Nonlinear Coupling Resonances in X-Y Coupled Betatron Oscillations Near the Main Coupling Resonance in VEPP-2000 Collider

S. Kladov and E. Perevedentsev

BINP SB RAS and NSU, Novosibirsk

Novosibirsk State University
*THE REAL SCIENCE

INTRODUCCTION

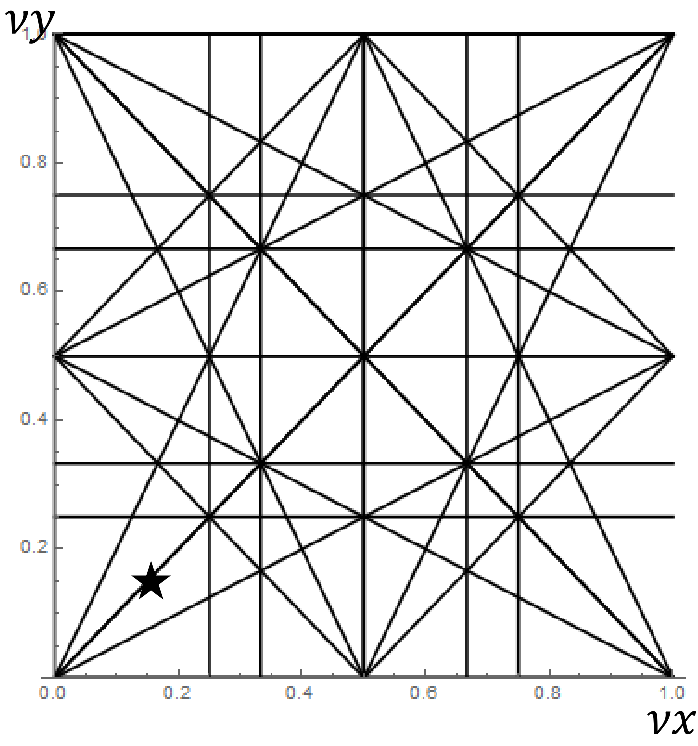
Linear beam dynamic in accelerators is well studied, but in the modern colliders nonlinear effects are gaining importance. The example of such an innovative accelerator is the e[+]e[-] collider VEPP-2000, which is designed to operate with round colliding beams and its working point is located on the main difference resonance.

It is easy to see from the basic resonance equation

$$l\nu_x + m\nu_y = n,$$

where l, m, n are integer, that such a working point actually stands not only on a single $(1-1)$ resonance $(l=1, m=-1, n=0)$, but also on an infinite number of nonlinear coupling resonances, such as $(2-2)$, $(3-3)$ and so on. These resonances result in beam dynamics whose details and consequences are not well understood.

The operating mode of VEPP-2000 can be called “strong coupling”, because two transverse dimensions whose tunes are very close are effectively mixed and form the betatron normal modes. In that situation, any perturbation in the accelerator lattice can excite two-dimensional resonances mentioned above.



Betatron resonance diagram. VEPP-2000 operating point is shown by star

In this article the contributions of nonlinearities to coupled betatron oscillation are considered with simultaneous action of $(1-1)$ and $(2-2)$ resonances in the VEPP-2000 collider.

Content

1. General Hamiltonian is derived in the variables of linear eigenvectors
2. The Hamiltonian is reduced to one-dimensional difference Hamiltonian
3. Some phase portraits are plotted
4. Center-of-charge motion of betatron coupled system is investigated and modeled for the system of interest
5. Fourier spectra of modeled motion are introduced
6. Experimental Fourier spectra are introduced
7. Different sources of nonlinearities in the VEPP-2000 colliders are estimated
8. Main findings are summarized in Conclusion.

Resonance 2-2 sources:

Nonlinear fields, produced by:

- colliding beam;
- Quadrupole fringe fields;
- Final focusing solenoids fringes

$1-1$ basis and linear decomposition. Normal modes

The 4-dimentional Floquet vectors, written in terms of no-coupling Twiss parameters, are very useful for analysis of $(1-1)$ resonance. They are used as a basis in this paper. Beam parameters can be expressed as a linear decomposition in this basis. $2-2$ is considered as a perturbation in this basis and is supposed to cause ``variation of constants'', namely of the complex variables $\hat{A}$ and $\hat{B}$

$$Y_a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e^{i\,\psi_a}; \qquad Y_b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} e^{i\,\psi_b}$$

$$\begin{cases} \hat{A} = A e^{i\,\varphi_a} \\ \hat{B} = B e^{i\,\varphi_b} \end{cases}$$

