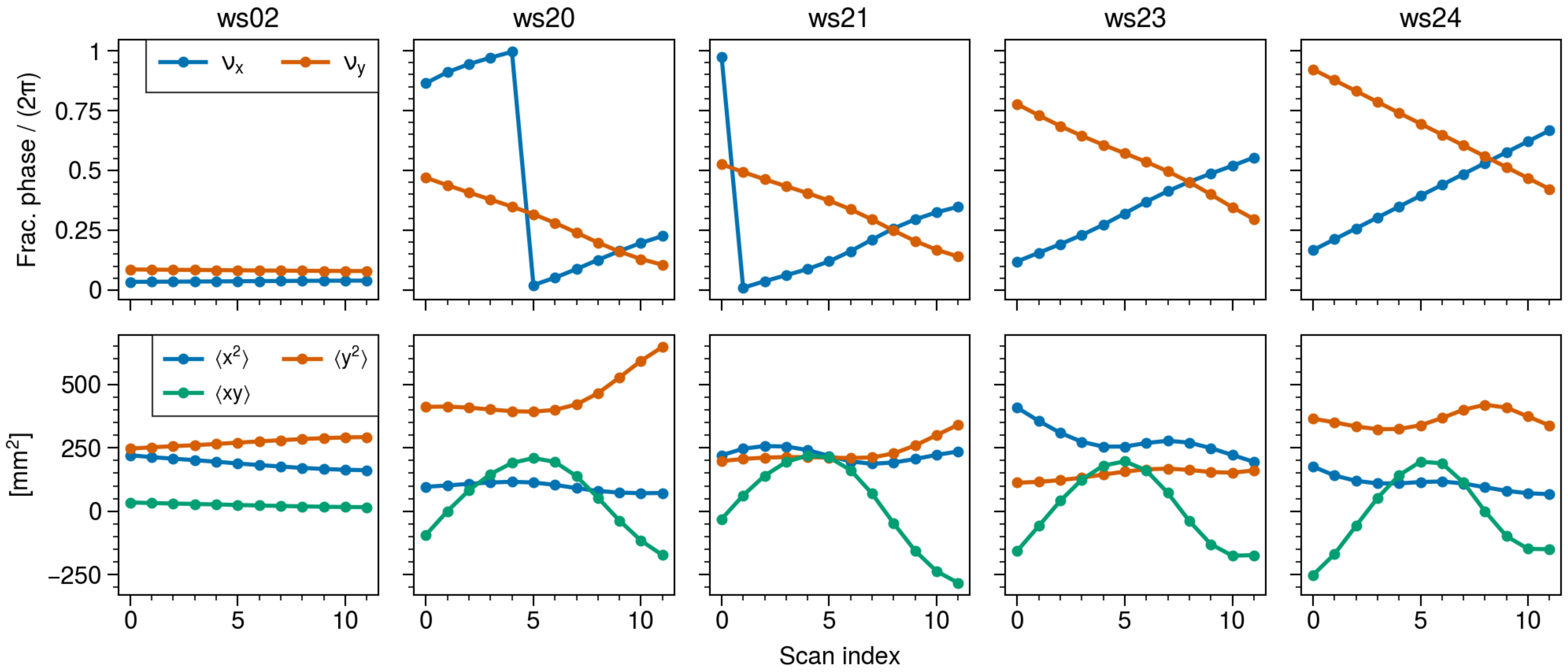


$$\langle x^2 \rangle_B = M_{11}^2 \langle x^2 \rangle_A + M_{12}^2 \langle x'^2 \rangle_A + 2 M_{11} M_{12} \langle x x' \rangle_A$$
$$\langle y^2 \rangle_B = M_{33}^2 \langle y^2 \rangle_A + M_{34}^2 \langle y'^2 \rangle_A + 2 M_{33} M_{34} \langle y y' \rangle_A$$
$$\langle xy \rangle_B = M_{11} M_{33} \langle xy \rangle_A + M_{12} M_{33} \langle y x' \rangle_A + M_{11} M_{34} \langle x y' \rangle_A + M_{12} M_{34} \langle x' y' \rangle_A$$

