

NUMERICAL MODELING OF THE OPTICAL STOCHASTIC COOLING EXPERIMENT AT IOTA*

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Abstract

A proof-of-principle optical-stochastic cooling (OSC) experiment is currently underway at Fermilab's IOTA ring. In support of this experiment, we recently implemented an OSC element in the *ELEGANT* tracking program. The model, based on a semi-analytic description of OSC, supports the simulation of a large number of macroparticles ($10^4 - 10^6$) over many turns (10^6). This paper showcases the simulation capabilities to investigate the beam dynamics in the presence of cooling (or self-interacting radiation field in general) and quantify the impact of various sources of error (e.g. transverse and phase jitter), guide data analysis.

INTRODUCTION

Optical stochastic cooling (OSC) is a process that reduces the phase-space of a particle beam over many turns in a storage ring [1, 2]. Particles traveling in a ring first pass through a "pickup undulator" (PU) where they produce radiation. The particles and radiation pulse are then separated as the particles travel through a magnetic-chicane bypass and then overlap downstream in the "kicker undulator" (KU). The bypass is optimized such that the radiation pulse applies a corrective kick based on each particle's phase-space deviation. While the energy exchange is purely longitudinal, the cooling force can also be coupled into the horizontal and vertical planes through appropriate design of the bypass and storage-ring optics. The modeling of OSC is intricate as it requires the simulation of charged-particle transport in the bypass beamline and the modeling of the radiation pulse propagation and manipulation through the optical beamline. An exact simulation would require the use of a finite-difference time-domain program to simulate the produced undulator radiation and its later coupling to the beam in the kicker undulator. Such an approach would be prohibitive in time especially given that the interaction repeats over 100s of thousands of turns; therefore, a reduced model was developed and implemented in *ELEGANT* as external *PYTHON* scripts [3]. Initially, the data exchange between *ELEGANT* [4] and the scripts were performed by writing files, which presented a significant bottleneck to perform large-scale simulations of the OSC process. We recently implemented a new element in *ELEGANT* to directly simulate the OSC process without

the need for external scripts. This contribution describes the status of the implementation, preliminary results, and planned improvements.

REDUCED MODEL OF OSC

ELEGANT is an open-source particle-tracking program capable of simulating many beamline elements of storage rings and linear accelerators [4]. Its tracking capabilities and flexibility make it a perfect tool for our simulations.

We implemented a custom element that uses a semi-analytic model of OSC to simulate the process inside *ELEGANT*. The element is built from the feedback element which uses a paired pickup and driver elements where the latter element applies a kick relative to beam parameters recorded at the pickup element location. In our preliminary implementation, we completed a simple one-dimensional transit-time model of OSC where the relative longitudinal-momentum kick applied in the kicker is given

$$(\delta p)/p = -\kappa \sin[\omega(t_0 - t_i) + \phi], \quad (1)$$

where κ is the kick peak amplitude, $\omega \equiv 2\pi c/\lambda$ with λ being the radiation wavelength, t_0 and t_i are the time-of-flight of the bunch recorded at the pickup and the kicker locations respectively, and ϕ is a phase that accounts for the delay between a particle and its associated radiation pulse. In our convention, $\phi = 0$ corresponds to the case when the particle propagates in the KU in phase with the optical pulse it originated in the PU and experiences the maximum kick. So far our model neglects kicks by neighboring particles since we are interested in the dynamics of a single classical particle in the accelerator. This choice is further justified in that the OSC test in IOTA will first be done passively where the kick amplitude is far from the optimal gain. Likewise, the kick parameters (κ , ϕ) are taken to be the same for all particles independently of their transverse positions such an assumption will eventually be refined to account for the undulator-radiation transverse-field structure [5].

By default, many of the elements in *ELEGANT* are capable of CPU parallel computing. When developing the new elements, we took advantage of the existing parallel capabilities which allowed us to dramatically improve the run time of the OSC simulation. The simulations outlined here were performed on a high-performance computing cluster at NIU. The run-time scales linearly with both the number of macroparticles and the number of revolutions around the ring with the mid-size simulation (10^5 macroparticles) running for approximately two hours.

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APPLICATIONS TO IOTA

The Integrable Optics Test Accelerator (IOTA) ring is a 40-m circumference storage ring at Fermilab used to test advanced concepts in accelerator science and technology including OSC. The IOTA lattice optimized for OSC [6] was implemented in ELEGANT to test the developed OSC element. In an ongoing experiment, a passive version of OSC is being tested with infrared undulator radiation ($\lambda = 950$ nm). The tracking simulations were performed using the canonical integrator.

ELEGANT has the ability to simulate incoherent synchrotron radiation (SR) due to the motion of particles through bending magnets. The energy lost through this process is restored each turn by an RF cavity and results in SR damping. In the IOTA ring, the SR damping times are $\tau_x = 1.06$ and $\tau_y = 1.05$ sec or ~ 7.5 sec million turns [6].

