

# TRANSVERSE EMITTANCE CHANGE AND CANONICAL ANGULAR MOMENTUM GROWTH IN MICE ‘SOLENOID MODE’ WITH MUON IONIZATION COOLING

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 on behalf of the MICE collaboration

## Abstract

Emittance reduction of muon beams is an important requirement in the design of a Neutrino Factory or Muon Collider. Ionization cooling, whereby beam emittance is reduced by passing a beam through an energy-absorbing material, requires tight focusing in the transverse plane which is achieved in many designs using solenoid focusing. In solenoid focusing, the beam acquires kinetic angular momentum due to the radial field in the solenoid fringe. Cooling in ‘flip’ mode, where the beam-focusing solenoid field changes polarity at the absorber, has already been demonstrated in the Muon Ionization Cooling Experiment (MICE). In this mode the absorber is near to the field flip, so the kinetic angular momentum is zero at the absorber. ‘Solenoid mode’ cooling, where the field polarity does not change across the absorber leading to a beam crossing the absorber with significant kinetic angular momentum, has been considered for the final section of the muon collider design due to potentially stronger focussing that it enables. In this paper, we present the performance of MICE in ‘solenoid mode’.

## THE MUON IONIZATION COOLING EXPERIMENT (MICE)

Future muon beam facilities like a Neutrino Factory or Muon Collider require significant reductions in the occupied phase-space volume (cooling) of the muon beams which supply them. Ionization cooling, proposed as the only viable emittance reduction method to meet these beam requirements, utilizes a low Z material absorber to reduce both longitudinal and transverse muon momentum before re-accelerating the beam longitudinally.

Ionization cooling has been demonstrated by the MICE collaboration in ‘flip mode’ [1], where the solenoid field polarity alternates across the absorber. In an extended cooling channel with successive absorber passes, reversing field polarity in this manner prevents build-up of canonical angular momentum and improves cooling performance [2], but is costly to implement. An alternative approach where the field polarity is kept constant throughout the cooling channel has been implemented by MICE, referred to as ‘solenoid mode’, with both field configurations shown in Fig. 1. Final stage cooling for the muon collider utilising ‘solenoid mode’ cooling channels can potentially achieve very low emittances [3]. Cooling performance in ‘solenoid mode’ is

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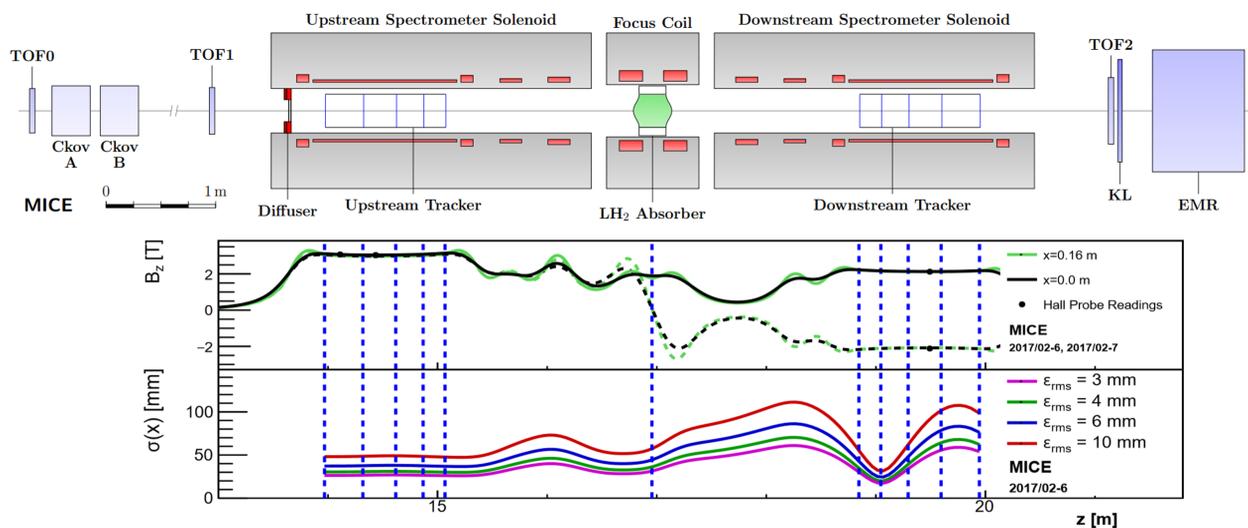


Figure 1: (Top) Layout of the MICE cooling channel, showing (red) magnet coils, (blue) tracker stations, time-of-flight (TOF) detectors, Cherenkov (Ckov) detectors, lead-scintillator sandwich (KL) detector, Electron-Muon Ranger (EMR); (Middle) Axial-component of the modelled solenoid field vs z for (solid) ‘solenoid’ mode and (dashed) ‘flip’ mode; (Bottom) Nominal ‘solenoid’ mode beam envelope in x for four emittance values, illustrating a well-constrained beam within tracker. Blue dashed lines indicate tracker stations and absorber centre. Hall probes are positioned at 0.16 m radial displacement.

presented in the following analysis, with measurement of the canonical angular momentum growth of the beam.

