A Dispersive Quadrupole Scan Technique for Transverse Beam Characterization

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Dispersive quadrupole scans

- Quadrupole scans are used to deduce transverse beam properties by scanning a quadrupole and fitting a parabola to the measured squared beam sizes

- However, the method is not directly applicable if dispersion is not negligible
  - Dispersive contribution must be removed in post-processing by assuming design values or measuring dispersion and propagating it for the different quadrupole settings

- We attempt to include fitting of the dispersive parameters in the quadrupole scan analysis

Dispersive quadrupole scans

- The squared beam sizes at a measurement point (MP) down stream of a point of interest (POI) including dispersion is

$$\sigma_{MP}^2 = m_{11}^2(\epsilon\beta_{POI} + (\eta_{POI}\sigma_\delta)^2) + 2m_{11}m_{12}\left(-\epsilon\alpha_{POI} + \eta_{POI}\eta'_{POI}\sigma_\delta^2\right)$$
$$+m_{12}^2(\epsilon\gamma_{POI} + (\eta'_{POI}\sigma_\delta)^2) + m_{13}^2\sigma_\delta^2 + 2m_{11}m_{13}\left(\eta_{POI}\sigma_\delta^2\right) + 2m_{12}m_{13}\left(\eta'_{POI}\sigma_\delta^2\right)$$

- Scanning one (or several) quadrupoles in a total of N steps will lead to a $N \times 6$ matrix on the form

$$\mathbf{M} = \begin{pmatrix} m_{11,1}^2 & 2m_{11,1}m_{12,1} & m_{12,1}^2 & m_{13,1}^2 & 2m_{11,1}m_{13,1} & 2m_{12,1}m_{13,1} \\ m_{11,2}^2 & 2m_{11,2}m_{12,2} & m_{12,2}^2 & m_{13,2}^2 & 2m_{11,2}m_{13,2} & 2m_{12,2}m_{13,2} \\ \vdots & & & \vdots & & \vdots \\ m_{11,N}^2 & 2m_{11,N}m_{12,N} & m_{12,N}^2 & m_{13,N}^2 & 2m_{11,N}m_{13,N} & 2m_{12,N}m_{13,N} \end{pmatrix}$$

- Resulting beam sizes can be written $\overrightarrow{\sigma^2}_{MP} = \boldsymbol{M}\vec{p}_{POI}$ with

$$\vec{p} = \left(\beta\epsilon + (\eta\sigma_\delta)^2, -\epsilon\alpha + \eta\eta'\sigma_\delta^2, \epsilon\gamma + (\eta'\sigma_\delta)^2, \sigma_\delta^2, \eta\sigma_\delta^2, \eta'\sigma_\delta^2\right)^T$$

and found from measured beam sizes using matrix inversion $\vec{p} = \boldsymbol{M}^{-1}\,\overrightarrow{\sigma^2}$

Dispersive quadrupole scans

- $m_{13} \neq 0$ in order to fit dispersive parameters
  – Best if m_{13} is not constant either

$$\mathbf{M} = \begin{pmatrix} m_{11,1}^2 & 2m_{11,1}m_{12,1} & m_{12,1}^2 & m_{13,1}^2 & 2m_{11,1}m_{13,1} & 2m_{12,1}m_{13,1} \\ m_{11,2}^2 & 2m_{11,2}m_{12,2} & m_{12,2}^2 & m_{13,2}^2 & 2m_{11,2}m_{13,2} & 2m_{12,2}m_{13,2} \\ \vdots & & & \vdots & & \vdots \\ m_{11,N}^2 & 2m_{11,N}m_{12,N} & m_{12,N}^2 & m_{13,N}^2 & 2m_{11,N}m_{13,N} & 2m_{12,N}m_{13,N} \end{pmatrix}$$

- We achieve that by scanning two quadrupoles, one of them after a dipole such that the transfer line is on the form: POI ⇨ Quad ⇨ Dip ⇨ Quad ⇨MP
  – Several other magnetic elements may appear in the transfer line

