PARAMETER ESTIMATION OF SHORT PULSE NORMAL-CONDUCTING STANDING WAVE CAVITIES

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Abstract
The linear accelerator ARES (Accelerator Research Experiment at SINBAD) is a new research facility at DESY. Electron bunches with a maximum repetition rate of 50 Hz are accelerated to a target energy of 155 MeV. The facility aims for ultra-stable sub-femtosecond arrival-times and high peak-currents at the experiment, placing high demands on the reference distribution and field regulation of the RF structure. In this contribution, we present the physical parameter estimation of key RF properties such as cavity detuning not directly measurable on the RF field decay. The method can be used as a fast monitor of inner cell temperature. The estimated properties are finally compared with the measured ones.

INTRODUCTION
The estimation of key parameters of an RF structure is essential for proper system setup and operation, but also for additional system checks e.g. fault detection and isolation. We propose the use of an estimation algorithm based on cavity white box model to estimate the coupling factor, the detuning and the half bandwidth. The commonly used detuning detection by the RF field delay is sometimes not usable due to missing RF circulator. This circulator blocks the backwards traveling wave and its superposition with the forward wave, while the decay approach relies on zero drive signal to detect the free response of the RF structure. Furthermore, the use of the entire RF pulse information with much more data-points should increase the resolution and precision of the parameters of interest. For that, the signals used for parameter estimation need to be accurately calibrated. This, together with the relying white box model is introduced in this paper.

THEORETICAL APPROACH
The estimation of physical quantities of a standing wave RF structure relies on the proper signal calibration of detected signals. In principle two signals, i.e. forward and the reflected signal, are sufficient to estimate the main properties. For that the signals need to be calibrated, often done by resonance circle and its complex reflexion coefficient [1].

The sum of calibrated forward and reflected signal yields a virtual probe signal corresponding to the stored energy in the cavity. At the ARES facility we have additionally the probe signal as detector mounted in the full cell of the 1.5 cell structure [2]. Hereby we do not have to scan the resonance frequency for calibration.

Signal Calibration
The detected forward, reflected and probe RF signals are expressed in the complex domain

$$\vec{V}_{\text{forw},t} + i\vec{V}_{\text{refl},t} = \vec{A}_{\text{forw}} \cdot e^{i\vec{\Phi}_{\text{forw}}} = \vec{V}_{\text{forw}} \in \mathbb{C},$$

$$\vec{V}_{\text{refl},t} + i\vec{V}_{\text{probe},t} = \vec{A}_{\text{refl}} \cdot e^{i\vec{\Phi}_{\text{refl}}} = \vec{V}_{\text{refl}} \in \mathbb{C} \text{ and }$$

$$\vec{V}_{\text{probe},t} + i\vec{V}_{\text{probe},Q} = \vec{A}_{\text{probe}} \cdot e^{i\vec{\Phi}_{\text{probe}}} = \vec{V}_{\text{probe}} \in \mathbb{C}.$$  

The probe signal can be calibrated with a spectrometer based measurement and its RF simulation. We can calibrate the forward and reflected signal by the complex coefficients $a$ and $b$ which need to be determined:

$$\vec{V}_{\text{probe}} = a\vec{V}_{\text{forw}} + b\vec{V}_{\text{refl}} = \vec{V}_{\text{forw}} + \vec{V}_{\text{refl}}.$$  

Hereby, the signals from klystron side are transformed to the cavity side. The probe signal is the sum of the calibrated forward $\vec{V}_{\text{forw}}$ and reflected $\vec{V}_{\text{refl}}$ signal. When all three signals are available, calibration can be carried out using linear regression

$$\begin{bmatrix} B \\ A \end{bmatrix} \cdot x = [\vec{V}_{\text{forw}} \vec{V}_{\text{refl}}] \begin{bmatrix} a \\ b \end{bmatrix}.$$  