## EVENT SELECTION

MICE operated a tertiary muon beam by capturing pions produced from proton interactions on a titanium target and allowing them to decay into muons. Dipoles momentum-selected the pions and their decay muons before they were transported to the MICE cooling channel. Figure 1 shows the cooling channel and detector suite.

A series of particle identification detectors upstream and downstream of the main cooling channel [4] provided rejection of pion or decay electron events. Scintillating fibre trackers contained within the superconducting solenoids were positioned upstream and downstream of the absorber module and measured particle trajectories in a uniform 2-4 T magnetic solenoid field, allowing reconstruction of particle positions and momenta through the trackers [5].

Several absorber configurations were used, including a 221 liquid hydrogen vessel in both empty (Empty LH<sub>2</sub>) and full (Full LH<sub>2</sub>) states, a 65 mm lithium hydride disk (LiH), and an empty drift space (No absorber). The empty LH<sub>2</sub> vessel and drift space provide controls for the emittance exchange measurement. Events were required to have time-of-flight consistent with the tracker reconstruction for a muon of between 135 and 145 MeV/c momentum. Each track was also required to be fully contained within the fiducial volume and to have a track goodness-of-fit  $\chi^2/NDF < 8$ . This analysis examines several beams with nominal normalised RMS emittances of 3, 4, 6 and 10 mm, referred to as ‘3-140’, ‘4-140’, ‘6-140’, and ‘10-140’.

## CANONICAL ANGULAR MOMENTUM

The canonical angular momentum of single particles inside of a cylindrically symmetric solenoid oriented along  $z$

is given by

$$L_{\text{canon}} = xP_y - yP_x + qr\mathcal{A}_\phi \quad (1)$$

where

$$\mathcal{A}_\phi \approx \frac{r}{2}B(z) - \frac{r^3}{16}B''(z) + \mathcal{O}(r^5), \quad (2)$$

$r$  is the radius,  $q$  is the particle charge, and  $B(z)$  is the magnetic field strength as a function of  $z$  [6].

The change in canonical angular momentum of muons passing through each absorber module configuration, calculated to first order using the MICE solenoid field model, is shown in Fig. 2. For the no absorber and empty LH<sub>2</sub> case, the distributions remain largely symmetric around 0, whereas a significant increase is induced by the presence of the LiH disk and full LH<sub>2</sub> vessel, indicating canonical angular momentum growth.

## EMITTANCE AND AMPLITUDE

The normalized transverse r.m.s emittance of a beam, calculated from the 4-dimensional covariance matrix,  $\Sigma$ , in  $x, p_x, y, p_y$  is

$$\varepsilon_\perp = \frac{\sqrt[4]{|\Sigma|}}{m_\mu c}, \quad (3)$$

with  $m_\mu$  the muon mass. Defining the single-particle amplitude at a point  $p = (x, p_x, y, p_y)$  as

$$A_\perp = \varepsilon_\perp (p - \bar{p})^T \Sigma^{-1} (p - \bar{p}), \quad (4)$$

with  $\bar{p}$  the centre of the distribution, then the normalized r.m.s emittance is proportional to the mean of the particle amplitude distribution. This estimates the emittance of a beam characterised by an ellipse passing through point  $p$ .

For a beam well-described by a multivariate Gaussian distribution, amplitudes are distributed as

$$f(A_\perp) = \frac{A_\perp}{4\varepsilon_\perp^2} \exp\left(\frac{-A_\perp}{2\varepsilon_\perp}\right) \quad (5)$$

