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MODELING SHORT RANGE WAKEFIELD EFFECTS IN A HIGH GRADIENT LINAC*

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Abstract

The interaction of charged beams with the surrounding accelerating structures requires a thorough investigation due to potential negative effects on the phase space quality. Indeed, the wakefields acting back on the beam are responsible for emittance dilution and instabilities, such as the beam break-up, which limit the performances of electron-based radiation sources and linear colliders. Here we introduce a new tracking code which is meant to investigate the effects of short-range transverse wakefields in linear accelerators. The tracking is based on quasi-analytical models for the beam dynamics which, in addition to the basic optics specified by the applied fields, include dipole wakefield forces and a simple approach to account for space-charge effects. Such features provide a reliable tool which easily allows to inspect the performances of a linac. To validate the model, a parallel analysis for a reference case is performed with well-known beam dynamics codes, and comparisons are shown. As an illustrative application, we discuss a study on alignment tolerances evaluating the emittance growth induced by misaligned accelerating sections.

INTRODUCTION

The demands on performance for applications such as photon sources and linear colliders require very intense particle beams in order to achieve the desired *brightness* or *luminosity*. Acceleration of such beams in high gradient linacs is accompanied by strong collective interaction due to the self-induced electromagnetic fields which may cause a significant loss of beam quality [1]. The *dipole wakefield* interaction is also responsible for instability effects such as the growth in amplitude of the transverse oscillations, known as beam breakup (BBU) [2, 3].

A recent work on the design of a C-band hybrid photoinjector for driving a Compton source [4] has motivated investigation of the effects of transverse wakefields in linacs. To fulfill this goal, we developed MILES (Modeling Instabilities in Linacs with Ellipsoidal Space charge), a custom tracking code for rf-linear accelerators. Based on simple analytical models, the code is computationally efficient and flexible, with typical run-times of a few tens of seconds. In this paper we describe the models employed to account for the main aspects of the beam dynamics and compare the results of MILES with those of pre-existing codes and theoretical formulations.

BEAM DYNAMICS

The code tracks a set of macro-particles propagating inside a linac which is described as a sequence of accelerating structures separated by field-free drifts. The tracking mainly focuses on the transverse dynamics since for short relativistic beams each particle experiences nearly the same longitudinal transformation. In addition, a smoothed linear energy growth is assumed inside the accelerating sections and is accounted for by the averaged gradient $\gamma' = \langle eE_z \rangle / mc^2$.

For our cases of interest, the only focusing force comes from the accelerating cells themselves, which posses a rich content of non-synchronous space harmonics [5]. The corresponding transverse mapping, with an energy change $\gamma_1 \mapsto \gamma_2$, is specified by the following matrix [6]

$$\begin{pmatrix} x \\ x' \end{pmatrix} \mapsto \begin{pmatrix} \cos\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) & \frac{\gamma_1}{\nu\gamma'} \sin\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) \\ -\frac{\nu\gamma'}{\gamma_2} \sin\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) & \frac{\gamma_1}{\gamma_2} \cos\left(\nu \ln \frac{\gamma_2}{\gamma_1}\right) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}, \tag{1}$$

where $\nu = \sqrt{\eta/8}/\cos\Delta\phi$, $\Delta\phi$ is the phase deviation from the crest and $\eta \approx 1.12$ is a measure of the higher space harmonics content [7, 8]. Further, when entering or leaving an accelerating section, particles experience a transition from a field-free to a non-zero field region, or vice versa. Such a process affects the radial momentum and can be described (*cf.* [6]) in terms of a transverse kick $\Delta x'$ from a thin lens of focal strength $\mp\gamma'/2\gamma$. The lens-like kicks and the transformation in Eq. (1) completely determine the *secular* trajectories for single particle dynamics and, thus, the basic transverse optics of the machine.

COLLECTIVE EFFECTS

The simple trajectories expected for individual particles are perturbed by two types of collective effects. First, the particles experience defocusing fields produced by the rest of the distribution which gives rise to space charge forces [9]. In addition, charged beams interact with the surrounding

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environment exciting wakefields which affect the motion of trailing particles [10]. In this section we describe the models adopted in MILES in order to account for space charge and dipole wakefield forces simultaneously.

Space Charge Forces

Beam dynamics simulations performed with GPT (General Particle Tracer) [11] have shown that, as the bunch propagates after the gun, it forms and maintains a nearly ellipsoidal shape [12]. The electrostatic field produced by a uniformly charged ellipsoid is known analytically and it is found in [13]. Such field grows linearly with the displacement from the center of mass, as does the force experienced by a charged particle that is responsible for the change in momentum.