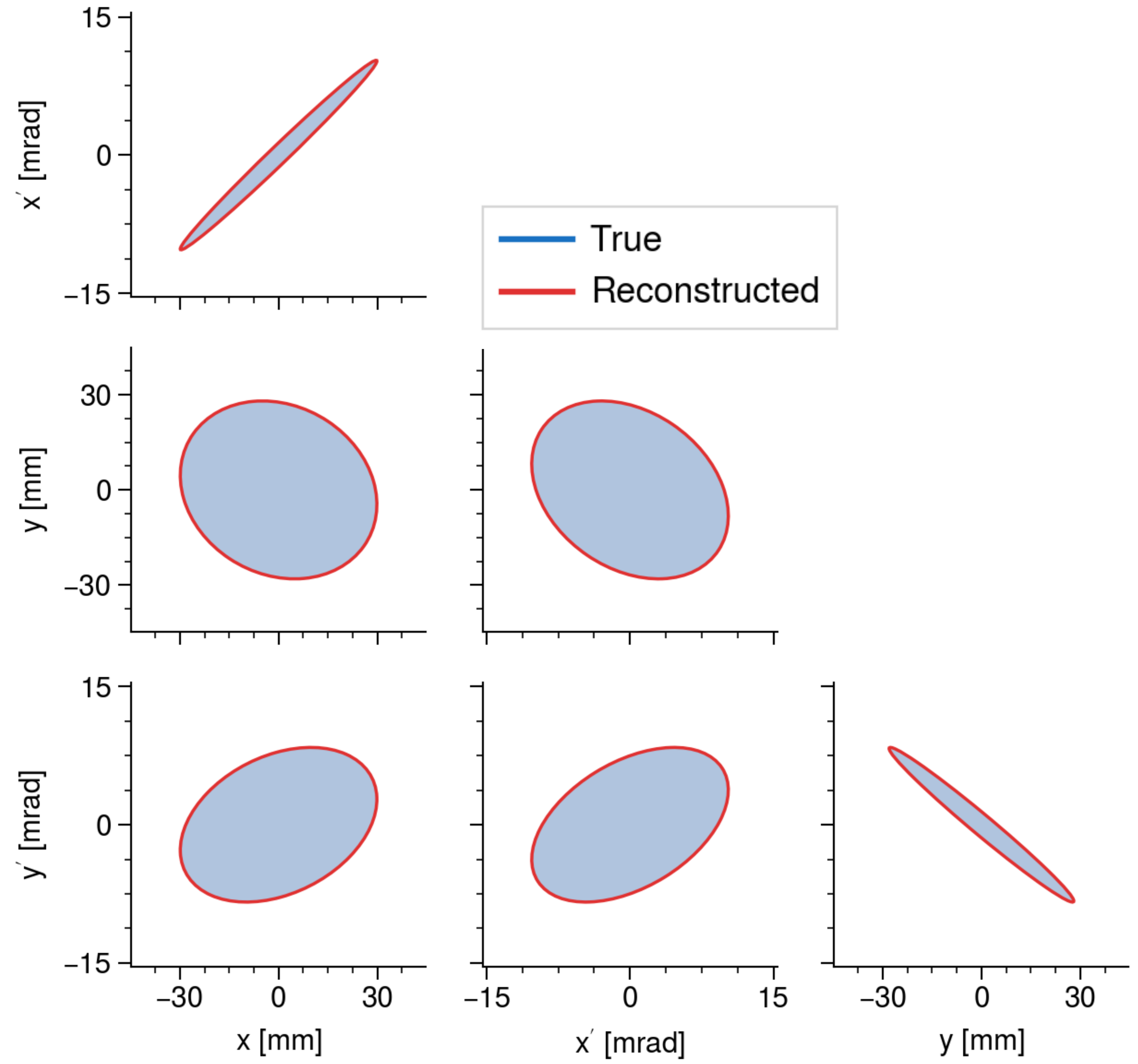


Example of simulated scan in PyORBIT

- Launch beam from start of RTBT (perfect Danilov distribution matched to design optics)
- Scan x and y phases simultaneously at WS24
  - Constrain beta function along transport line to < 40 m/rad
  - Enforce production values of beta functions on the target

