Simulations of AGS Boosters Imperfection Resonances for Protons and helions[1]

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May 24, 2021

[1]Funding Agency Work supported by Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy.

Helions in the Booster

Polarized helions are injected into the Booster at $|G\gamma| = 4.19$ from the EBIS and are ideally extracted at $|G\gamma| = 10.5$. In this range, polarized helions will encounter:

- six imperfection resonances ($|G\gamma| = 5,\ 6,\ 7,\ 8,\ 9,\ 10$) and
- two intrinsic resonances ($|G\gamma| = 12 - \nu_y$ and $6 + \nu_{\mathrm{y}}$)

as they are accelerated to $|G\gamma| = 10.5$. To preserve polarization through the Booster intrinsic resonances, an AC dipole has been installed [2].

Polarization through the imperfection resonances will be used the harmonic correction method with the ring corrector magnets.

Imperfection Resonances

Imperfection resonances are caused by non-zero closed orbit primarily arising from misaligned quadrupoles that causes the particles to sample the horizontal field in quadrupoles. These resonances occur when the spin tune is equal to an integer,

$$\nu_s = |G\gamma| = k \tag{1}$$

where k is an integer, G is the anomalous magnetic moment ($G_{helions} = -4.18415$ and $G_{protons} = 1.79285$), γ is the Lorentz factor, and ν_s is the spin tune. The imperfection resonance strength is [5],

$$\epsilon_k = \frac{1+G\gamma}{2\pi} \oint \frac{\frac{\partial B_x}{\partial y}}{B\rho} y_{co} e^{iK\theta} ds \tag{2}$$

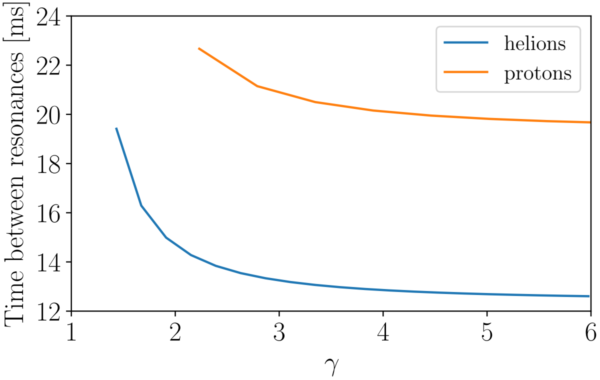
where $\partial B_x/\partial y$ is the quadrupole gradient field, $B\rho$ is the rigidity, K is the resonance condition (which in this case corresponds to K=k), y_{co} is the vertical closed orbit, and θ is the orbital angle.

Imperfection Resonance Spacing

Helion imperfection resonances are separated by

$$\frac{M_o/u}{G} = 223.73 \text{ MeV/u}. \quad (3)$$

In time, these resonances are separated as shown in Fig. 4.



Resonance Correction

For correcting the $|G\gamma| = k$ resonance, the h=k harmonic of the corrector dipoles is used. The harmonic h=k must either be corrected so no polarization is lost, or enhanced to induce a full spin-flip.

The resonance strengths are calculated by simulation, and Eq. 2 using optics outputs. The Froissart-Stora formula allows predicting the final polarization at a given resonance, k, and harmonic, h=k, as a function of corrector current using [5],

$$\frac{P_f}{P_i} = 2e^{-\frac{(I_{k,S}-I_{k,oS})^2}{2\sigma_{k,S}^2}} e^{-\frac{(I_{k,C}-I_{k,oC})^2}{2\sigma_{k,C}^2}} - 1 \tag{4}$$

where P_i and P_f are the asymptotic values of the polarization before and after crossing the resonance, $I_{k,S}$ and $I_{k,C}$ are the corrector dipole current for the sine and cosine components, $I_{k,oS}$ and $I_{k,oC}$ are the optimal corrector currents for the sine and cosine components, and $\sigma_{k,S}$ and $\sigma_{k,C}$ are the RMS widths for the two families. The currents I_S and I_C for harmonic h will be referred to as sinhv and coshv.

Imperfection Resonance Strengths

Table: Summary of imperfection resonance strengths for protons and helions with quadrupole alignment based on Fig. 8a.