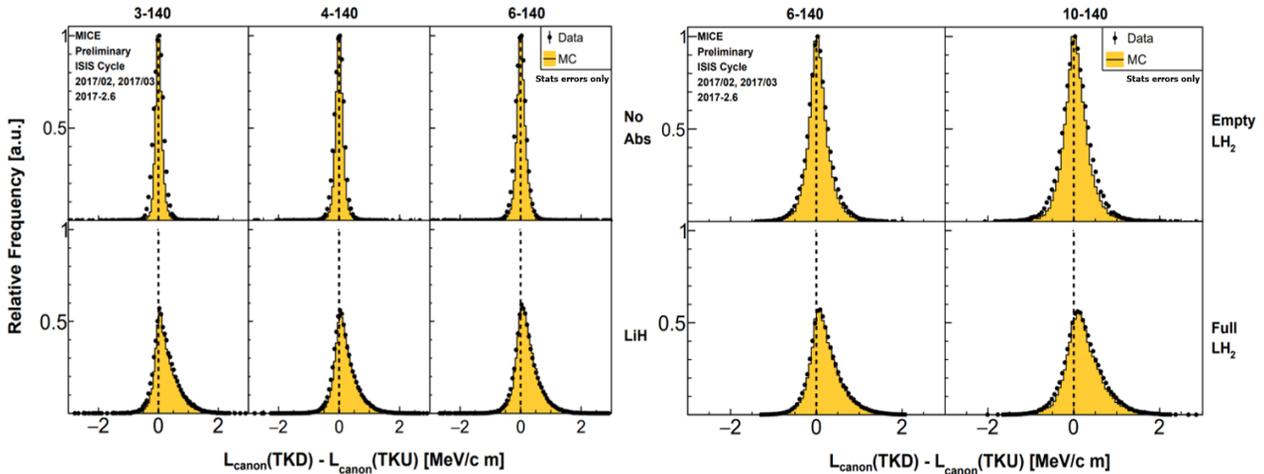


Figure 2: Canonical angular momentum growth of muons after passing through the absorber module, showing the no absorber and lithium-hydride with nominal 3, 4, 6 mm emittance beams at 140 MeV/c and the effect of the empty and full LH<sub>2</sub> vessel for nominal 6 and 10 mm emittance beams at 140 MeV/c.