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \frac{1}{2}\left(\hat{A}Y_a + \hat{A}^*Y_a^*\right) + \frac{1}{2}\left(\hat{B}Y_b + \hat{B}^*Y_b^*\right)$$

$$Y_A = \begin{bmatrix} \mathrm{e}^{i\left(\chi_x + 2\pi n\frac{n_{\text{рез}}}{2}\right)} w_x \cos\gamma \\ \mathrm{e}^{i\left(\chi_x + 2\pi n\frac{n_{\text{рез}}}{2}\right)}\left(w_x' + \frac{\mathrm{i}}{w_x}\right)\cos\gamma \\ \mathrm{e}^{i\left(\chi_y - 2\pi n\frac{n_{\text{рез}}}{2} - \alpha\right)} w_y \sin\gamma \\ \mathrm{e}^{i\left(\chi_y - 2\pi n\frac{n_{\text{рез}}}{2} - \alpha\right)}\left(w_y' + \frac{\mathrm{i}}{w_y}\right)\sin\gamma \end{bmatrix} e^{2\pi i n(\nu_x + \nu_y + \eta)/2}$$

$$Y_B = \begin{bmatrix} -\mathrm{e}^{i\left(\chi_x + 2\pi n\frac{n_{\text{рез}}}{2} + \alpha\right)} w_x \sin\gamma \\ -\mathrm{e}^{i\left(\chi_x + 2\pi n\frac{n_{\text{рез}}}{2} + \alpha\right)}\left(w_x' + \frac{\mathrm{i}}{w_x}\right)\sin\gamma \\ \mathrm{e}^{i\left(\chi_y - 2\pi n\frac{n_{\text{рез}}}{2}\right)} w_y \cos\gamma \\ \mathrm{e}^{i\left(\chi_y - 2\pi n\frac{n_{\text{рез}}}{2}\right)}\left(w_y' + \frac{\mathrm{i}}{w_y}\right)\cos\gamma \end{bmatrix} e^{2\pi i n(\nu_x + \nu_y - \eta)/2}$$

Amplitude and phase derivatives equations

Using the usual differential equations, $\begin{cases} \hat{A}' = -i\, {F_a^*}^T SG \\ \hat{B}' = -i\, {F_b^*}^T SG \end{cases}$

where G – vector of the Hill's equations perturbations, $G_x = \frac{1}{B\rho}\left(y' H_z - H_y\right),\ G_y = -\frac{1}{B\rho}(x' H_z - H_x),$

and substituting the octupole fields, we obtain: $A' + i\, A\, \varphi_a' = \sum_{k,l=-4}^{4} c_{kl} e^{i(k\,(\phi a+\psi a)+l\,(\phi b+\psi b))}.$

After the one-turn averaging $\overline{c_{kl}} = \frac{1}{2\pi\bar{R}} \oint_0^{2\pi\bar{R}} c_{kl} e^{i(n_q \frac{s}{\bar{R}})} ds$

we obtain averaged equations $A'(n) + i\, A(n)\varphi_a'(n) = 2\pi\bar{R} \sum_{k,l=-4}^{4} \overline{c_{kl}} e^{i(k\,\varphi_a + l\,\varphi_b + \nu 2\pi n)},$

from which one can pick out necessary terms with slow phases.

In our case, besides constant terms ($k, l = 0$), only the difference resonances $1-1$ and $2-2$ ($k = -l = \pm 1, \pm 2$) will remain.

Hamiltonian

$$H = \left(k_{11}J_a^{\frac{3}{2}}\sqrt{J_b} + k_{12}J_b^{\frac{3}{2}}\sqrt{J_a}\right)\cos(\varphi_a - \varphi_b + 2\pi n\,\eta + \alpha) + (pJ_a^2 + rJ_b^2 + 2qJ_aJ_b) - f_2J_aJ_b(\cos(2\varphi_a - 2\varphi_b$$

$$+ 4\pi n\,\eta)),$$

$J_a = \frac{A^2}{2},\ \varphi_a,\ J_b = \frac{B^2}{2}, \varphi_b$ – action-angles variables

- Sum of the actions is invariant, i.e. the 2nd integral of motion additionally to the Hamiltonian itself. So, from the Liouville theorem on integrable dynamical systems we conclude that the system is integrable and motion is regular.

