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TMCI THEORY OF FLAT CHAMBERS REVISITED

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Abstract

By accounting for the transverse impedance' quadrupolar component according to the work of R. Lindberg [1], no TMCI-instability can be observed in case of a pure horizontal resistive wall impedance of flat vacuum chambers. In order to study this effect more closely, TMCI-theory is reviewed and Lindberg's work is further developed by including the resonator model as impedance type. The theory is applied to the ALBA-impedance model for the calculation of horizontal TMCI-detuning and threshold. Moreover, a couple of example cases are presented including vertical TMCI-detuning and threshold. Results on both planes are compared to simpler descriptions which account for the quadrupolar impedance effect only by tune shift.

INTRODUCTION

The transverse motion of particles in a single bunch can be described by the (linearized) Vlasov equation [2–6]. The effect of dipolar impedance on the transverse motion of bunch particles was described by [7] by decomposition into azimuthal and radial modes. Around 2000 it was found that for flat vacuum chambers quadrupolar impedance has a sensible effect on the transverse motion [8–10]. Its effect was typically included in the mode detuning by a tune shift. However, R. Lindberg showed [1] that its effect has to be fully accounted for in the dynamics of the bunch motion. Reference [11] already shows that the latter description agrees better with HEADTAIL simulations [12] for not too high frequencies in a two azimuthal mode system than the description that only uses the tune shift caused by effective quadrupolar impedance. In this work Lindberg's theory will be applied on a larger number of modes and include quadrupolar broadband impedance in the theory which was not yet done in [1]. For this purpose a program (LINDE) was written which is able to treat 10×10 azimuthal × radial modes under the influence of dipolar and quadrupolar resistive wall (RW) and broadband (BBR) impedance whose frequency dependence we recall briefly:

$$Z_{RW}^{\beta}(\omega) = (sgn(\omega) - i)\frac{Z_{RW}}{\sqrt{|\omega|}}, Z_{BBR}^{\beta} = \frac{(\beta R)_{\perp}}{\frac{\omega}{\omega_r} + iQ\left(1 - \frac{\omega_r^2}{\omega^2}\right)},$$

where Z_{RW} is the β-function weighted strength of the RW and $(\beta R)_{\perp}$ is the β-weighted shunt impedance, Q the quality factor and ω_r the resonator frequency. Parameters with β-superscript are supposed to be β-function weighted in the following. The treatment of RW-impedance of vacuum elements – important for the ALBA impedance model – that don't follow the infinitely thick RW-model is included. Only Gaussian bunch profiles with length $\sigma_\tau = 15.4$ ps are considered. Zero chromaticity is assumed.

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LINDBERGS'S MODE EVOLUTION THEORY

In [1] the Vlasov equation is linearized with the Planck-Fokker terms included. In the following it is assumed that the TMCI is strong enough for the disregard of the Planck-Fokker terms. This leads to the following equation[1]:

$$\Delta\omega^m a_p^m + \sum_{n,q}(D+Q)_{p,q}^{m,n} a_q^n = 0,$$

with $\Delta\omega^m = \Delta\Omega - m\omega_s$ with ω_s as synchrotron ang. frequency, with dipolar and quadrupolar matrix elements with $C_{p,q}^{m,n} = \sqrt{(p+|m|)!(q+|n|)!}$, $\epsilon_m = (-1)^{m(1-\delta_{|m|}^m)}$, $\Lambda := \frac{I}{4\pi(E/e)}$ as intensity parameter (I bunch current, E particle energy):

$$D_{p,q}^{m,n} = \frac{\Lambda i^{m-n+1}\epsilon_m\epsilon_n}{\sqrt{p!q!}C_{p,q}^{m,n}} \int_{-\infty}^{\infty} Z_D^{\beta}(\omega) e^{-(\omega\sigma_\tau)^2} \left[\frac{\omega\sigma_\tau}{\sqrt{2}}\right]^{2(p+q)+|m|+|n|} d\omega,$$

with $Z_D^{\beta}(\omega)$ as dipolar impedance and:

$$Q_{p,q}^{m,n} = \frac{\Lambda i^{m-n+1}\sqrt{p!q!}}{C_{p,q}^{m,n}} \int_{-\infty}^{\infty} Z_Q^{\beta}(\omega) e^{\frac{-(\omega\sigma_\tau)^2}{2}} I_{p,q}^{m,n}(\omega) d\omega,$$

with $Z_Q^{\beta}(\omega)$ as quadrupolar impedance and the following abreviation:

$$I_{p,q}^{m,n}(\omega) = \int_0^{\infty} dr e^{-r} r^{\frac{-(|n|+|m|)}{2}} J_{m-n}(\omega\sigma_\tau\sqrt{2r}) L_p^{|m|}(r) L_q^{|n|}(r),$$

