

# ELECTROMAGNETIC MODELLING OF KICKER MAGNETS TO DERIVE EQUIVALENT CIRCUITS

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## Abstract

An equivalent circuit model of a kicker magnet system is an invaluable tool for predicting and optimizing performance. The frequency content of pulses associated with a transmission line kicker magnet generally extends up to a few tens of MHz: hence, it is feasible to accurately model such a kicker magnet using lumped elements. This technique is powerful as it, in general, has a run time several orders of magnitude less than a full wave electromagnetic simulation. In this paper, we determine values, including those of parasitic components, using modern simulation tools, for use in the lumped equivalent circuit. In addition, a method to simulate coupling between beam and the electrical circuit of a kicker magnet system, at relatively low frequencies, is described: this allows circuit analysis tools to be used to study means of mitigating beam induced resonances.

## INTRODUCTION

Each stage of an accelerator system has a limited dynamic range and therefore a chain of accelerators is required to reach high energy. A combination of septa and kicker magnets is frequently used to inject and extract beam from each stage. The kicker magnets typically produce rectangular field pulses with fast rise- and/or fall-times: the field must rise/fall within the time period between batches of beam [1, 2]. In addition, the magnetic field must not significantly deviate from the flat-top value or from zero between pulses. Circuit simulation and finite element (FEM) codes greatly assist the goal of obtaining high performance kicker systems. It is feasible to accurately model a kicker magnet using lumped elements: simulation of circuits which include almost all known parasitic elements and non-linearities is possible [3].

The CERN Proton Synchrotron Booster (PSB) has been known to suffer from horizontal instabilities since its early operation in the 1970s [4]. The instability is due to coupling of the beam with the kicker magnets [4, 5]. Thus, to study mitigating measures, it is necessary to simulate the beam coupling with the kicker magnet, including the electrical circuit in which the magnet is connected.

## EQUIVALENT CIRCUIT

A transmission-line magnet consists of a few to many 'cells'. A cell typically consists of a C-shaped ferrite core sandwiched between high-voltage (HV) capacitance plates: ground plates are interleaved between the HV plates to form a capacitor [1]. Capacitors can be installed between the HV and ground plates to increase the capacitance value [6].

The pulse must not significantly degrade while travelling through the magnet. The cut-off frequency,  $f_c$ , is a key parameter [3], and is given by:

$$f_c = \frac{1}{\pi \sqrt{(L_{mc} + 4 \cdot L_{cs}) \cdot C_c}}, \quad (1)$$

where:  $L_{mc}$  is the cell inductance (see below);  $C_c$  is the value of the cell capacitance; and  $L_{cs}$  is the parasitic inductance of the cell capacitance, including inductance of the capacitors and the negative forward magnetic coupling between the cells [3]. There are frequency dependent losses in the ferrite core of each cell, which can help damp oscillations [6].

## CST simulations

At CERN the individual cells of a transmission line kicker magnet have generally been modelled using 2D FEM codes, where the equivalent cell inductance is determined per unit length. A 2D model does not take into account end-effects. In addition, the 2D modelling has previously not taken into account frequency dependent properties of the ferrite permeability. By modelling the cells in a full wave solver, such as *CST MWS* [7], the finite length as well as material frequency dependency are taken into consideration. This allows for the derivation of lumped element values which cannot be determined from a 2D FEM model.