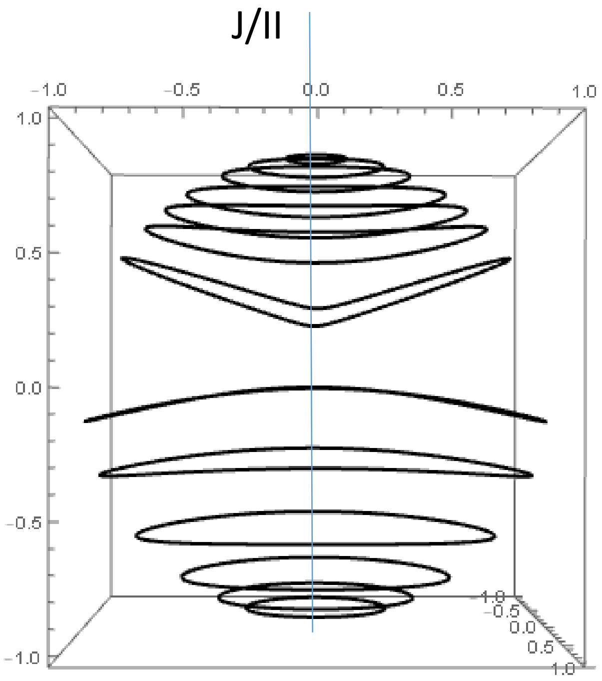
- The sum-of-actions preservation also allows us to switch to difference variables ($J = J_a - J_b, \varphi$=φ_a-φ_b) and reduce the problem to one dimension.

- Reduction of the problem to one dimension leads to the possibility of 2D-phase space plotting and, therefore, allow to analyze the system qualitatively.

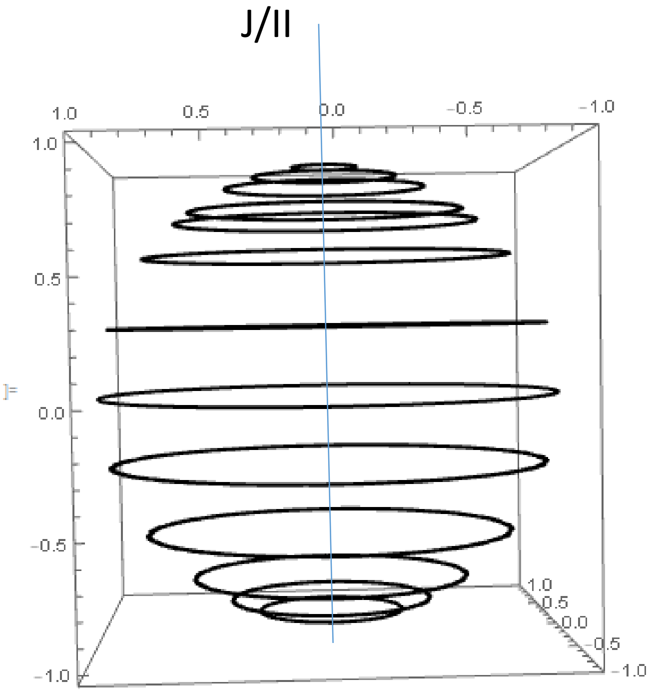
- All terms in the Hamiltonian ~ J^2; all coefficients are the same order.

- Magnitude estimation of terms: $p \sim 10\ \text{см}^{-1}, J \sim 10^{-4}\ cm$

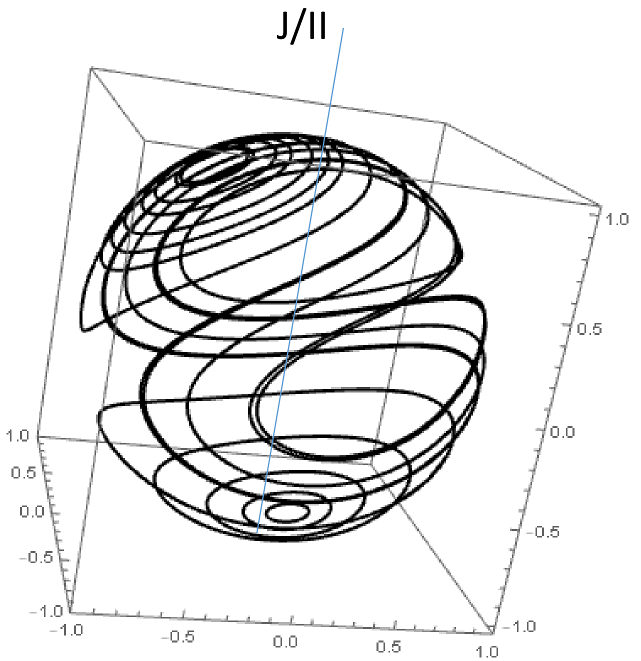
Phase portraits "on sphere" in difference variables