Most of the OSC dynamics can be captured by the one-dimensional model described by Eq. (1). The kick is applied relative to the difference in each particle's time-of-flight from the bunch. This has the effect of slowing down fast particles and speeding up slower ones and reducing the longitudinal phase space. The horizontal cooling is produced via the combined effect of coupling terms between the horizontal and longitudinal planes in the bypass along with the horizontal dispersion in the undulators. The partitioning between longitudinal and horizontal cooling can be tuned over a wide range using a single quadrupole magnet located in the center of the bypass chicane. This coupling is directly related to the design parameters of the OSC system and gives us another useful metric to benchmark the simulation against. While vertical phase-space deviations do not enter into the single-pass cooling force Eq. (1), operation of the lattice on an $x - y$ coupling resonance will result in vertical cooling as well.

Two of the most important considerations in implementing OSC in a storage ring are the stability and cooling range. If the delay of either the particle beam or the undulator radiation between the pickup and kicker undulators is too unstable, the cooling will be greatly diminished, and if the phase-space deviations are larger than the cooling range, then the OSC force can heat the beam and push it towards higher-order attractors [7]. This simulation allows us to investigate the impact of the two parameters in Eq. (1) (κ and ϕ) that control the OSC dynamic.

SIMULATION RESULTS

Most of the simulations presented in this section were performed using 10^5 macroparticles over $10^4 - 10^6$ turns in the IOTA ring including SR effects and OSC and all calculations consider a radiation wavelength of 950 nm.

SR Damping

In our ELEGANT model SR effects are implemented on an element-by-element basis and uses Gaussian distributions for the energy scattering of the electrons [8]. The time associated with 10^5 revolutions of ~ 13 ms is too short for

the beam to reach its equilibrium emittance via SR damping. Consequently, the impact of SR is small but can be quantified by comparing the beam dynamics in simulations where SR effects are turned off and inferring the SR damping time from the difference between the two simulations. By examining the turn-by-turn evolution of individual particles, we can estimate the cooling rate with only a few thousand passes. The rate of damping is determined by the equation $\frac{d\varepsilon}{dt} = -k[\varepsilon(t) - \varepsilon_{eq}]$ where k is the damping coefficient and ε_{eq} the equilibrium emittance. A fit of the initial emittance-evolution slope $\left. \frac{d\varepsilon}{dt} \right|_{t=0} = k\varepsilon(0) + \varepsilon_{eq}$ obtained from simulation done for several initial emittances provides the SR-damping decrement k with a fitted transverse cooling rate to be $\tau_x^{SR} \approx 0.1$ s.

OSC Cooling Rates and Ranges

For optimum particle-radiation overlap in the KU, the cooling rate of OSC is controlled by the amplitude parameter κ . In Equation 1 κ is given in unit of γ the Lorentz factor so that a value $\kappa = 10^{-7}$ correspond to a momentum kick of $\delta pc \approx 1$ eV (which is comparable to the energy kick an electron experiences in the KU for the case of active OSC with an amplifier gain of ~ 100). Figure 1 illustrates the impact of κ on the horizontal emittance with $\phi = 0$ (corresponding to the optimum cooling phase). As expected larger values of κ result in a faster cooling rate. In the limit $\kappa = 0$, no cooling is observed as SR damping time is much longer than the number of revolutions used in our simulations. In the simulations, the injection emittance is taken to be 200 pm i.e. one order of magnitude smaller than the nominal injection emittance in the FAST-IOTA complex. Such a lower value was selected to avoid particle loss and be more efficient in capturing the OSC dynamics from the early stage.

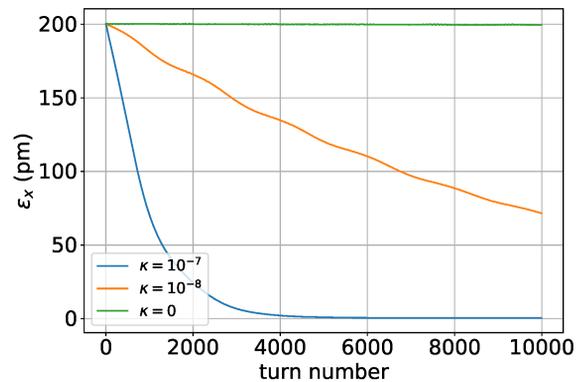


Figure 1: Impact of strength parameter κ on emittance evolution as a function of turn number. The simulations also include SR damping ($\kappa = 0$ on has SR damping).

In the second set of studies, we explore the impact of the phase ϕ . For the non-optimal phase, it is expected that some of the particles will be outside of the cooling range resulting in a heating of a fraction of the beam. Particles are ultimately kicked to large amplitudes and clustered on a peripheral

attractor. The effect of the phase ϕ on transverse cooling is summarized in Fig. 2. As expected, when $\phi \approx 0$, significant cooling is achieved. Cooling is produced up to phase values corresponding to $\phi \approx 0.5\pi$ (quarter wavelength phase shift) albeit with a reduced cooling rate. For $\phi = 0.8\pi$ (corresponding to an out-of-phase kick) the beam is heated and asymptotically reaches a large “equilibrium” emittance corresponding to a fraction of the beam bifurcating to a stable attractor.