Species	k	$B\rho\ [T \cdot m]$	ϵ_k	
			Simulation	Eq. 2
Protons	3	4.198	0.000714	0.000644
	4	6.240	0.002367	0.002396
Helions	5	3.064	0.004605	0.004492
	6	4.814	0.000701	0.000716
	7	6.282	0.001299	0.001158
	8	7.633	0.003582	0.003834
	9	8.920	0.000226	0.000239
	10	10.167	0.006252	0.006646

Harmonic Orbit Correction

The Booster has 24 vertical orbit correctors, placed adjacent to vertically focusing quadrupoles, and are used for creating and correcting orbit harmonics. These corrector magnets are 10 cm long with an excitation of 0.975 $\mathrm{G \cdot m/A}$, where the supplies have a maximum current of $\pm$25 A [4]. These correctors are powered according to [3]

$$B_{j,h} = a_h \sin(h\theta_j) + b_h \cos(h\theta_j) \tag{5}$$

where j is corrector number, θ_j is the location in the ring, a_h and b_h are the amplitudes for harmonic h.
The total current on corrector j is

$$I_j = \sum_h I_{h,S} \sin(h\theta_j) + I_{h,C} \cos(h\theta_j) \tag{6}$$

where $I_{h,S}$ and $I_{h,C}$ are the equivalent of sinhv and coshv. This is used to determine the total current on each of the correctors, where the maximum current of all correctors is

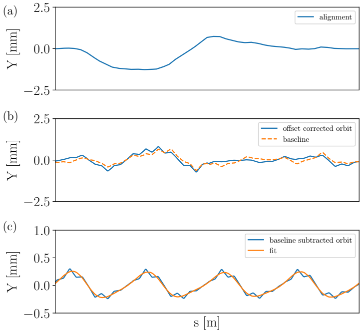
$$I_{max} = \max[|I_j|]. \tag{7}$$

This is an important parameter so as to avoid exceeding the maximum current of the supplies.

Booster Alignment Errors

Quadrupole alignment errors from survey seen in (a) and placed into the zgoubi input files using pyzgoubi using the CHANGREF keyword [1].

- a Vertical quadrupole misalignments in the Booster are scaled to 65% to match h=4 data;
- b Orbit after incorporating misalignments with sin4v=5.22 A and baseline orbit with sin4v=6.97 A, and all other harmonic strengths being the same;
- c Baseline subtracted orbit for helions crossing the $|G\gamma| = 8$ resonance.



This example orbit at $|G\gamma| = 8$ has a corrector current with respect to h=8 of cos8v=5 A, sin8v=13 A. The components of the fit results are: [sin4, cos4, sin5, cos5, sin8, cos8]=[0.1997 mm, 0.07796 mm, 0.01137 mm, 0.00031849 mm, -0.01263 mm, -0.04177 mm].

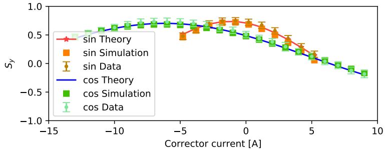
Protons Crossing $|G\gamma|$=3,4

Results from these simulation and comparison to experimental data is shown on right. The harmonic scan data is fit with a Gaussian defined as

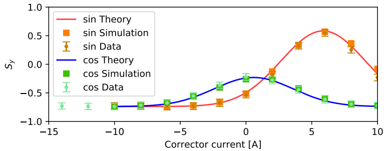
$$S_y(I) = B\exp\left[-\frac{(I_{of}-\mu_f)^2}{2\sigma_f^2}\right]-1 \quad (8)$$

with f is the sine or the cosine corrector family, μ_f is the location of the peak corresponding to the optimal corrector current to correct the harmonic, σ_f is the width of the response, and B is the normalization. These fit parameters are used to determine the shape of each corrector currents and the optimal corrector currents for correction.