The unknown parameter $x = [a \ b]^T \in \mathbb{C}^{2\times1}$ can be estimated using a least squares method by

$$B = A \cdot x \iff x = (A^T A)^{-1} A^T B.$$  

System Model
For an RF structure operated in standing wave mode, the static cavity probe signal is given as

$$\vec{V}_{\text{probe}} = \frac{\beta}{\beta + 1} \cdot \cos(\psi) e^{i\psi} \cdot \left(2 \cdot \vec{V}_{\text{forw}} + \frac{R}{\beta} \vec{I}_b \right),$$  

with tuning angle $\psi$, coupling factor $\beta$ and the transformed forward signal $\vec{V}_{\text{forw}}$ [3]. The beam loading $\vec{I}_b$ for a single pC bunch with picoseconds duration given in Eq. (2) can be neglected. Only the forward and probe signal of Eq. (2) is directly measurable. To first order approximation the detuning angle $\psi$ is proportional to the resonance frequency offset $\Delta f$ and can be computed by the forward and probe phase if the system is in steady state condition [1]. If this is not the case, the static equation must be replaced by the
The dynamic transfer function mapping the forward to the probe signal
\[ \frac{V_{\text{probe}}}{V_{\text{forw}}} = G(s) = \frac{\beta}{s + \omega \frac{1}{2}} \frac{2\omega \frac{1}{2}}{s + \omega \frac{1}{2} - j\Delta\omega} = \frac{K}{s - p_1}, \]
with static gain \( K \) and complex eigenvalue \( p_1 \) as
\[ K = \frac{2\beta \omega \frac{1}{2}}{\beta + 1} \quad \text{and} \quad p_1 = (-\omega \frac{1}{2} - j\Delta\omega). \]
The eigenvalue consists of the cavity detuning \( \Delta\omega \) and the half cavity bandwidth \( \omega \frac{1}{2} = \omega_0 / (2 \cdot Q_L) \). The gain factor \( \beta \) hosts the coupling parameter \( \beta \). Since a whole RF pulse is used to estimate the parameters its usability is limited to following assumptions:

- Short RF pulse in microsecond range.
- No intra-pulse heating/detuning variation of the RF structure nor the input coupling changing the coupling coefficient and half bandwidth.
- Delay of the signal detection for the forward, reflected and probe signal are of same size.
- Signals used for parameter estimation physically next to each other to avoid additional effects like signal damping etc.

### Physical Parameter Estimation

The approach used in this paper is based on an estimation method for system identification known as the Prediction Error Method (PEM) [4]. The input and output data, i.e. complex forward and probe signal, is used for an optimization resulting in a system model in Laplace or s-domain. The model order is chosen to be one to estimate one complex eigenvalue \( p_1 \) of the system model \( G_{\text{est}}(s) \). The system model is converted into continuous time using the zero order hold (‘zoh’) approximation. The relation to physical properties is given by the real and imaginary part of the complex eigenvalue as
\[ f_1 = \omega_1 / (2\pi) = -\Re(p_1) \quad \text{and} \quad \Delta f = \omega_1 / (2\pi) = -\Im(p_1). \]
The steady state solution to a step response of the transfer function is computed as the low frequency (static) gain of the identified system model and given as
\[ G_0 = G(s)|_{s \to 0} = \frac{\beta}{(\beta + 1) \omega_1 \frac{1}{2} - j\Delta\omega}. \]
Sorting this by the remaining unknown parameter \( \beta \) yields
\[ \beta_{\text{est}} = \frac{G_0}{2\omega_1 \frac{1}{2}} \frac{\omega_1 \frac{1}{2}}{G_0} = (\omega_1 \frac{1}{2} - j\Delta\omega). \]
The estimated coupling factor \( \beta_{\text{est}} \) can further be used to determine the resonance circle given by complex reflection coefficient and the two intersection points with the real axis, i.e. -1 as full reflection and \( \Gamma_{\text{min}} = (\beta_{\text{est}} - 1) / (\beta_{\text{est}} + 1) \).