The introduction of transverse space charge forces is fundamental in order to provide a meaningful description of the beam dynamics. As an example, let us consider a beam produced by a hybrid gun and then injected into a ~ 4 m long linac such as those described in [4]. Figure 1 shows that, when the space charge force is removed, the input kick received at low energy is excessive, leading to over focusing of the beam and causing its trajectories to cross the axis. After such a waist, the focusing forces are not strong enough to prevent the rms-envelope to expand. Conversely, inclusion of space charge defocusing forces allows to preserve a laminar motion: a quasi-equilibrium with the applied forces is established and the beam is gently focused in the linac as predicted by the invariant envelope theory [14]. In this case, the rms-envelope from our code, using the simplified ellipsoidal model, and the corresponding GPT simulation exhibit similar behaviors, with small disagreements due to the model imperfections.

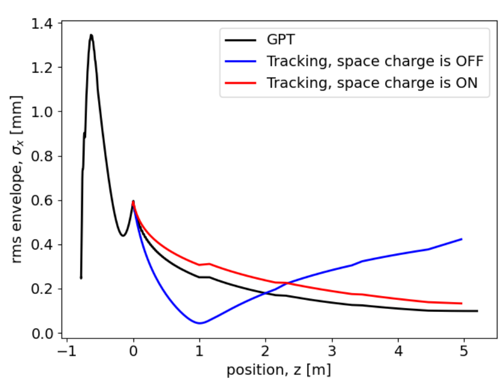

Figure 1: The effect of transverse space charge forces. Injection in the 4 section linac occurs at $z = 0$.

Dipole Wakefields

One of the main purposes of MILES is to study BBU effects induced by dipole wakefields within the accelerating sections. Following the approach described in [15, 16] for circular machines, wakefields are accounted for by means of a matrix formalism. Indeed, a set of effective resonant modes can be introduced to represent an arbitrary transverse wake-function and, for each mode, the binary interaction between two macro-particles is given by the so-called *wakefield matrix*.

As a demonstration for such formalism, a classic beam breakup example is considered. In Ref. [3], A. Chao provides an asymptotic formula describing the oscillations of the beam tail for bunches subjected to strong BBU in linacs. The model assumes a linear growth of the transverse wakefield from the head to the tail where the slope quantifies the strength of the interaction. In Fig. 2 the wakefield matrix has been used to account for the same wake-function in a 3 km linac and the results from MILES are compared with the asymptotic formula for a bunch injected $x_0 = 3$ mm off-axis.

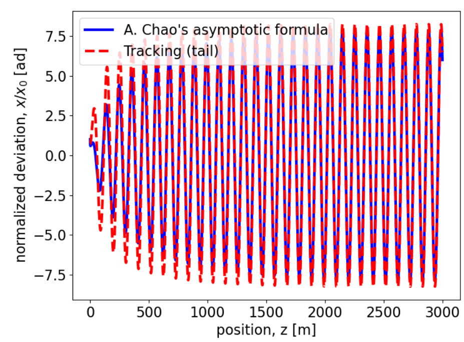

Figure 2: Transverse oscillations of the tail in strong BBU regime: comparison with the asymptotic formula in [3].

For linacs, the main source of wakefields are contributed by periodic accelerating structures. The short range dipole wake-function of such devices, under conditions that our case obeys, can be evaluated by use of diffraction theory, an asymptotic model valid for short bunches [17–20]. Thus, the wake-function of interest is known (see, *e.g.*, Bane [21]) and can be fit by a set of superposed resonators in order to apply the wakefield matrix formalism in the tracking code.

MISALIGNMENT INDUCED EMITTANCE GROWTH

It is well known that particles traveling off-axis inside an accelerating structure excite dipole wakefields. The excitation process can be dramatically enhanced when the machine is subjected to alignment errors which will cause deviations from the nominal trajectory. Such imperfections can lead to a significant phase space quality dilution and, therefore, it is important to investigate alignment tolerances in order to ensure the design performance.

Misaligned Sections

There are two main types of alignment errors that can occur either separately or combined.

1. **Transverse offset**. The center of the input surface of an accelerating section is shifted from the nominal axis.
2. **Tilt angle**. The axis of a section forms a non-zero angle with the direction of the nominal machine axis.

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Trajectories inside a misaligned section can be described in terms of a local coordinate system. If we let (x, z) be the transverse and longitudinal coordinates with respect to the nominal machine axis and (x_i, z_i) be the coordinates according to the i-th section, we can account for misalignments by means of the following transformation

$$\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} \cos\delta_i & \sin\delta_i \\ -\sin\delta_i & \cos\delta_i \end{pmatrix} \begin{pmatrix} x_i \\ z_i \end{pmatrix} + \begin{pmatrix} \Delta x_i \\ \Delta z_i \end{pmatrix} \tag{2a}$$

$$x' = \frac{x'_i + \tan\delta_i}{1 - x'_i \tan\delta_i}, \tag{2b}$$

where $(\Delta x_i, \Delta z_i)$ is the center of i-th structure's input face and δ_i its tilt angle.