Harmonic scan of protons crossing $|G\gamma| = 3$ with a comparison between theory, simulations, and experimental data,



and for $|G\gamma| = 4$



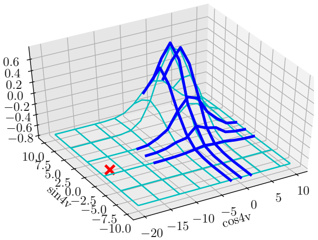
Proton Harmonic Scan summary

Summary of fit data to proton harmonic scans.

k	source	μ_{sin}	σ_{sin}	μ_{cos}	σ_{cos}
3	scan data	-1.1821	3.8390	7.5322	-6.1607
3	simulation	-1.2997	3.5643	7.8536	-6.2140
4	scan data	5.8646	3.2160	0.5740	3.2160
4	simulation	6.0330	3.2482	0.7019	3.3593

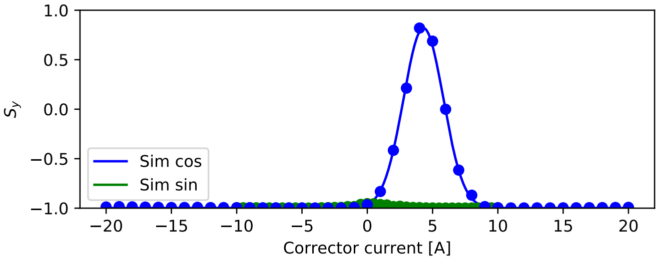
A scan using zgoubi at various initial currents of the sine and cosine corrector families shows the reliance on initial currents in the scan shown in Fig on right:

- Dark blue lines are from simulations of protons crossing the $|G\gamma| = 4$ resonance with initial currents at [-2,0,2] for the sine and cosine components.
- The light blue surface grid is reconstructed by fitting to the experimental data and extrapolating it to a larger range.
- The red 'X' marks corrector currents used in Run17, [sin4v,cos4v]=[0, -18].



Helions Crossing $|G\gamma|$=5

With the experimental data of protons being well matched with simulation, the treatment is extended to helions. Figure below shows a harmonic scan for helions crossing $|G\gamma| = 5$.



To correct the orbit harmonic h=4, [sin4, cos4]=[2.797, 0.669] and to spin-flip: h=5 currents are [sin5, cos5]=[10.0, -18.0] and I_{max}=23.086.
Harmonic scans for the remaining imperfection resonances and fit results are summarized in Table and corrector family currents and corresponding I_{max} on next slides.

Helion Harmonic Scan Summary

For helions, the h=4 orbit is corrected at $|G\gamma|$=5 and the correction from the h=5 harmonic scan is scaled to all higher order resonances by the ratio of rigidity. That is

$$I(h=5, |G\gamma|=k) = I(|G\gamma|=5)\frac{B\rho(|G\gamma|=k)}{B\rho(|G\gamma|=5)} \tag{9}$$

which allows all helion imperfection resonances to be studied with the same orbit baseline orbit. These corrector strengths are [sin4v, cos4v, sin5v, cos5v]=[2.797 A, 0.669 A, 0.520 A, 4.296 A]

Table: Summary of fit data for helion harmonic scans.

k	$\mu_{\sin}$	$\sigma_{\sin}$	$\mu_{\cos}$	$\sigma_{\cos}$
5	0.5200	1.5750	4.2955	1.5288
6	1.2226	3.6268	-0.2896	2.7384
7	3.1077	4.4358	1.8801	4.5166
8	-4.8460	4.8366	10.6646	5.5313
9	-1.1232	5.2331	-0.3165	3.9495
10	-23.6518	5.5783	-0.4287	5.4708

These fit parameters are used to determine optimal correction currents.

Summary of Corrector Strength Requirements

Table: Current scaling factor to correct the two major orbit harmonics (h=4, 5), current to correct or amplify the orbit harmonic corresponding to the resonance (h=k), and the resulting maximum current on any single dipole corrector.

Species	k	$\frac{B\rho(k)}{B\rho(5)}$	sinkv	coskv	I_{max} [A]
Protons	3	-	0.9	-6.468	6.530
	4	-	0.0	18.0	24.306
Helions	5	-	10.0	-18.0	23.086
	6	1.571	15.0	-10.0	24.672
	7	2.051	-10.0	-10.0	24.246
	8	2.491	4.0	-13.0	24.491
	9	2.911	-1.1	0.0	17.924
	10	3.318	10.0	10.0	23.459

Conclusion

- Simulations of imperfection resonances using misaligned quadrupoles from survey data match experimental scan data for protons crossing $G\gamma=3$ and $G\gamma=4$.
- This method was extended to simulate these resonances for helions.
- These simulations determined there is sufficient corrector current to correct each of the imperfection resonances crossed as helions are accelerated to $|G\gamma|=10.5$.

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