### EXAMPLE FOR THE ARES RF-GUN

The ARES RF-gun is a 1.5 cell normal conducting RF structure operated at 2.998 GHz in a standing wave mode [2]. The RF-gun is typically operated with up to 6 µs long RF drive signal in the MW power range. We use a MicroTCA.4 based LLRF system in a single cavity regulation scheme to keep the overall signal processing latency small (typically below 700 ns) [5, 6].

#### Signal Detection and Calibration

The signal detection for the forward, reflected and probe signal is done on the same ADC (SIS8300-L2) board. The cable lengths from the directional coupler (forward and reflected signal) and the RF-gun probe pickup to the ADC card are almost identical. The distance between directional coupler and the RF structure is less than 2 m such that the signal delay on the waveguide distribution is neglectable [5]. The ADC ranges have been optimized for the signals to operate at about 70% for nominal operation. This ensures the proper detection of outliers and reserves margin for an increase in operating gradient. The calibration coefficients \( a \) and \( b \) of Eq. (1) are used to correct the measured forward and reflected signals with relation \[ V_{\text{probe}} = aV_{\text{forw}} + bV_{\text{refl}}. \]
The signals are typically re-calibrated only when it becomes necessary, Fig. 1.

![Figure 1: Calibrated signals of the ARES RF-gun in amplitude and phase for the probe, forward reflected signal together with the virtual probe.](image-url)
Parameter Estimation

The parameter estimation and additional data recording and saving is carried out by a Matlab script with an update time of 0.2 s to 0.3 s. The processing time is limited by the estimation of the system model as a function of the forward and probe signal. The extraction of the coupling factor $\beta$, the half bandwidth $\omega_{1/2}$ and the detuning $\Delta\omega$ follows the equations given in Physical Parameter Estimation. The physical parameters together with the calibration coefficients are shown for a long-term analysis in Fig. 2. The calibration coefficients vary slightly in phase, which is currently not understood and requires further investigation. The coupling factor have been estimated with $\beta = 1.25$ and the half bandwidth as $f_{1/2} = 244$ kHz. The variation of the detuning is forced by the change of the water inlet temperature into the RF-gun. The water inlet temperature set-point has been changed first by -0.05 K and to a later time by +0.05 K. Steady state conditions are typically reached after 5 minutes. The water temperature sensors for the water inlet and outlet temperature are with 24-bit resolution and an update frequency of 1 Hz. Comparing the measured and estimated water temperature using static conversion factor of 48.5 kHz/K shows very nice agreement. This conversion factor is a result of a correlation plot. It can be seen that the steady state value for the measured input and output water temperature is almost identical to the estimated value using the detuning. A delay between the water temperature and the estimated temperature has been observed for rapidly changing temperatures. This could result from the measurement points, i.e., the water temperature sensor as a measure of the external RF gun temperature and the RF-based temperature estimate as the internal temperature. The standard deviation for data in steady state (time range of 60 min to 95 min) is $\sigma(\beta)/\beta \approx 0.1\%$, $\sigma(f_{1/2}) \approx 275$ Hz and $\sigma(\Delta\omega) \approx 300$ Hz corresponding to $\sigma(T_{\text{est}}) \approx 5.7$ mK. The standard deviation for the water inlet and the outlet temperature sensor shows a standard deviation of 2.2 mK each.

CONCLUSION

The usability of a physical parameter estimation based on white box model for RF structures operated with a short RF pulse has been studied. The underlying restrictions such as no intra-pulse detuning nor coupling factor variation allows to estimate in each RF pulse the detuning (if normalized with half bandwidth), coupling factor and half bandwidth at 0.1% precision. This makes the approach suitable for fault detection and/or fault isolation. The detuning parameter has been varied and the temperature estimation has been experimentally validated. It has been observed that the calibration coefficients for the forward and reflected signal vary slightly in phase. This is currently not understood and requires further investigation. A server implementation with a 10 Hz to 50 Hz update rate of the physical parameter estimate is targeted for the inner cell temperature estimate. Ideally, this estimated temperature is combined with the water temperature control scheme to improve the thermal RF-gun regulation.

REFERENCES