As an example, let us consider a four-section linac for which the second section is misaligned. In particular, we assume $\Delta x_2 = 50\,\mu\text{m}$ and $\delta_2 = 50\,\mu\text{rad}$ while $\Delta x_i = 0$, $\delta_i = 0$ for $i \neq 2$. In Fig. 3 the center of mass trajectory from our code is compared with that from GPT, showing a good agreement. Since the example is meant to investigate the optics properties in presence of misalignments, simulations do not include wakefield effects.

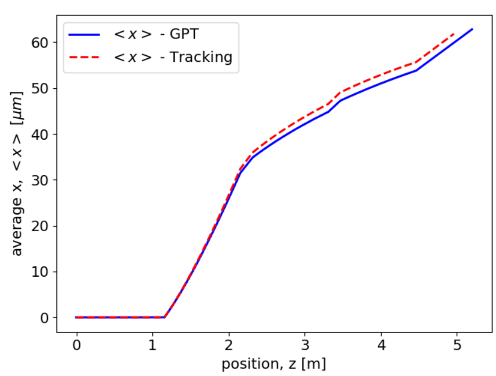
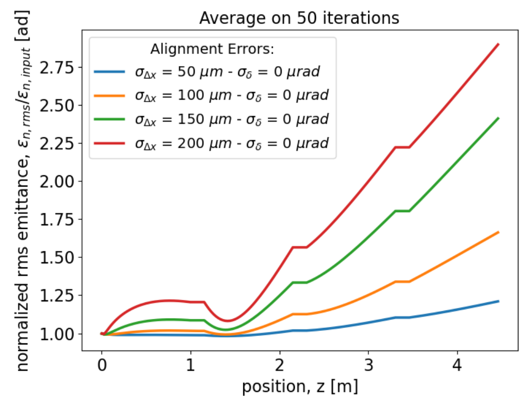


Figure 3: Center of mass transverse displacement induced by a misaligned section ($\Delta x_2 = 50\,\mu\text{m}$, $\delta_2 = 50\,\mu\text{rad}$).

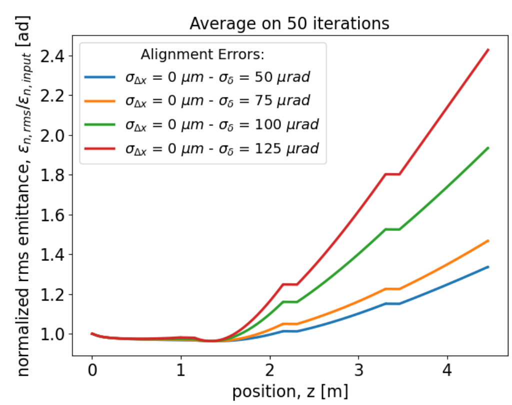
Random Alignment Errors

In the following, a study on tolerances to random misalignments for a 4 section linac is presented. For simplicity, the analysis investigates separately the cases of sections affected by random offsets and rotations however the procedure is the same.

Let us now concentrate on the case of sections whose axis is parallel to the nominal one but shifted in the transverse direction. We start by assuming that the deviations Δx from the nominal axis follow a gaussian distribution with standard deviation $\sigma_{\Delta x}$. The latter can be thought as a measure of the characteristic alignment error. For a given $\sigma_{\Delta x}$ we perform 50 simulations, each time using a new set of random offsets (one offset per section) according to the distribution we fixed. The rms emittance is then averaged on the whole ensemble of simulations in order to mitigate statistical fluctuations. In Fig. 4 the emittance, normalized to its value at injection, is shown for different values of the characteristic error. A similar analysis made on structures with zero offset but whose axis has a tilt error described by a gaussian distribution of standard deviation σ_δ is shown as well. The results suggest that, due to the strength of the dipole wakefields, the machine is quite sensitive to misalignments: errors of a few tens of micro-meters or micro-radians can lead to notable emittance growth. As a consequence, correction schemes employing steering magnets are being examined in order to preserve the phase space quality.



(a) Random shifts from the nominal optical axis.



(b) Random tilt angles with respect to the optical axis.

Figure 4: Emittance growth induced by randomly misaligned sections, for differing alignment errors $\sigma_{\Delta x}$ and σ_δ.

CONCLUSIONS

In this paper we introduce MILES, the tracking code developed to investigate transverse wakefield effects in high brightness beams linacs. The code successfully describes the main aspects of the beam dynamics and exploits simple time-saving models to include self-induced fields. Among the ongoing activities, an upgrade of the code is foreseen to improve the description of the longitudinal dynamics which are responsible for energy spread and chromatic effects. Further studies on alignment tolerances will be performed to help guide the design of a possible correction scheme.