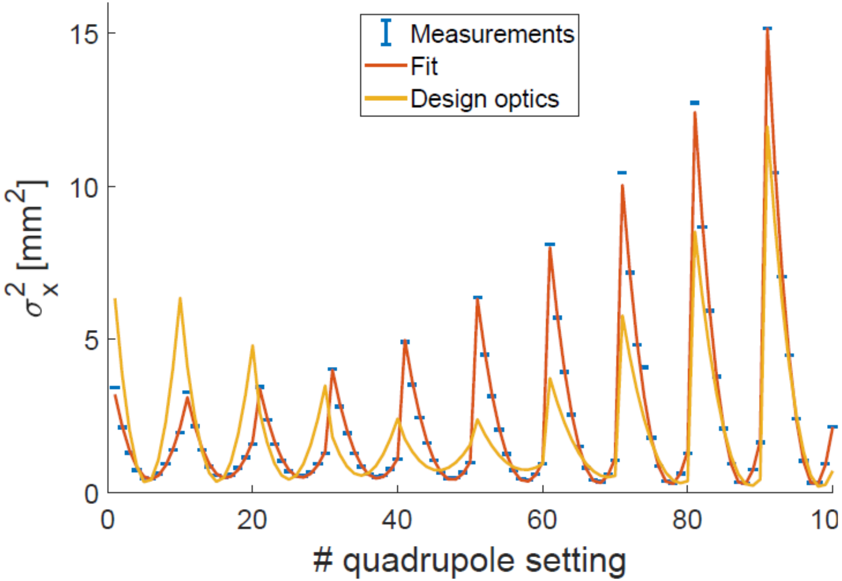
- Care must be taken to ensure that $\boldsymbol{M}$ is well-conditioned

ESRF TL2

- Method tested in ESRF TL2, scanning two quadrupoles simultaneously, leading to 100 quadrupole settings

- Results reasonably close to design values
- However, η_x not in good agreement with direct dispersion measurements ($\eta_x = -0.27\ [m]$)

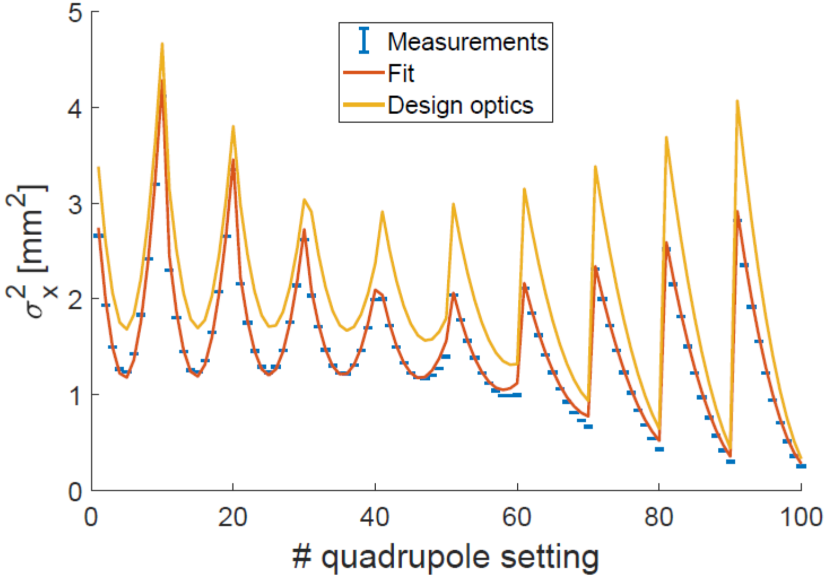
- The different value of α_x should be corrected for when matching to the storage ring



		β_x [m]	α_x	ϵ_x [nm rad]	η_x [m]	η'_x [$\times 10^{-3}$]	σ_δ [$\times 10^{-4}$]
ESRF	Design	6.73	-2.21	85	-0.14	-0.09	13.0
	Fit	6.83 ± 0.12	-0.89 ± 0.04	90 ± 2	-0.05 ± 0.02	-0.06 ± 0.01	12.8 ± 0.4

SLS BRTL

- Method tested in SLS BRTL
- Reasonable results but ambiguous when redoing measurements
- $\beta_x \epsilon_x$ found to be robust
  – $\beta_x \approx 30.5\ [m]$ found by assuming design emittance
- However, no conclusive results...



		β_x [m]	α_x	ϵ_x [nm rad]	η_x [m]	η'_x [$\times 10^{-3}$]	σ_δ [$\times 10^{-4}$]
SLS	Design	31.5	-5.60	9.6	0.22	-0.01	7.3
	Fit	22.4 ± 5.8	-3.89 ± 1.05	12.1 ± 3.2	0.37 ± 0.02	0.01 ± 0.00	6.6 ± 0.1

Discussion

- Camera calibration errors directly influence fit results
- The presented method suffers from ambiguous fit results when redoing measurements or removing a few data points
- Additional scans with more measurement points must be done to provide conclusive results