Nonlinearities near critical



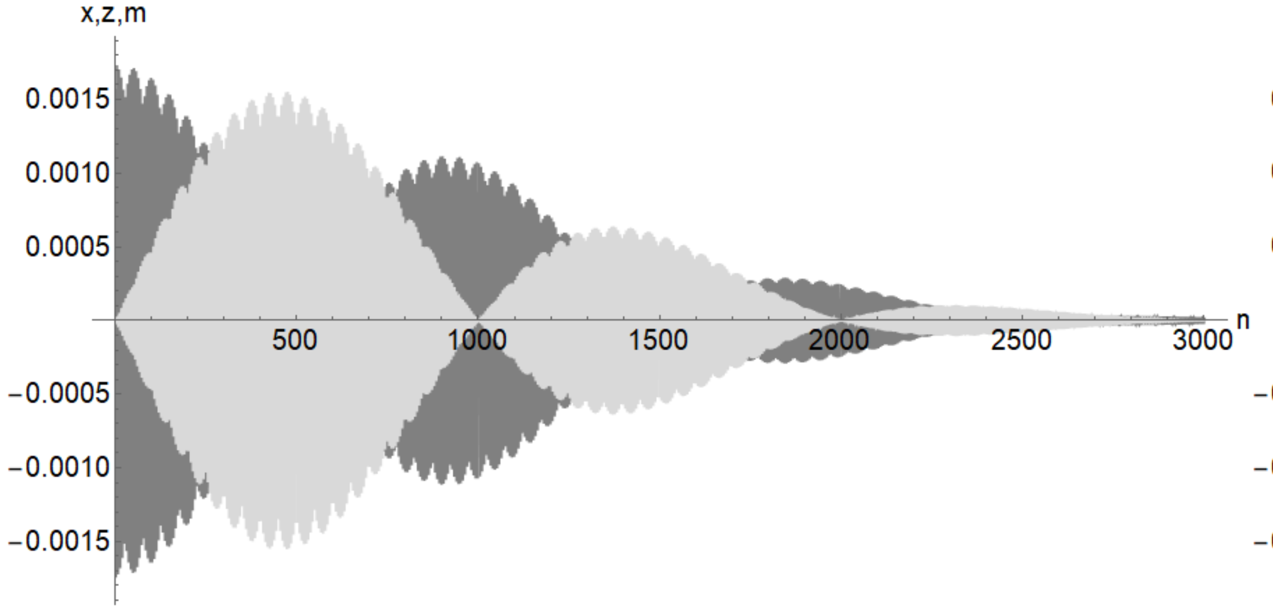
Without nonlinearities

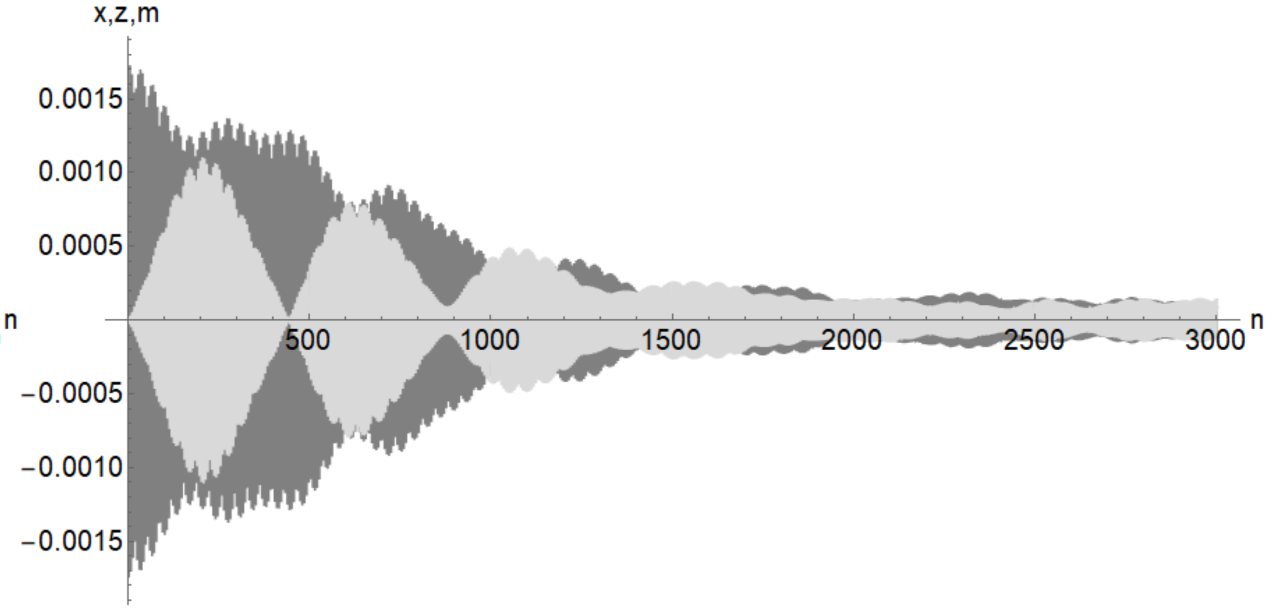


Nonlinearities greater than critical

Analysis of the system in difference variables shows some critical condition, which lower estimation is expressed as $c \equiv \frac{|f_2|\, II}{\eta} = 1$ (so as with substitution $f_2 \rightarrow k_{11}, k_{12}$). At this point the phase portrait is altered and a few additional fixed points may appear.

Signal modeling. Whole beam