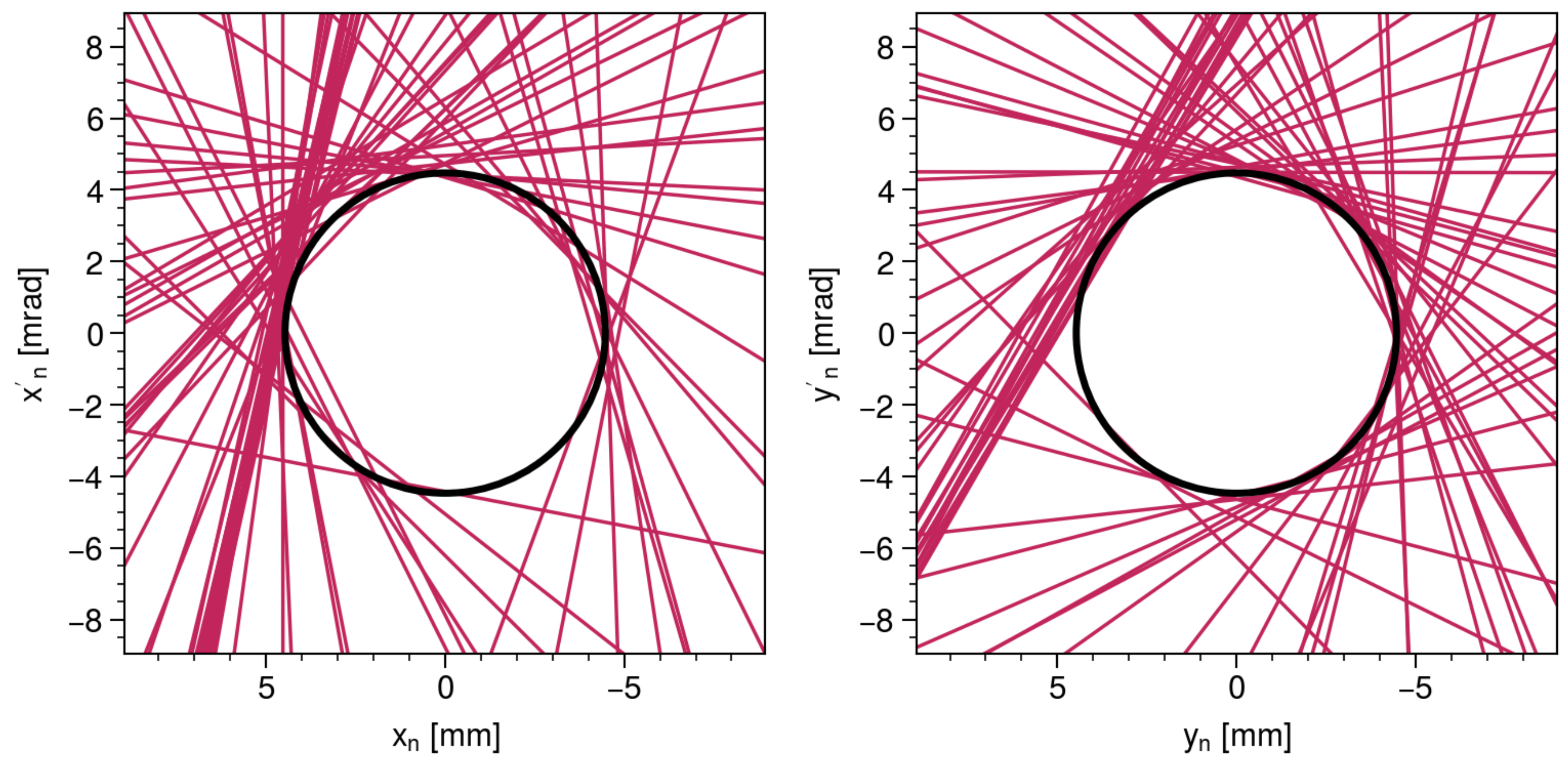


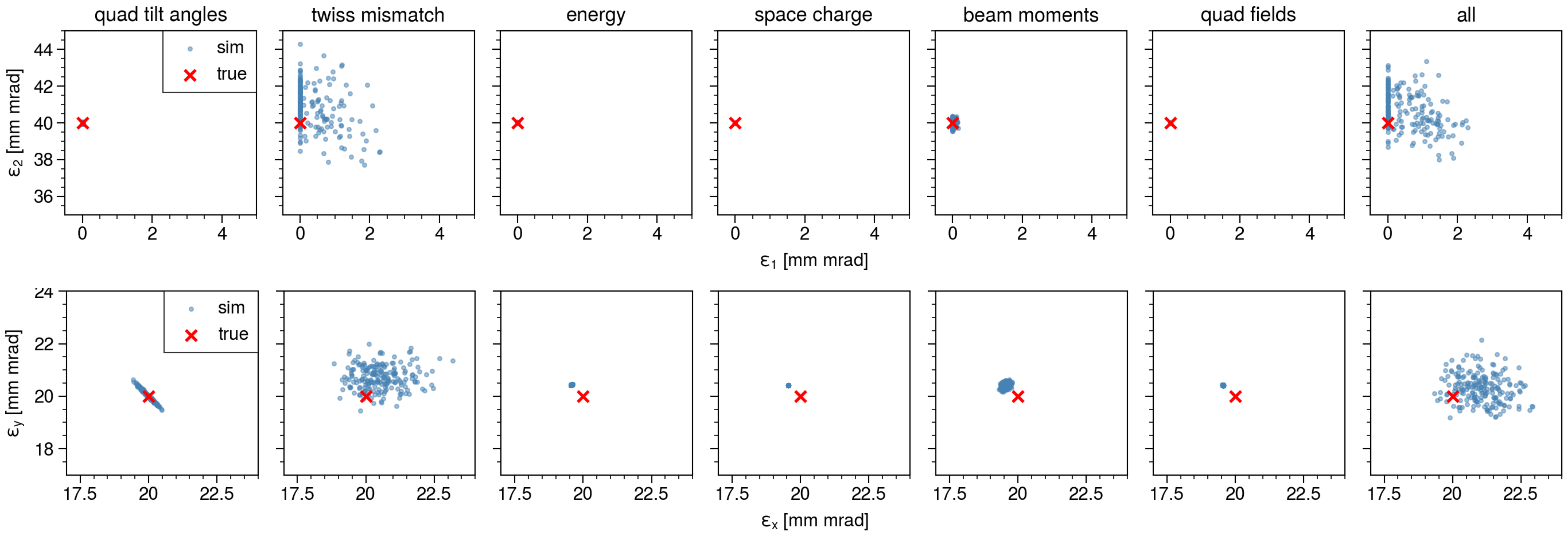
Perfect agreement without including any sources of error



Error analysis

- Add random errors and track envelope: repeat 200 trials
- Most errors are small; largest is from mismatched beam at reconstruction point
- Rough estimates need to be refined
- Other sources of error to consider (energy spread, fringe fields, wire-scanner resolution, etc.)