The approach used here is to create a 3D model of a single cell of the transmission line magnet, in *CST MWS*, and run an S-parameter simulation. The cell is fed and terminated using discrete ports that match the characteristic impedance of the cell. The transmission parameter,  $S_{21}$ , is then compared with that of an equivalent circuit of the cell (Fig. 1, inset). The value of parasitic inductance  $L_{cs}$  and its parallel resistance  $R_{cs}$  are modified until the circuit reproduces the required frequency response. The resistance, which represents the frequency dependent losses of the ferrite ( $\mu_r''$  from the datasheet), over a limited frequency range, was initially modelled in parallel with the cell inductance  $L_{mc}$ : however, a better fit is obtained for  $S_{21}$  by modelling the resistance in parallel with  $L_{cs}$ . Figure 1 shows that an equivalent circuit, with fixed values, can be tuned to match the full wave 3D simulations very well. Figure 1 also shows the effects that different boundary conditions have on  $S_{21}$ . Using a perfect electrical conductor (PEC) boundary close to the cell ( $\ll L_{cell}$ ), where  $L_{cell}$  is the physical length of the cell, represents a central cell. A PEC boundary far away from the cell ( $\gg L_{cell}$ ), increases the fringe fields at one end of the cell and represents an end cell of the magnet.

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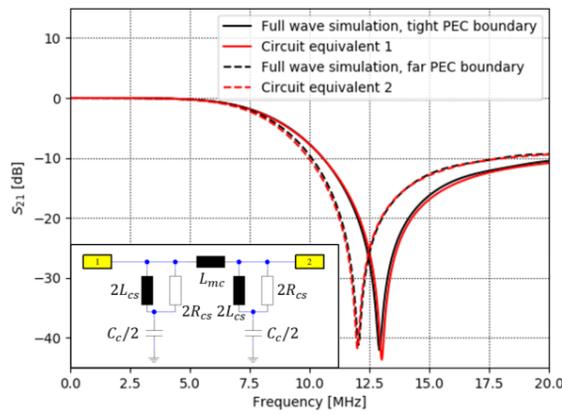


Figure 1: Full wave simulations of an LHC injection kicker magnet cell compared to circuit equivalents. The inset depicts the  $\pi$ -network used to model the cell.

### Cell Inductance

Inductance is only defined for closed loops, and the inductance depends on the return path of the current. A circuit can be treated as being made up of “partial inductances”, but this is only valid when a complete loop is considered and all the mutual terms between the partial inductances are included [8]. For a kicker magnet the inductance for the Go and Return conductor ( $L_{mc}$ ), for a central cell, can be calculated using the following equation [1]:

$$L_{mc} = \mu_0 \cdot \frac{N^2 \cdot H_{ap}}{V_{ap}} \cdot l_{eff}, \quad (2)$$

where:  $\mu_0$  is the permeability of free space;  $N$  is the number of turns, typically 1 for a fast kicker magnet;  $V_{ap}$  is the distance between the legs of the ferrite in the magnet aperture;  $H_{ap}$  is the distance between the Go and Return conductors in the magnet aperture; and  $l_{eff}$  is the effective length of the cell. Equation (2) assumes that the relative permeability of the ferrite is very high. The cell inductance calculated from Eq. (2) is the total inductance, i.e. the self of both the Go and Return conductors and the mutual between these conductors.

The cross-section of the kicker magnet can be modelled using an electromagnetic code such as Opera-2d [9], with current in the Go and Return conductors: these simulations assume that the model is infinitely long in the z-direction, with x and y components of flux, and z-directed currents. Total inductance can be calculated from the predicted stored energy: as per Eq. (2), this value is the sum of the self and mutual inductances of both conductors.

In order to simulate coupling between beam and a kicker magnet in Opera-2d, the beam is represented as a conductor: however, to properly model the interaction of the beam with the kicker magnet, it is necessary to determine both self and mutual partial inductances associated with the ‘conductors’. Figure 2 shows a simplified equivalent circuit of one cell of a kicker magnet, with an inductance for each of the Go ( $L_{go}$ ), Return ( $L_{return}$ ) and Beam ( $L_{beam}$ ) ‘conductors’. The partial self inductance of a conductor and partial mutual

inductance of one conductor to another can be calculated using the flux linkage: considering two points in the cross-section, the flux linking lines parallel to the z-direction, through the two points, is given by the difference in the vector potential ( $A_z$ ) between the points [10]. Calculating inductance from flux linkage is a powerful method when multiple conductors are involved and the return path of a current is not well defined.