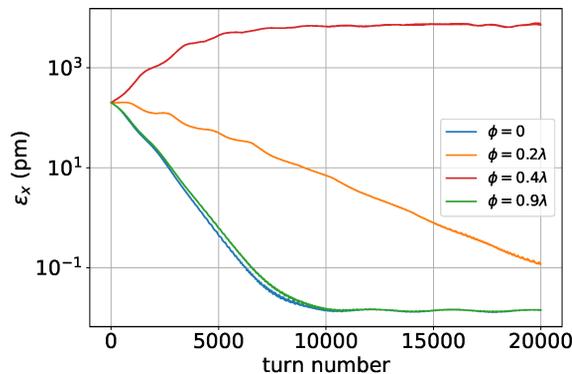


Figure 2: Impact of the relative phase between particle and radiation in the KU ϕ on emittance evolution as a function of turn number. The red trace corresponds to the case where a significant fraction of the bunch is bifurcated ($\kappa = 10^{-7}$).

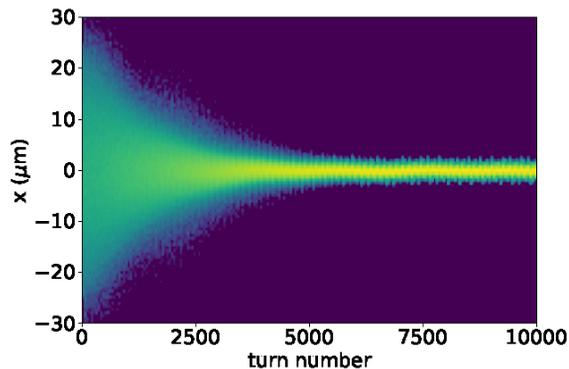


Figure 3: Histogram of the horizontal particle distributions as a function of turn number when OSC is set for optimum cooling ($\kappa = 10^{-7}$, $\phi = 0$).

Phase Space Tracking and Control

To examine the evolution of the phase-space distribution we record phase-space coordinates at different points along the lattice on a turn-by-turn basis in ELEGANT. Figure 3 shows the evolution of the horizontal particle distribution as the beam circulates and with OSC set for cooling. As expected the horizontal distribution (histogram of the x coordinate for all particles) coalesces resulting in a higher beam density. A similar observation hold for the conjugated variable p_x .

However, as we discussed previously, radiation arriving out of phase $\phi = \pi$ with the particle will result in the particle

receiving kick that exacerbates its offset w.r.t. the reference particle. Eventually, the process will result in beam heating accompanied by the formation of a stable particle cluster. To quantify this process we consider the normalized phase space coordinate (x_n, x'_n) obtained from (x, x') through the transformation $x_n = x$ and $x'_n = x\alpha_x + x'\beta_x$ in such a coordinate system the quantity $J_x \equiv [x_n^2 + x_n'^2]^{1/2}$ is the particle action which is nominally invariant.

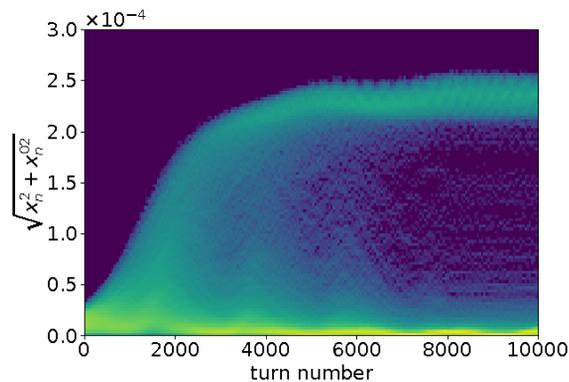


Figure 4: Histogram of the horizontal normalized invariant as a function of turn number when the OSC parameter are ($\kappa = 10^{-7}$, $\phi = 0.8\pi$).

Figure 4 provides a turn-by-turn evolution of the histograms of the action for $\phi = 0.8\pi$ and demonstrates the bifurcation process in the phase space where the heated particles are attracted to a fixed-point in action. The initial and final normalized phase spaces are also compared in Fig. 5 for the case of beam heating.

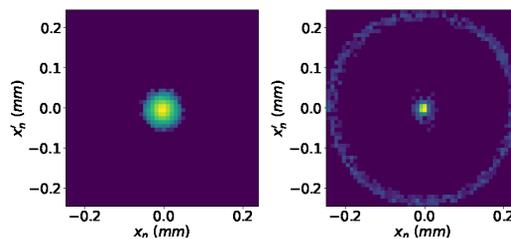


Figure 5: Injected (left) and final (right) horizontal phase-space distributions in normalized coordinates. The final distribution corresponds to turn number 10^5 in Fig. 4.

CONCLUSION

ELEGANT was modified to model OSC and preliminary studies have been performed in support of the passive-OSC experiment at IOTA. The implemented algorithm, once validated, will guide future developments related to an active-OSC scheme. Future work will focus on improving the model of the OSC kick to include transverse effects and allow for turn-by-turn control of the OSC element parameters. Such a capability will enable the exploration of phase-space manipulation techniques using OSC beamlines.

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