Theoretical Analysis of the Conditions for an Isochronous and CSR-Immune Triple-Bend Achromat with Stable Optics*

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Abstract: Transport of high-brightness beams with minimum degradation of the phase space quality is pursued in modern accelerators. For the beam transfer line which commonly consists of bending magnets, it would be desirable if the transfer line can be isochronous and coherent synchrotron radiation (CSR)-immune. For multi-pass transfer line, the achromatic cell designs with stable optics would bring great convenience. In this paper, based on the matrix transfer formalism and the CSR point-kick model, we report the detailed **theoretical analysis and derivation process** of the condition for a triple-bend achromat **(TBA)** with **stable optics** in which the first-order longitudinal dispersion (i.e., R_{56}) and the CSR-induced emittance growth can be eliminated **(i.e., R_{56}=0 and $\Delta\varepsilon_n$=0)**. The derived condition suggests a new way of designing the bending magnet beamline that can be applied to the FEL spreader and ERL recirculation loop.

Derivation of the R_{56}=0 and $\Delta\varepsilon_n$=0 conditions for a TBA

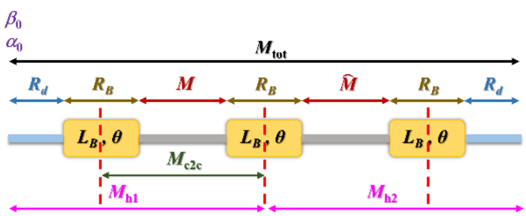

Fig. 1 Schematic of the TBA structure with three identical dipole magnets of length L_B and bending angle θ.

A midpoint symmetric TBA structurethat consists of three identical dipole magnets (see Fig.1) is studied. Here only the transverse motion in the horizontal plane and the longitudinal motion of the particle are considered for simplicity. The transfer matrix for the phase-space coordinate vector (x, x', z, δ) is used, which is part of the typical 6×6 transfer matrix that describes the motion of (x, x', y, y', z, δ) though we still adopt the subscripts 5 and 6 for the longitudinal dispersion functions R_{56}.

$$R_B=\begin{pmatrix}\cos\theta & \frac{\sin\theta L_B}{\theta} & 0 & \frac{(1-\cos\theta)L_B}{\theta}\\ -\frac{\theta\sin\theta}{L_B} & \cos\theta & 0 & \sin\theta\\ \sin\theta & \frac{(1-\cos\theta)L_B}{\theta} & 1 & \frac{(\theta-\sin\theta)L_B}{\theta}\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$M=\begin{pmatrix}m_{11} & m_{12} & 0 & 0\\ m_{21} & m_{22} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$M_{\rm h1}=R_{B/2}MR_BR_d$$

achromatic condition $(M_{\rm h1})_{26}=0$

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symplectic condition $(M_{\rm h1})_{11}(M_{\rm h1})_{22}-(M_{\rm h1})_{12}(M_{\rm h1})_{21}=1$

$$m_{12}=\frac{\sec\left(\frac{\theta}{2}\right)^2L_B\left\{\theta\left[1+m_{11}-m_{11}^2+\cos\theta\left(1+m_{11}^2\right)\right]+\sin\theta L_Bm_{11}m_{21}\right\}}{2\theta\left[-L_Bm_{21}+\theta m_{11}\tan\left(\frac{\theta}{2}\right)\right]}$$

$$m_{22}=\frac{\sec\left(\frac{\theta}{2}\right)^2\left[\theta^2\sin\theta+\theta L_B(1-m_{11}+\cos\theta m_{11})m_{21}+\sin\theta L_B^2m_{21}^2\right]}{2\theta\left[-L_Bm_{21}+\theta m_{11}\tan\left(\frac{\theta}{2}\right)\right]}.$$

1st order isochronous condition

$$(M_{\rm h1})_{56}=0$$

$$m_{21}=\frac{\theta\sin\left(\frac{\theta}{2}\right)\left[-2\sin\theta+(-3\theta+2\sin\theta)m_{11}\right]}{\left[-3\theta\cos\left(\frac{\theta}{2}\right)+3\sin\left(\frac{\theta}{2}\right)+\sin\left(\frac{3\theta}{2}\right)\right]L_B}$$

CSR-induced emittance growth cancellation condition

$$(M_{\rm c2c})_{11}-\frac{2\theta^2\sin\left(\frac{\theta}{2}\right)}{\theta\cos\left(\frac{\theta}{2}\right)L_B-2\sin\left(\frac{\theta}{2}\right)L_B}(M_{\rm c2c})_{12}=-\frac{1}{2}$$

$$\begin{aligned}m_{21}&=-\frac{\theta\cos\left(\frac{\theta}{2}\right)\csc(\theta)}{4\sqrt{2}L_B\left(\theta-2\tan\left(\frac{\theta}{2}\right)\right)}S_1{}^{1/2},\\ S_1&=\sec^4\left(\tfrac{\theta}{2}\right)\left[-2\left(7\theta^2-16\cos(2\theta)+8\cos(3\theta)+8\right)\right]\\ &+\sec^4\left(\tfrac{\theta}{2}\right)\{8\left[8\sin(\theta)+\sin(2\theta)-4\sin(3\theta)\right]\}\theta\\ &+\sec^4\left(\tfrac{\theta}{2}\right)\left[-9\theta\cos(\theta)+18\theta\cos(2\theta)+13\theta\cos(3\theta)\right]\theta\\ &+\sec^4\left(\tfrac{\theta}{2}\right)\left[\left(\theta^2-2\right)\cos(2\theta)+2\theta\sin(\theta)-2\right]16m_{11}\\ &+\sec^4\left(\tfrac{\theta}{2}\right)\left\{\left[\theta^2-6\theta\sin(\theta)+4\right]\cos(\theta)\right\}16m_{11}\\ &+\sec^4\left(\tfrac{\theta}{2}\right)\left[\theta\cos\left(\tfrac{\theta}{2}\right)-2\sin\left(\tfrac{\theta}{2}\right)\right]^2 32m_{11}^2.\end{aligned}$$