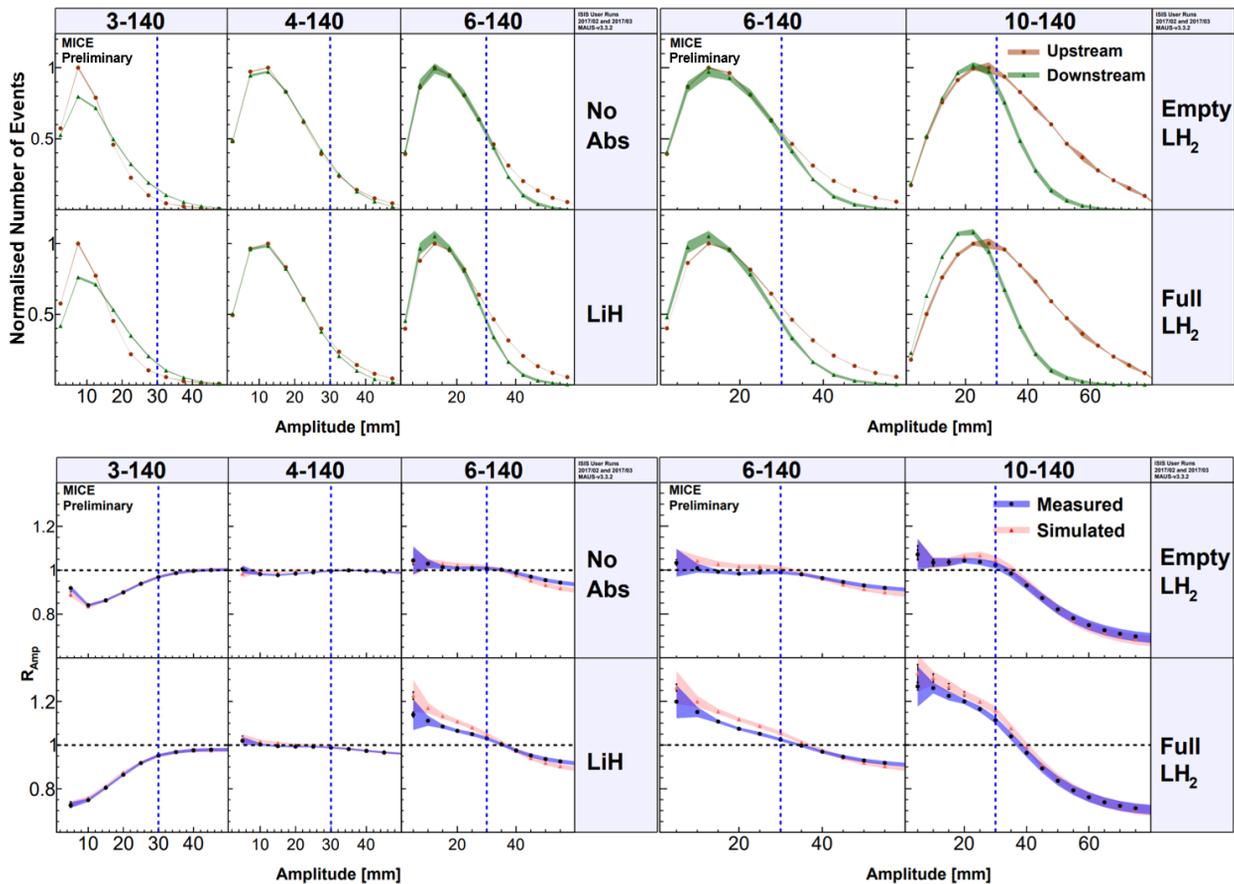


Figure 3: (Top) Normalised amplitude distributions for (left) no absorber and LiH, and (right) LH<sub>2</sub> empty and LH<sub>2</sub> full. Coloured bands show combined statistical and systematic errors for (red) upstream and (green) downstream distributions. (Bottom) Ratios of the corresponding cumulative distributions of the above, showing (left) no absorber and LiH, and (right) LH<sub>2</sub> empty and LH<sub>2</sub> full. Coloured bands show combined statistical and systematic errors for (blue) measured data and (red) simulated MC.

and hence the upstream and downstream amplitude distributions  $f^u(A_{\perp})$ ,  $f^d(A_{\perp})$  are related to the upstream and downstream emittances  $\varepsilon_{\perp}^u$ ,  $\varepsilon_{\perp}^d$  by

$$\frac{f^d(A_{\perp})}{f^u(A_{\perp})} = \left( \frac{\varepsilon_{\perp}^u}{\varepsilon_{\perp}^d} \right) \exp \left[ -\frac{A_{\perp}}{2} \left( \frac{1}{\varepsilon_{\perp}^d} - \frac{1}{\varepsilon_{\perp}^u} \right) \right]. \quad (6)$$

Figure 3 (top) shows distributions of amplitude upstream and downstream of the absorber for reconstructed data events. The downstream over upstream ratio of cumulative distributions of these amplitudes, integrated from zero, are shown in Fig. 3 (bottom) for both reconstructed data and reconstructed Monte-Carlo simulation using Geant4. Ratios above 1 demonstrate an increased muon density in the beam's core for 6 and 10 mm emittance beams, indicating a clear cooling signal in both simulated events and data. Ratios below 1 in the region above 30 mm indicate muons lost from striking the beam pipe or outside the downstream tracker fiducial volume. Heating is visible in the 3 mm beams where the ratio of cumulative distributions is below 1, also consistent in both simulated events and data.

## SUMMARY

These preliminary results show clear signals of ionization cooling in the presence of the lithium hydride and liquid hydrogen absorbers for a cooling channel operating in 'solenoid mode', with results well-described by simulations. Additionally, canonical angular momentum growth is demonstrated for muons passing these absorbers in 'solenoid mode'.

## REFERENCES

- [1] M. Bogomilov *et al.*, "Demonstration of cooling by the Muon Ionization Cooling Experiment", *Nature*, vol. 578, pp. 53-59, 2020. doi:10.1038/s41586-020-1958-9
- [2] R. C. Fernow and R. B. Palmer, "Solenoidal ionization cooling lattices", *Phys. Rev. ST Accel. Beams*, vol. 10, p. 064001, 2007. doi:10.1103/PhysRevSTAB.10.064001
- [3] D. Neuffer, "New approaches to final cooling", *AIP Conf. Proc.*, vol. 1777, p. 100005, 2016. doi:10.1063/1.4965686
- [4] M. Bogomilov *et al.*, "The MICE muon beam on ISIS and the beam-line instrumentation of the Muon Ionization Cooling Experiment", *J. Instrum.*, vol. 7, p. 05009, 2012. doi:10.1088/1748-0221/7/05/P05009

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- [5] M. Ellis *et al.*, “The design, construction and performance of the MICE scintillating fibre trackers”, *Nucl. Inst. Phys. A*, vol. 659, pp. 136-153, 2011. doi:10.1016/j.nima.2011.04.041
- [6] G. Penn and J. S. Wurtele, “Beam envelope equations for cooling of muons in solenoid fields”, *Phys. Rev. Lett.*, vol. 85, pp. 764-767, 2000. doi:10.1103/PhysRevLett.85.764