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COOLING AND DIFFUSION RATES IN COHERENT ELECTRON COOLING CONCEPTS

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Abstract

We present analytic cooling and diffusion rates for a simplified model of coherent electron cooling (CEC), based on a proton energy kick at each turn. This model also allows to estimate analytically the rms value of electron beam density fluctuations in the "kicker" section. Having such analytic expressions should allow for better understanding of the CEC mechanism, and for a quicker analysis and optimization of main system parameters. Our analysis is applicable to any CEC amplification mechanism, as long as the wake (kick) function is available.

INTRODUCTION

Let us consider a 1D longitudinal coherent electron cooling (CEC) scheme as proposed in [1–5]. Figure 1 presents a simplified schematic of CEC. The electron bunch picks up density modulations from co-propagating protons in the "Modulator" section. These density modulations are then amplified by some mechanism in the "Amplifier" section (blue).

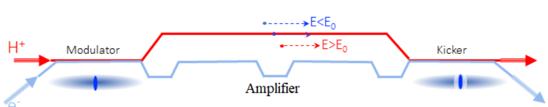

Figure 1: A simplified schematic of CEC.

The proton beam line (red) is arranged in such a way that when protons arrive at the "Kicker" section, faster (slower) protons overcome (lag behind) a reference on-energy particle.

In our simplified model we will assume that at the end of the "Kicker" section, the proton energy experiences a kick as shown in Fig. 2. For convenience, we will call the proton energy change dependence versus z the *wake function*—apart from a different normalization, it is the same as the conventional longitudinal wake in accelerator physics. For simplicity, we will assume that the proton's longitudinal position, z, in the "Kicker" section does not change and is equal to $z = R_{56}\delta$, where $\delta = \frac{\delta p}{p_0}$ is the proton's relative momentum deviation and R_{56} is the proton-line linear transfer matrix element from the end of the "Modulator" section to the "Kicker" section, i.e. z depends only on the proton momentum deviation. One can now see from Fig. 2 that faster (slower) protons would lose (gain) energy after the "Kicker" section passage.

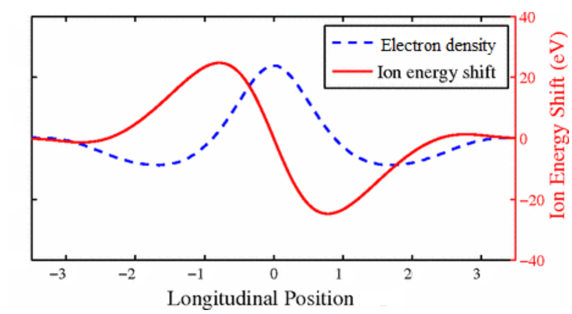

Figure 2: The electron beam density modulation due to a single proton (arb. units) and a corresponding energy kick (in eV) after the "Kicker" section as a function of the proton's longitudinal position (μm).

The wake function, introduced above, is the main element in various modifications of coherent electron cooling. For the microbunched electron cooling (MBEC) concept it was calculated in [5]; for the plasma-cascade (PCA) cooling concept, the wake function can be found in [6]. In what follows, we will use the wake function calculated for an MBEC cooler currently being designed for the electron-ion collider (EIC) (for details see [7]) and shown in Fig 2.

Table 1: CEC System Parameters (Example)

Parameter	Symbol	Value	Unit
Proton Energy	E_0	275	GeV
Lorentz Factor	γ	290	
Ring Circumference	C	3834	m
Revolution Frequency	f_0	78.3	kHz
Protons per Bunch	N_p	6.9	10^{10}
Prot. RMS Moment. Spread	δ_p	6.8	10^{-4}
Prot. RMS Bunch Length	σ_{pz}	6.0	cm
Electrons per Bunch	N_e	6.3	10^9
El. RMS Bunch Length	σ_{ez}	4.0	mm
El. RMS Beam Size (vert)	σ_{ey}	0.6	mm
El. RMS Beam Size (hor)	σ_{ex}	0.6	mm
Kicker Section Length	L_k	40	m

Table 1 gives an example of system parameters, used in our calculations, and may differ somewhat from [7]. We will discuss both the cooling rate and the diffusion rate due to neighboring protons producing random kicks and, thus,

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creating a heating mechanism. Other diffusion mechanisms will also be considered.

ENERGY KICK

To allow for analytical treatment of the problem, we will use the following model expression for the proton energy kick in the ”Kicker” section,

$$w(z) = -V_0 \sin\left(2\pi \frac{z}{z_0}\right) \exp\left(-\frac{z^2}{{\sigma_0}^2}\right), \quad (1)$$

where we introduced three adjustable parameters: V_0, the amplitude of the kick, z_0, the characteristic wavelength, and σ_0, the characteristic width. The negative sign reflects the fact that a particle, moving faster than the reference particle ($z > 0$), loses its energy after the kick. For example, the energy kick, calculated using the system parameters in Table 1 and shown in Fig. 2, is presented in Fig. 3 (red curve) together with our model, Eq. (1) (blue curve). One can see from Fig. 3 that the proposed approximation slightly underestimates the far tales of the wake. This does not affect the cooling rate but slightly underestimates the diffusion rate.