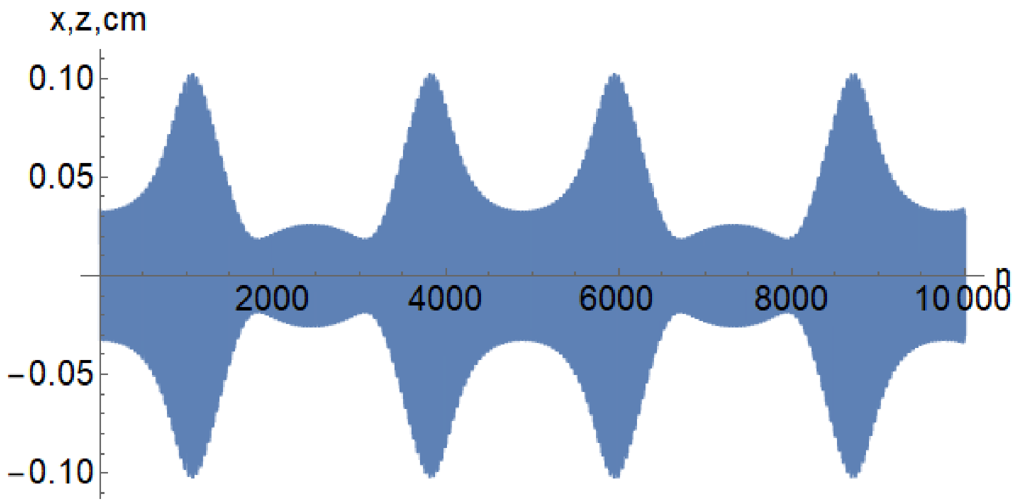
Signal modeling, its decay is due to decoherence for the case with only 0-mode nonlinearities. This will be useful for the experiment and model comparison – from the decoherence time, we can estimate constant nonlinearities, which are the same order as the 2-2 resonance terms

The signal modeling for the full Hamiltonian is important for comparison with the experiment. This comparison can be performed qualitatively using the center-of-charge history (modeling plotted above), or using the oscillations spectra