$$\begin{aligned}m_{21}&=\frac{1}{8L_B\left[\theta-2\tan\left(\frac{\theta}{2}\right)\right]}\theta\csc(\theta)\{4\theta\left[\cos(\theta)-2m_{11}\cos(\theta)-2\right]\}\\ &+\frac{1}{8L_B\left[\theta-2\tan\left(\frac{\theta}{2}\right)\right]}\theta\csc(\theta)\left\{8\tan\left(\tfrac{\theta}{2}\right)\left[2m_{11}\cos(\theta)+1\right]\right\}\\ &+\frac{1}{8L_B\left[\theta-2\tan\left(\frac{\theta}{2}\right)\right]}\theta\csc(\theta)\cos\left(\tfrac{\theta}{2}\right)S_2{}^{1/2},\\ S_2&=2\sec^4\left(\tfrac{\theta}{2}\right)\left\{-9\theta^2\cos(\theta)+2\left[9\theta^2+16\right]\cos(2\theta)\right\}\\ &+2\sec^4\left(\tfrac{\theta}{2}\right)\left\{-2\left[7\theta^2+8\cos(3\theta)+8\right]\right\}\\ &+2\sec^4\left(\tfrac{\theta}{2}\right)\{\theta\left[8\left(8\sin(\theta)+\sin(2\theta)-4\sin(3\theta)\right)+13\theta\cos(3\theta)\right]\}\\ &+64m_{11}\left[\theta-2\tan\left(\tfrac{\theta}{2}\right)\right]\sec^2\left(\tfrac{\theta}{2}\right)\left[-\theta-2\sin(\theta)+2\theta\cos(\theta)\right]\\ &+64m_{11}\left[\theta-2\tan\left(\tfrac{\theta}{2}\right)\right]\sec^2\left(\tfrac{\theta}{2}\right)\left[\theta m_{11}-2\left(m_{11}-1\right)\tan\left(\tfrac{\theta}{2}\right)\right].\end{aligned}$$

Cross points of 1st order isochronous & CSR-induced emittance growth cancellation condition

The two cross points locate at

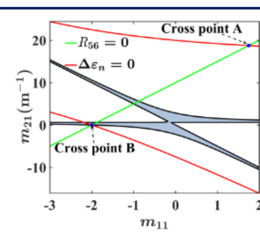
$$\begin{aligned}m_{11}&=-1-\cos\theta,\\ m_{11}&=\frac{P_1}{P_2},\\ P_1&=-9\theta^3+\theta\left[\left(3-9\theta^2\right)\cos(\theta)+26\theta\sin(\theta)\right]\\ &+\theta\left[7\theta\sin(2\theta)+10\cos(2\theta)+\cos(3\theta)\right]\\ &-14\theta+8\sin(\theta)-4\sin(2\theta),\\ P_2&=4\left(\theta^2+2\right)\sin(\theta)-10\theta-4\sin(2\theta)\\ &+8\theta\cos(\theta)+2\theta\cos(2\theta).\end{aligned}$$

When $\theta \ll 1$, Taylor expand with respect to θ and to the lowest order

$$m_{11}=\frac{7}{4}\quad m_{21}=\frac{15}{2L_B},\ \text{[Cross point A]}$$

$$m_{11}=-2\quad m_{21}=0.\ \text{[Cross point B]}$$

Discussion in the stable area



The cross point B satisfies the stability criterion

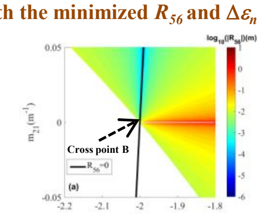
$$|(M_{\rm tot})_{11}+(M_{\rm tot})_{22}|\leqslant 2$$

However, the periodic solution of β_0 will diverge at cross point B

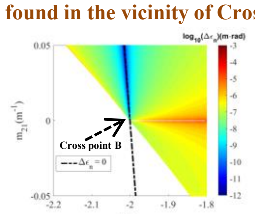
$$\begin{aligned}\beta_0&=\frac{\sqrt{(-1+2dm_{21}+L_Bm_{21})}}{2\sqrt{-m_{21}^2(2m_{11}+L_Bm_{21})}}B_0^{1/2},\\ B_0&=-4+L_B^2m_{21}^2+L_Bm_{21}(-1+2dm_{21})\\ &+2m_{11}(-1+2dm_{21}+L_Bm_{21}),\\ \alpha_0&=0\\ \gamma_0&=1/\beta_0.\end{aligned}$$

But both the minimized R_{56} and $\Delta\varepsilon_n$ can be found in the vicinity of Cross point B

R_{56}



$\Delta\varepsilon_n$



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