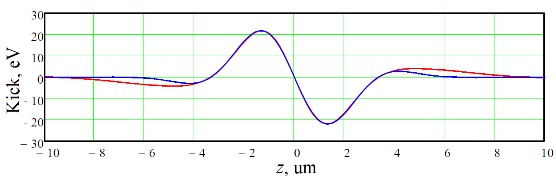

Figure 3: The energy kick (eV) after the “Kicker” section as a function of the proton's longitudinal position $z = R_{56}\delta$ (μm) with respect to the reference on-energy proton. The red curve is a calculated wake, based on [8, Eq. (C7)]. The blue curve is the proposed approximation, Eq. (1).

For the calculated energy kick, the following model parameters provide the best fit: $V_0 = 28$ eV, $z_0 = 6.7$ μm, and $\sigma_0 = 3.0$ μm. One can notice that at $|z| > \frac{z_0}{2}$ the energy kick changes its sign and cooling becomes anti-cooling. This determines the so-called cooling range, the number of “sigmas” n such that $nR_{56}\delta_p = z_0/2$.

FOKKER-PLANCK EQUATION

To describe the evolution of the proton momentum distribution function, we will use the Fokker-Planck equation in the action-angle variables, (J, ϕ) [9, 10]:

$$\frac{\partial \psi}{\partial t} = -\sqrt{2\beta}\frac{\partial}{\partial J}\left(\sqrt{J}\tilde{F}(J)\psi\right) + \beta\frac{\partial}{\partial J}\left(J\tilde{D}(J)\frac{\partial \psi}{\partial J}\right), \quad (2)$$

where β is the so-called longitudinal beta function, $\beta = \sigma_{pz}/\delta_p \approx 88$ m for the parameters in Table 1. For the detailed derivation of this equation and for the definitions of the cooling force $\tilde{F}$ and diffusion $\tilde{D}$ see [11].

We will now multiply both sides of Eq. (2) by J and integrate in order to obtain the evolution of the rms longitudinal emittance, $\epsilon_L = \int_0^\infty \psi \, J dJ$ (here we assume that ψ is normalized by unity, $\int_0^\infty \psi \, dJ = 1$).

COOLING RATE

To obtain the cooling rate, τ_c, from the fokker-Plank equation, Eq. (2), we will evaluate the following integral:

$$\frac{1}{\tau_c} = \frac{\sqrt{2\beta}}{\epsilon_L}\int_0^{+\infty} J\frac{\partial}{\partial J}\left(\sqrt{J}\tilde{F}(J)\psi\right) dJ, \quad (3)$$

where $\tilde{F}(J)$ is cooling force, averaged over the angle variable ϕ, as defined in [11], and the distribution function

$$\psi = \frac{1}{\epsilon_L}\exp\left(-\frac{J}{\epsilon_L}\right). \quad (4)$$

The resulting cooling rate is

$$\frac{1}{\tau_c} = \lambda\left(1 + \frac{{z_0}^2}{2n^2{\sigma_0}^2}\right)^{-3/2} \exp\left(-\frac{\pi^2}{2n^2 + {z_0}^2/{\sigma_0}^2}\right), \quad (5)$$

where n is the cooling range, such that $nR_{56}\delta_p = z_0/2$ and λ is the small-amplitude cooling rate,

$$\lambda = \frac{\pi f_0 V_0}{\delta_p n E_0}\frac{\sigma_{ez}}{\sqrt{2}\sigma_{pz}}. \quad (6)$$

Figure 4 shows the cooling time, τ_c, as a function of R_{56}, expressed through the cooling range, n, for the CEC system parameters in Table 1 and $V_0 = 28$ eV, $z_0 = 6.7$ μm, and $\sigma_0 = 3.0$ μm.

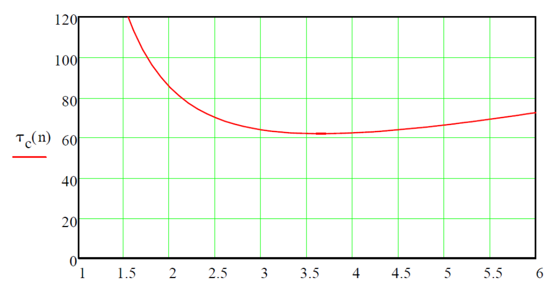

Figure 4: Cooling time (in minutes) as a function of the cooling range n defined as $nR_{56}\delta_p = \frac{z_0}{2}$.

One can see that there is a shallow minimum of about 60 minutes for n in the range 3.5 to 4.5. For example, choosing $n = 3.7$ results in $R_{56} = Z_0/(2n\delta_p) \approx 1.3$ mm. This value should be compared to the kinematic portion of the R_{56} element. If the proton path length between the “Modulator” and the “Kicker” section is $L \approx 100$ m, the kinematic portion of the R_{56} element is $L/\gamma^2 \approx 1.2$ mm. Thus, the proton beam line has to provide an additional 0.1 mm increase to the R_{56} matrix element. It also means that the Kicker section cannot be too long as its length[1] increases the effective value of the R_{56} element. For $n \to \infty$, the cooling time increases linearly with n and becomes: $\tau_c^{-1} \approx \lambda$, as expected.