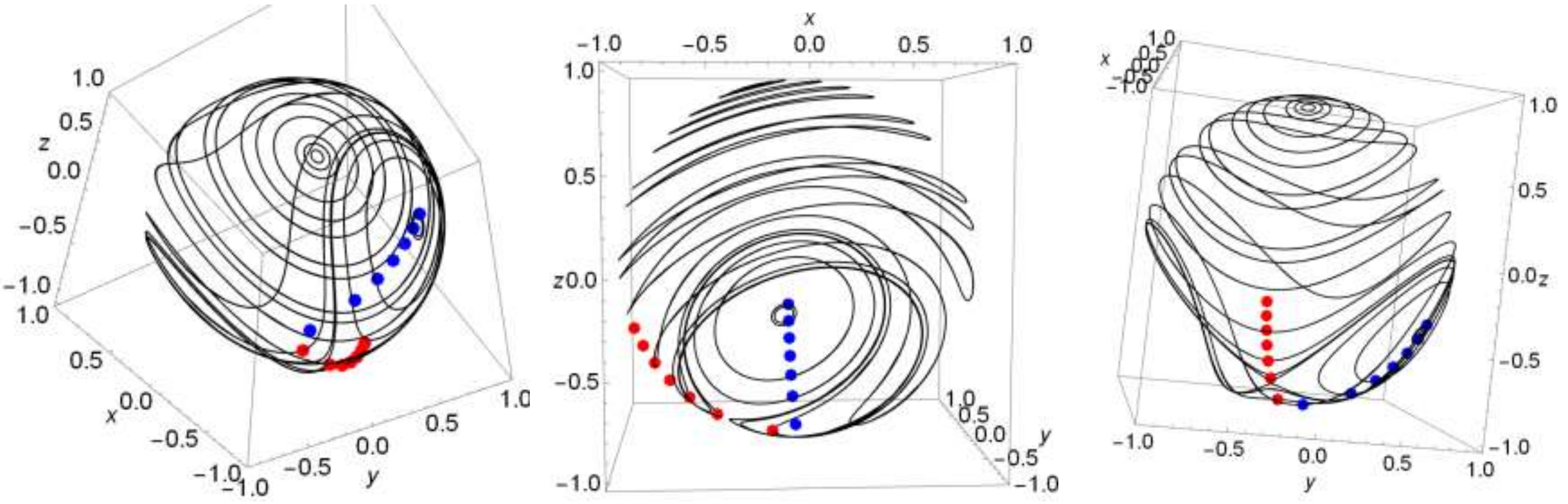
Signal modeling. Solo particle

The single particle simulations are easier to make, and they contain all the necessary information except decoherence.
For the solo particle we can choose initial conditions in the difference variables, mark it on the phase portrait and visualize oscillations for all important cases separately.

On the bottom figure, the initial conditions are plotted as red and blue points. They can be deep inside the auto-phasing region, near the saddle point, or somewhere in the middle.



Single particle oscillations. The trajectory is near the saddle-point



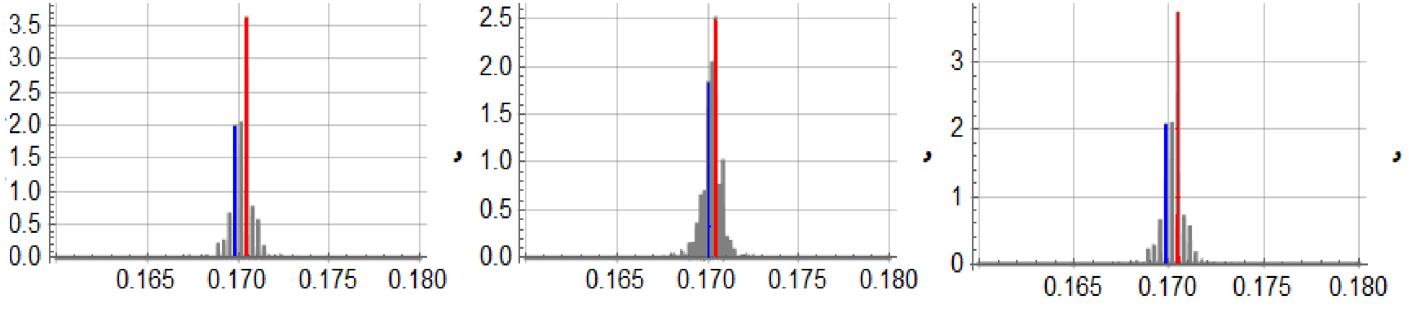
Initial conditions in the difference variables phase space, plotted on the sphere

The oscillations of single particle with initial conditions near the saddle-point line are plotted on the figure above. It can be seen, that these oscillations exhibit several frequencies and the spectrum should be non-trivial