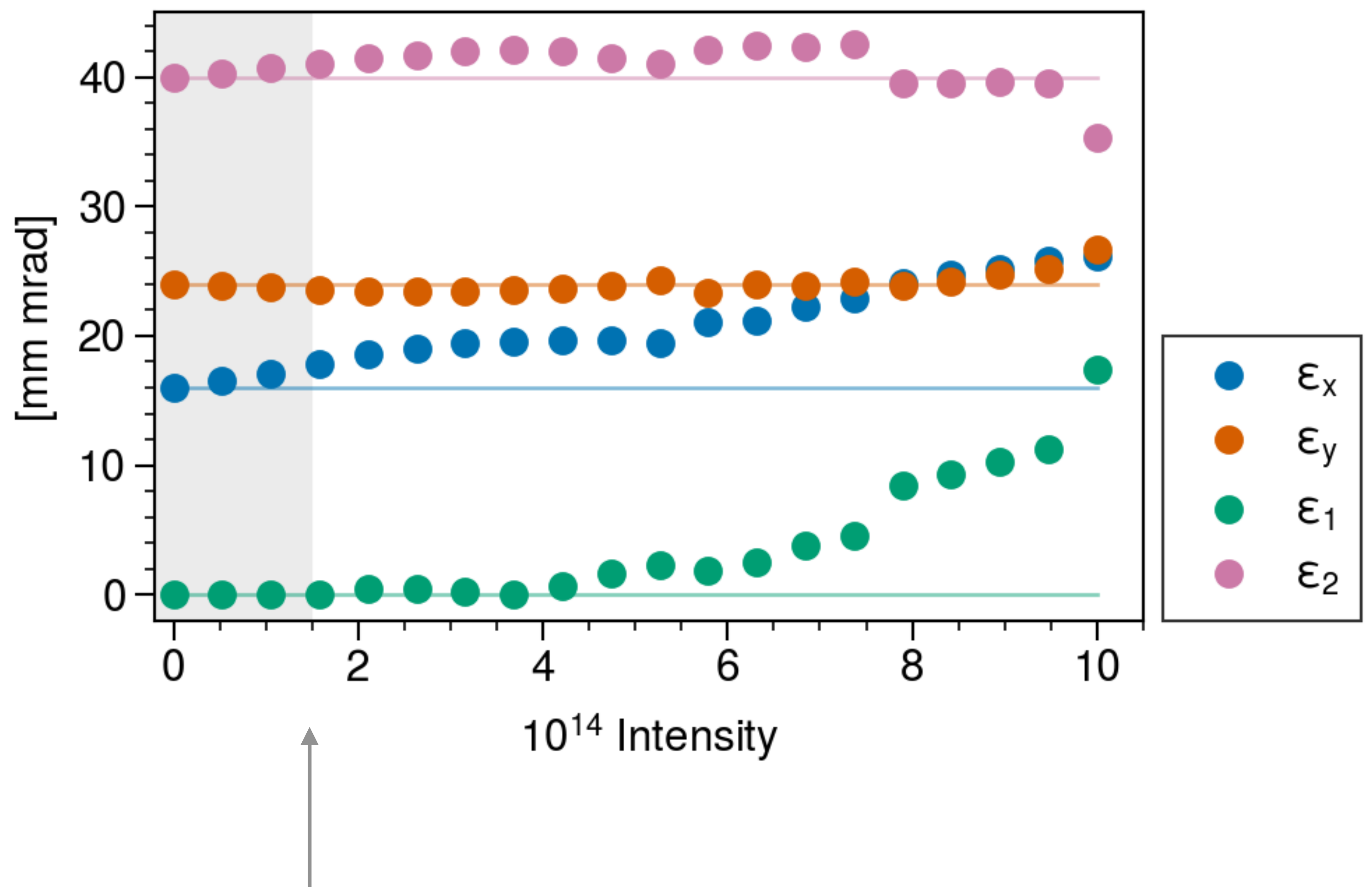




Sim [mm mrad]	True [mm mrad]
$\varepsilon_1 = 0.57 \pm 0.63$	$\varepsilon_1 = 0.0$
$\varepsilon_2 = 40.66 \pm 1.04$	$\varepsilon_2 = 40.0$
$\varepsilon_x = 21.04 \pm 0.75$	$\varepsilon_x = 20.0$
$\varepsilon_y = 20.31 \pm 0.55$	$\varepsilon_y = 20.0$

Error analysis

- Plot obtained by tracking envelope with space charge — no other errors
- Near SNS intensities, the smaller intrinsic emittance stays near zero



Max SNS intensity is 1.5×10^{14}

Outlook

- Measurement seems feasible to perform in the SNS
  - Diagnostics and optics sufficient for reconstruction
  - Initial simulations including several sources of error lead to acceptable accuracy
- Work in progress
  - Refine error estimates
  - Make predictions for realistic painted beam and compare with production beam
  - Carry out measurement experimentally