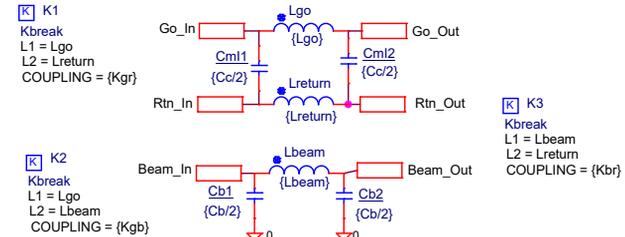


Figure 2: Central cell partial inductance circuit model, including a conductor for the beam.

**Partial Self Inductance** Partial self inductance of a conductor is modelled by simulating a current flow through a conductor, without current in other conductors: self inductance is calculated from the energy or the flux linkage method. The latter involves calculating the average vector potential over the area of the conductor carrying current, and then dividing by the area of this conductor [10].

**Partial Mutual Inductance** Partial mutual inductance of a conductor is modelled by simulating a current flow through one conductor, without current in other conductors. Mutual inductance with the current carrying conductor, is determined by calculating the average vector potential over another conductor, and then dividing by the area of this second conductor [10].

**Partial Inductance Matrix** The procedure is repeated for each conductor in turn, simulating current in only one conductor at a time and then calculating the partial self-inductance ( $L$ ) of this conductor, and the partial mutual inductances ( $M$ ) to other conductors. For a kicker magnet and beam, the following partial inductance matrix can be constructed, with self inductances on the main diagonal:

$$\begin{pmatrix} L_{go} & M_{(beam,go)} & M_{(return,go)} \\ M_{(go,beam)} & L_{beam} & M_{(return,beam)} \\ M_{(go,return)} & M_{(beam,return)} & L_{return} \end{pmatrix}, \quad (3)$$

where:

$$L_{mc} \approx (L_{go} - M_{(return,go)} - M_{(go,return)} + L_{return}). \quad (4)$$

The coupling coefficient value ( $k_{(1,2)}$ ) between conductors ‘1’ and ‘2’, see Fig. 2, is given by:

$$k_{(1,2)} = M_{(1,2)} / \sqrt{(L_1 \cdot L_2)}. \quad (5)$$

In Fig. 2 all the inductors are drawn electrically in the same sense (dot at the left hand side): whether the mutual inductances sum with or subtract from the self inductances is dependent upon the directions of the currents which flow.

## Time Domain Simulations

To validate the equivalent circuit with the partial inductances against a ‘normal’ model (which represents total inductance per cell), the coupling coefficients between ‘Lbeam’ and both ‘Lgo’ and ‘Lreturn’ were set to zero - it only makes sense to simulate a non-zero coupling when the interaction of the beam with the electrical circuit of the off-state kicker system is being studied. The resulting time domain predictions for the two models were identical [11].

## COUPLING OF BEAM TO THE KICKER MAGNET SYSTEM

### Methodology

Figure 2 shows inductor ‘Lbeam’ for simulating beam current in the aperture of a cell of a kicker magnet. On both sides of the ‘Lbeam’ a capacitor to ground, of value ‘{Cb/2}’, is shown. The purpose of this is to give an appropriate propagation delay of the ‘beam’ through each cell.

Either frequency or time domain simulations of the beam interaction with the electrical circuit can be carried out: here frequency domain simulations are reported. The ‘beam conductor’ is excited using a current source and the frequency is swept up to 12 MHz. A resistance of  $\sqrt{\frac{\{Lbeam\}}{\{Cb\}}}$ , is modelled in parallel with the current source and also in series with the output of the ‘beam conductor’.

Reference [12] describes techniques used to measure the longitudinal and transverse impedance of accelerator components. The Opera-2d model, with the beam represented as a conductor in the aperture, can be considered to be analogous to the measurement technique used in [12]. Thus, to interpret the predictions of PSpice simulations with the ‘beam conductor’, for both longitudinal and transverse impedance, equations from [12] are used [11]. The Opera-2d simulations are run with various horizontal or vertical offsets of the ‘beam conductor’, in the magnet aperture, to determine the coupling coefficients. The appropriate coupling coefficients are simulated in PSpice to allow the influence of the electrical circuit on longitudinal or transverse beam coupling impedance to be evaluated [11].