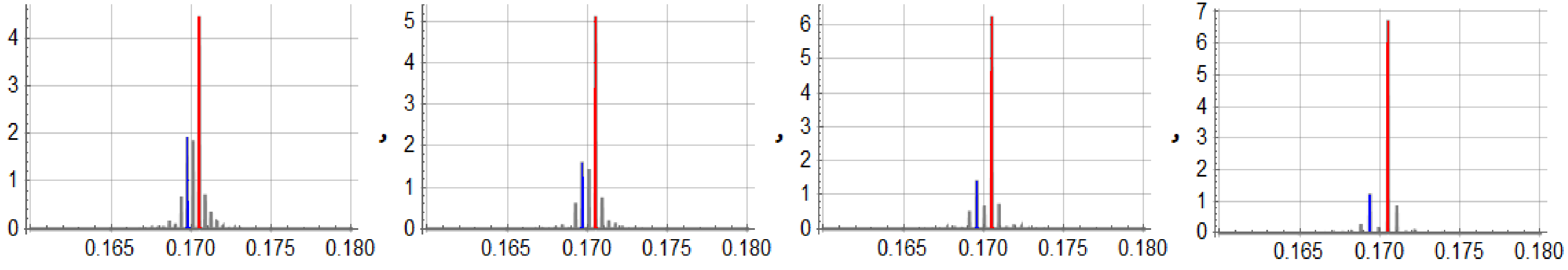
Signal modeling. Spectra

In the presence of $2-2$ resonance, actions cease to be constants, and oscillate at the double frequency 2η as well as phases. The amplitude of these oscillations and frequencies shifts are proportional to c^2. The same effect is produced by $1-1$ resonance, exciting oscillations on single η.

For different parameters, the frequencies of modes a and b are shifted in different directions (minus the total shift), increasing or decreasing the distance between the main peaks, denoted below as η_1. As a result, in the Fourier analysis of the dipole moment oscillations, two fundamental frequencies ν_x and ν_y are visible, shifted due to the presence of nonlinearities, resonances and dephasing, and there appear one or more equidistant satellites responsible for phase oscillations in the regions of auto-phasing on single and double η_1.



One of the modeling series for betatron oscillations spectra with the initial conditions from the previous slide (red points) is introduced here. The η_1 parameter decreases when approaching the saddle-point motion line, and vice-versa.



The x-frequencies are plotted in red, and y's are shown in blue

Series of modeled spectra for different initial conditions. The main x and y frequencies are shown in red and blue respectively