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REFERENCES

[1] L. Palumbo, V. G. Vaccaro, and M. Zobov, "Wake fields and impedance", CAS Accelerator School, Geneva, Switzerland, Rep. CERN 95-06, 1995.

[2] W. K. H. Panofsky and M. Bander, "Asymptotic theory of beam breakup in linear accelerators", *Rev. Sci. Instrum.* vol. 39, no. 2, pp. 206–212, Feb. 1968. doi:10.1063/1.1683315

[3] A. W. Chao, *Physics of collective beam instabilities in high energy accelerators*. New York, USA: Wiley, 1993.

[4] L. Faillace *et al.*, "Beam dynamics for a high field C-band hybrid photoinjector", presented at the 12th Int. Particle Accelerator Conf. (IPAC'21), Campinas, Brazil, May 2021, paper WEPAB051.

[5] S. Tantawi, M. Nasr, Z. Li, C. Limborg, and P. Borchard, "Design and demonstration of a distributed-coupling linear accelerator structure", *Physical Review Accelerators and Beams*, vol. 23, no. 9, pp. 092001, Sep. 2020. doi:10.1103/PhysRevAccelBeams.23.092001

[6] J. Rosenzweig and L. Serafini, "Transverse particle motion in radio-frequency linear accelerators", *Physical Review E*, vol. 49, no. 2, pp. 1599–1602, Feb. 1994. doi:10.1103/PhysRevE.49.1599

[7] S. C. Hartman and J. Rosenzweig, "Ponderomotive focusing in axisymmetric rf linacs", *Physical Review E*, vol. 47, p. 2031, Mar. 1993. doi:10.1103/PhysRevE.47.2031

[8] R. Robles, O. Camacho, A. Fukasawa, N. Majernik, and J. Rosenzweig, "Versatile, high brightness, cryogenic photoinjector electron source", (to be published).

[9] M. Ferrario, M. Migliorati, and L. Palumbo, "Space charge effects", in *Proc. of the CAS-CERN Accelerator School,*, Trondheim, Norway, Aug. 2013, pp. 331-356. doi:10.5170/CERN-2014-009.331

[10] M. Ferrario, M. Migliorati, and L. Palumbo, "Wake fields and instabilities in linear accelerators", in *Proc. of the CAS-CERN Accelerator School*, Trondheim, Norway, 19–29 Aug. 2013, paper CERN-2014-009, pp. 357-375. doi:10.5170/CERN-2014-009.357

[11] Pulsar Physics, http://www.pulsar.nl/gpt

[12] M. Carillo *et al.*, "Three-dimensional space charge oscillations in a hybrid photoinjector", presented at the 12th Int. Particle Accelerator Conf. (IPAC'21), Campinas, Brazil, May 2021, paper WEPAB256.

[13] O. Kellogg, *Foundation of Potential Theory*. Dover, UK: Springer, 1953.

[14] L. Serafini and J. Rosenzweig, "Envelope analysis of intense relativistic quasilaminar beams in rf photoinjectors: A theory of emittance compensation", *Physical Review E*, vol. 55, p. 7565, Jun. 1997. doi:10.1103/PhysRevE.55.7565

[15] M. Migliorati and L. Palumbo, "Multibunch and multiparticle simulation code with an alternative approach to wakefield effects", *Phys. Rev. ST Accel. Beams*, vol. 18, p. 031001, Mar. 2015. doi:10.1103/PhysRevSTAB.18.031001

[16] M. Migliorati *et al.*, "Instability studies at the CERN Proton Synchrotron during transition crossing", *Phys. Rev. ST Accel. Beams*, vol. 21, p. 120101, Dec. 2018. doi:10.1103/PhysRevAccelBeams.21.120101

[17] J. D. Lawson, "Radiation from a ring charge passing through a resonator", Rutherford High Energy Laboratory, Chilton, England, Rep. RHEL/M144, 1968.

[18] S. A. Heifets and S. A. Kheifets, "High frequency limit of the longitudinal impedance of an array of cavities", SLAC, Menlo Park, USA, Rep. SLAC PUB-4625, May 1988.

[19] R. L. Gluckstern, "High-frequency behavior of the longitudinal impedance for a cavity of general shape", *Phys. Rev. D*, vol. 39, p. 2773, May 1989. doi:10.1103/physrevd.39.2773

[20] K. Bane and M. Sands, "Wakefields of very short bunches in an accelerating cavity", *Particle accelerators*, vol. 25, pp. 73-95, 1990.

[21] K. Bane, "Short-range dipole wakefields in accelerating structures for the NLC", SLAC, Menlo Park, USA, Rep. SLAC-PUB-9663, Mar. 2003.