To solve equation 2.5 of [12], there is a need for a reference  $S_{21,REF}$  ‘measurement’.  $S_{21,REF}$  for a homogeneous matched line corresponds to the electrical length of the Device Under Test [12]. Hence, the reference is derived from PSpice simulations with zero mutual coupling between the ‘beam conductor’ and other conductors modelled. In the PSpice model, the ‘beam conductor’ has distributed capacitance to ground to give an appropriate time of flight of the beam through a magnet [11], i.e. the relative velocity,  $\beta$ .

Figure 3 shows the PSpice prediction for the real part of the beam coupling impedance for a PSB extraction kicker magnet: the magnet is terminated in a short-circuit at one end and, at the other end, has 18 m of transmission cable connected to an off-state thyratron [11]. Filters [4] are included in the equivalent circuit [11]. The first high impedance resonance occurs at 1.8 MHz, and subsequent resonances are at

odd integer multiples of 1.8 MHz. The prediction shown in Fig. 3 agrees well with measurements [4].

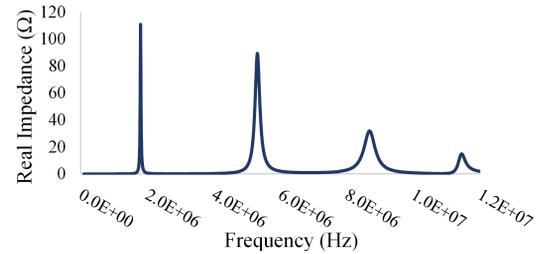


Figure 3: PSpice prediction for the real part of the longitudinal beam coupling impedance, for a short-circuit PSB extraction kicker magnet.

### Verification Against CST

In order to verify the equivalent circuit, of the beam coupling to the electrical circuit, both PSpice and CST have been run with the same beam current profile (single-bunch beam, at injection ( $\beta = 0.52$ ), of  $1 \times 10^{13}$  protons at 160 MeV with a full bunch length of 480 ns (full width half magnitude of  $\sim 330$  ns)), in the time domain. In PSpice the beam current is simulated as a piece-wise-linear current source. In both simulations, the magnet Go conductor was modelled as being open circuit at the pulse input end - i.e. no transmission cables attached - and short circuit at the magnet output. Figure 4 shows the beam current for both simulations (green, dashed): the figure also shows the voltage predicted at the open-circuit end of the kicker magnet, from both CST (courtesy C. Zannini) and PSpice - the two predictions are in excellent agreement [13].

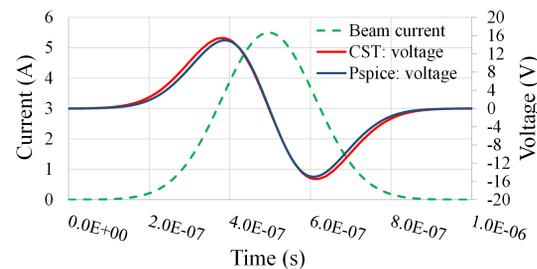


Figure 4: Beam current (green, dashed) modelled and voltage predicted at the open circuit end of the kicker magnet, using both CST (red) and PSpice (blue).

## CONCLUSION

A 3D model has been used to derive values for both parasitic inductance and frequency dependent ferrite losses, for a lumped element equivalent circuit of a cell of a kicker magnet. In addition, a 2D model has been used to derive values for a partial inductance matrix, for simulating low frequency coupling between beam and the electrical circuit of a kicker magnet. The partial inductance matrix has been included in a PSpice equivalent circuit, and the method for interpreting predictions for beam coupling impedance has been described: predictions have been validated. This modelling technique allows means of mitigating beam induced resonances, in a kicker system, to be studied.

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