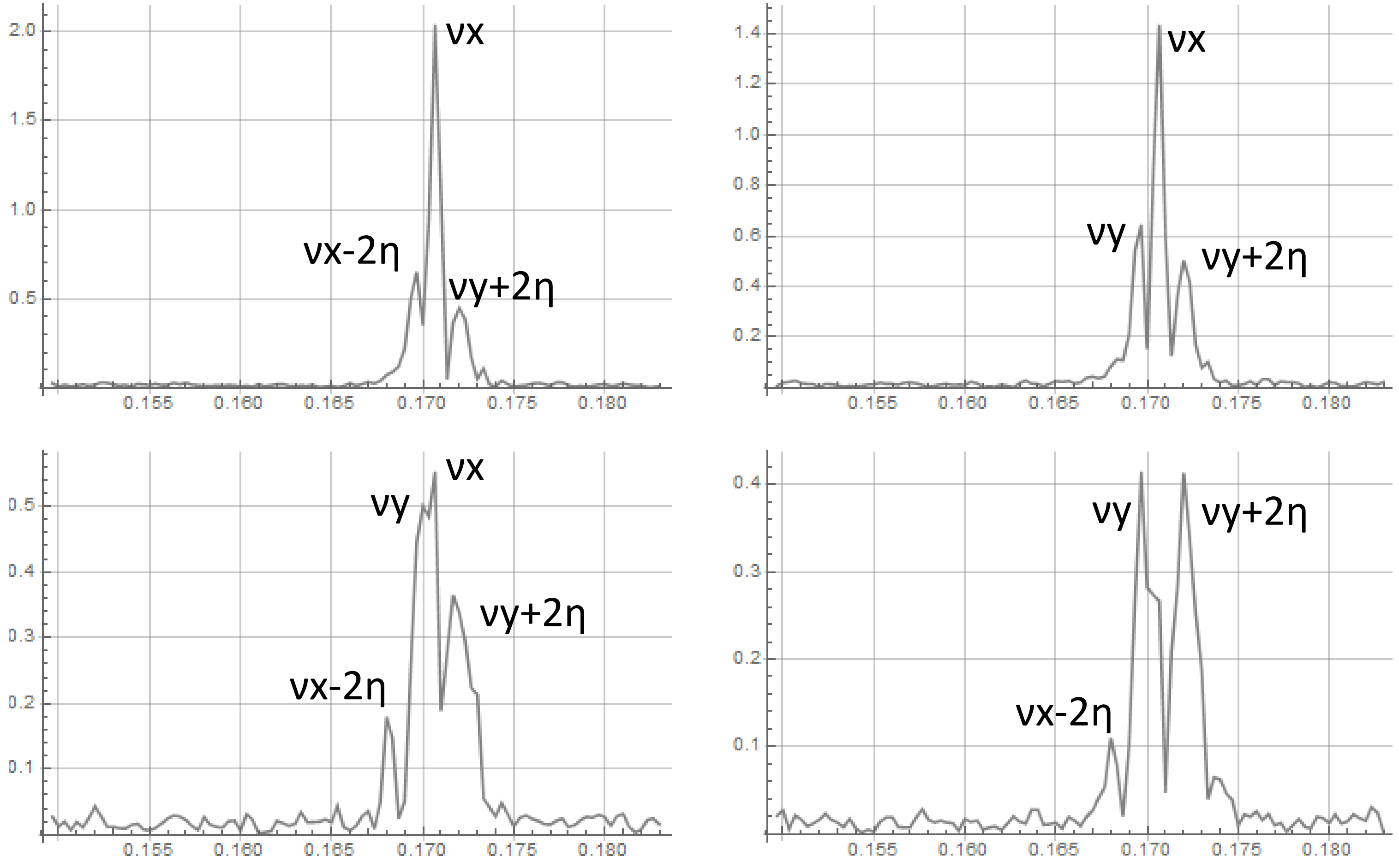
Experimental observations

Predicted spectrum patterns were prepared to compare with the experimental observables.

Presently but a little time was available for this experiment.

The measured spectra are not rich for satellites, and comparing with the series of predicted spectra, it can be concluded, that the resonance terms are weaker or slightly above the critical values, as was estimated from the decoherence time.

Sure, more experimental data is needed to prove the resonance 2-2 influence on the particle motion on the VEPP-2000 collider.



x (on the top), and y (on the bottom) spectra, observed after kicks on the VEPP-2000

Estimation of VEPP-2000 cubic nonlinearities

Final focusing solenoids

$$H_x = H'(z)x + \frac{H'''(z)}{16}(x^3 + y^2 x)$$
$$H_y = H'(z)y + \frac{H'''(z)}{16}(x^2 y + y^3)$$

$$f_2 \sim L \frac{H'''(s)}{16H\rho}\beta^2 \sim 100 \text{ см}^{-1}$$

Quadrupole lenses

$$H_x = G(z)y - \frac{G''(z)}{12}(3x^2 y + y^3)$$
$$H_y = G(z)x - \frac{G''(z)}{12}(x^3 + 3\mathrm{x}y^2)$$
$$H_z = G'(z)xy$$

$$\Delta\varphi_y = \left(\frac{G_0}{B\rho}\right)^2 \frac{L}{4} w_y^2 \left(2w_x^2 J_x + w_y^2 J_y\right)$$

$p \sim 2.5; q \sim 1.5; r \sim 1.6$

Colliding beam

$$F_r(r) = -2n\, e^2\, (1 + \beta^2) \frac{1}{r}\left(1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right)$$

$$\Delta r' = -f\frac{\sigma^2}{r}\left(1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right), f = \frac{4Nr_e}{\gamma\sigma^2}$$

$$f_2 \sim \frac{\xi\beta^{*2}}{\sigma^2} \sim 10^5 \text{ см}^{-1}$$

Conclusions

- A system with several closely spaced resonances can be reduced to integrable form using fast-phase averaging, so that its dynamics is regular.
- The simultaneous action of resonances 1-1 and 2-2 was considered. The phase portraits were plotted and analyzed, the qualitative features, auto-phasing areas and fixed points were found.
- Strengths of VEPP-2000 cubic nonlinearities resulting in the resonance term amplitude were estimated. The comparison with decoherence time showed the qualitative agreement of estimations and measurements.
- Extensive series of modeling the betatron motion were performed with different phase-space patterns. Fourier spectra of those oscillations were predicted in both normal-mode and observable coordinates.
- Series of spectra from the routine operating mode of the collider were compared with predicted ones. The comparison showed a fairly good agreement, but a dedicated validation experiment is required to get more data.