[1] There are additional constraints on the Kicker section length, due to plasma oscillations in the electron beam, for example.

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DIFFUSION RATE

It was shown in [12] that in the case of stochastic cooling with a strong Schottky band overlap, the diffusion coefficient due to random kicks from neighbouring protons is independent of particle momentum, δ, and proportional to the local density of protons. The CEC method, having the typical frequencies of $c/z_0 \approx 45$ THz, is in the regime of a strong Schottky band overlap. In this regime, the momentum diffusion coefficient at the center of the electron bunch can be written as

$$D_0 = \frac{\langle (w(z)/E_0)^2 \rangle}{T}, \tag{7}$$

where the angular brackets $\langle ... \rangle$ indicate averaging of random energy kicks from neighboring ions, and $T = 1/f_0$ is the revolution period in the ring. Taking into account that the number of ions per unit length at the center of a Gaussian bunch is $N_p/(\sqrt{2\pi}\sigma_{pz})$, we obtain (see [11])

$$D_0 = \frac{N_p f_0 {V_0}^2}{4{E_0}^2} \frac{\sigma_0}{\sigma_{pz}} \left(1 - \exp\left(-2\pi^2 \frac{{\sigma_0}^2}{{z_0}^2}\right)\right). \tag{8}$$

As expected, the diffusion rate is independent of the cooling range n and is proportional to the width of the kick, σ_0, which can be viewed as the inverse band-width of the system. The length of electron bunch is much smaller than for the proton one. This simplifies the averaging over angle ϕ (see [11]). As a result we obtain

$$\tilde{D} = \frac{D_0}{2} \frac{\sigma_{ez}}{2\sigma_{pz}}. \tag{9}$$

From Eq. (2) the evolution of the longitudinal rms emittance, ϵ_L is determined by

$$\frac{1}{\epsilon_L}\frac{d\epsilon_L}{dt} = -\frac{1}{\tau_c} + \frac{\tilde{D}\beta}{\epsilon_L}, \tag{10}$$

with τ_c from Eq. (5) and $\tilde{D}$ from Eq. (9). For the CEC system parameters in Table 1 and for $V_0 = 28$ eV, $z_0 = 6.7\,\mu$m, and $\sigma_0 = 3.0\,\mu$m, the diffusion time is $(\tilde{D}\beta/\epsilon_L)^{-1} \approx 660$ minutes, which is much greater than the cooling time for the same parameters and $n = 3.7$, $\tau_c \approx 62$ minutes. This indicates that, in theory, the overall sum of cooling and diffusion rates in Eq. (10) can still be increased by increasing the kick amplitude, V_0. For the so-called "optimal gain" [13] condition we have:

$$\frac{1}{\tau_c} = \frac{2\tilde{D}\beta}{\epsilon_L}. \tag{11}$$

From this we can obtain the optimal kick amplitude, $V_{opt} \approx 150$ eV, for maximum cooling. With this optimal kick amplitude, the achievable cooling time becomes $2\tau_c \approx 24$ minutes (the factor of 2 is due to Eq. (11)). We note that only the diffusion due to neighboring protons is taken into account. Other diffusion mechanisms can be added to analyze the effective cooling rate in Eq. (10).

ELECTRON BEAM NOISE

In this section we assume that the electron beam has the noise equal to the shot noise of non-interacting electrons. Since the electron longitudinal charge density is similar to that of protons (see Table 1), we can estimate the electron shot-noise contribution to the diffusion to be similar to the proton beam contribution, Eq. (8). This doubles the effective diffusion coefficient and gives the effective diffusion time of $(\tilde{D}\beta/\epsilon_L)^{-1} \approx 330$ minutes, still much greater than the cooling time, ≈ 60 minutes. One can see that exceeding the shot-noise value in the electron beam by a factor of 2-3 is acceptable for the chosen parameters, with further increase the electron beam noise becomes a dominant diffusion factor. One has to remember, however, that doubling the diffusion coefficient increases the rms electron beam density fluctuations by a factor of $\sqrt{2}$, to over 20%. Thus, this may also become a limiting factor. A detailed calculation of electron beam contributions to the diffusion rate requires a separate investigation and is outside of the scope of this note.

CONCLUSIONS

In this paper, we considered a simplified cooling wake (kick) model, given by Eq. (1). This model allows to derive analytic expressions for cooling and diffusion rates, as well as for the electron beam rms density fluctuations. Having such analytic expressions should allow for better understanding of the CEC mechanism, and for a quicker analysis and optimization of main system parameters.

We would like to emphasize that even though we have used the wake calculated for the MBEC amplification scheme, our analysis can be easily applied to other coherent cooling techniques, for example, to the PCA concept, as long as the wake (kick) function, similar to the one shown in Fig. 3, is available.

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