with $L_p^n(x)$ as general Laguerre-polynomials. Interesting special cases can be found in solving the azimuthal 2-mode system under the effect of RW- or BBR-impedance where the mode frequencies follow:

$$0 = \begin{bmatrix} \Delta\Omega + Ib(1+\rho) & I\alpha b(1-\rho) \\ -I\alpha b(1+\rho) & \Delta\Omega + \omega_s + \rho Ib + I\frac{\beta}{2}b(1-\rho) \end{bmatrix} \begin{bmatrix} a_0^0 \\ a_0^{-1} \end{bmatrix},$$

whose eigenvalues representing the mode frequencies yield:

$$\Delta\Omega = -\frac{Ib(1+2\rho) + \frac{\beta}{2}Ib(1-\rho) + \omega_s}{2} \pm \frac{1}{2}\sqrt{[Ib - \frac{\beta}{2}Ib(1-\rho) - \omega_s]^2 - 4\alpha^2 I^2 b^2(1-\rho^2)}, \quad (1)$$

where $b = \kappa_{\perp}^{\beta}/(2(E/e))$ – the numerator representing the trans. kick factor, $\rho = \frac{Z_Q^{\beta}(\omega)}{Z_D^{\beta}(\omega)} = const$, α and β (Fig. 1) parameters characterizing the used impedance model (RW or BBR) in the context of the 2-mode system. In the following 3 sections we simplify a bit by assuming that dipolar and quadrupolar impedance are spectrally equal with either $\rho = 0, 0.5$ (vertical) or -1 (horizontal).

[1] Actually we stick to the mode expansion of [13].

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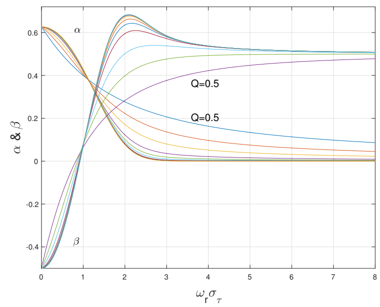

Figure 1: α and β for different quality factors Q as a function of bunch length weighted resonance frequency of the broadband impedance.

COMPARISON WITH OTHER SIMULATION PROGRAMS

Since LINDE-theory is MOSES-theory [7] for pure dipolar impedance it agrees with the azimuthal and radial mode decomposition of MOSES in that case (Fig. 2). Good agreement is also achieved with HEADTAIL (Fig. 3).

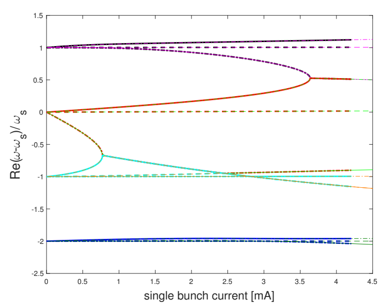

Figure 2: Mode detuning LINDE compared to MOSES for 1BBR: f_r = 3 GHz, $(\beta R)_\perp$ = 100 MΩ, Q = 2.3.

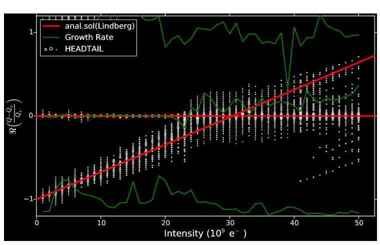

Figure 3: Mode detuning LINDE compared to HEADTAIL for one horizontal (dipole + quadrupole) BBR-resonator .

VERTICAL PLANE

Considering only 2 azimuthal modes the TMCI-threshold is found by putting Eq. (1) radicand to zero:

$$I_{thres} = \frac{\omega_s}{b(1 - \frac{\beta}{2}(1-\rho) + 2\alpha\sqrt{1-\rho^2})}. \tag{2}$$

For a pure dipolar impedance that is supplemented if applicable by a tune shift of quadrupolar origin the TMCI-threshold yields (ρ = 0):

$$I_{thres} = \frac{\omega_s}{b(1 - \frac{\beta}{2} + 2\alpha)},$$

whereas in Lindberg's theory with an assumed ratio of quadrupolar to dipolar impedance of ρ = 0.5 (applying RW-impedance theory of flat chambers [8] also on BBR-impedance) it yields:

$$I_{thres} = \frac{\omega_s}{b(1 - \frac{\beta}{4} + \sqrt{3}\alpha)}.$$

The difference between α and β is the largest at $\omega_r\sigma_\tau = 0$ (Fig. 1) leading to a threshold increase with respect to the pure dipolar one (Fig. 4) whereas the largest reduction of the threshold (-18%) can be reached at $\omega_r\sigma_\tau = 2.3$ (Fig. 5). This might reduce the gap found at many synchrotrons between the measured TMCI-thesholds and notoriously too high predicted ones [14]. However, it is based on the idealization of a quadrupolar BBR-impedance that has the same spectral distribution as the dipolar one. Moreover, for quadrupolar RW-impedance, a large contributor to the total impedance, actually β = 0.25 and α = 0.239 the TMCI-threshold slightly increases by $\Delta I = 8.4 \cdot 10^{-4}\omega_s/b$. But in case of quadrupolar BBR-impedance spectrally different from the dipolar impedance further possibilities for a reduction of the TMCI-threshold open up which can be only discussed in a larger publication. Finally the Eq. (2) for the vertical threshold assumes only one radial mode whereas the true threshold actually yields only considering many radial modes best to be found in an iterative procedure.

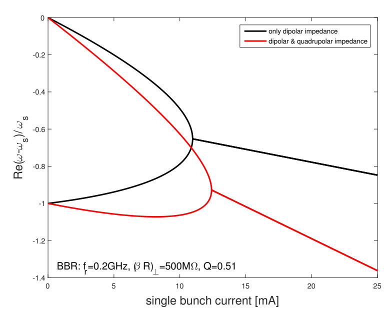

Figure 4: The TMCI-threshold for a dipolar+quadrupolar BBR-impedance is larger than for a purely dipolar impedance for a very small $\omega_r\sigma_\tau$.

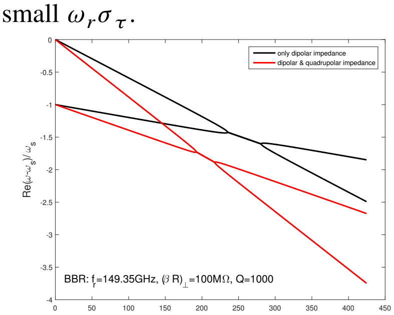

Figure 5: The TMCI-threshold dipolar+quadrupolar BBR-impedance is smaller than for a purely dipolar impedance.

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HORIZONTAL PLANE

Due to $\rho = -1$ (applying RW-impedance theory of flat chambers [8] also on BBR-impedance) in the 2 azimuthal mode case the coupling term disappears making the mode-coupling impossible. Both modes only cross each other at

$$I_{modecrossing} = \frac{\omega_s}{b(1-\beta)}, \tag{3}$$

without any coupling. This astonishing result was checked in [11] for RW- and BBR-impedance (Fig. 3). It is also remarkable that the positive slope of mode $m = -1$ is weaker than in case of pure dipolar impedance. For resonator frequencies > 5 GHz the agreement between Lindberg's theory and HEADTAIL-simulations is less good in case of the 2 azimuthal mode model. By taking more azimuthal and radial modes into account the agreement again is good (Fig. 6).

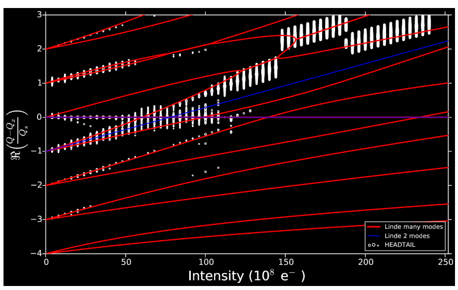

Figure 6: Large number of azimuthal and radial modes (red) compared to fewer modes (blue) make HEADTAIL and LINDE agree better for a BBR impedance $(\beta R)_\perp = 100\,\mathrm{M\Omega}$ and $Q = 2.3$ of higher frequency: $f_{res} = 10$ GHz.

APPLICATION ON THE ALBA IMPEDANCE MODEL

For the study of more realistic HT-mode evolution (we don't aim here at an impedance budget comparison) the newest transverse impedance model of the ALBA-synchrotron with a relaxed $\sigma_\tau = 22$ ps is input to LINDE. It consists of vertical and horizontal dipolar and quadrupolar BBR-impedance spectra plus an important RW-impedance part consisting of a thick wall RW-model plus the a couple of special chambers' RW-impedance. Element-wise GdfidL-simulations [15] of the storage ring vacuum chamber geometry provide wakefields which are after detuning-wake separation are Fourier-transformed into dipolar and quadrupolar impedance that are separately accumulated. A series of 3-6 broadband models are fitted to the obtained impedance spectra. It was checked that the impedance spectra and the obtained fits yield about the same kick factor to assure the same detuning slopes. Spectrally different dipolar and quadrupolar BBR-impedance now are part of the model, so ρ looses its meaning. The BBR-models obtained by the fits serve as input into LINDE. In the vertical plane a reduction of the TMCI-threshold is achieved with the inclusion of many radial modes (Fig. 7). In the horizontal plane the ratio of effective quadrupolar and dipolar impedance only reaches -0.7 (comparable with the ρ-value). Therefore modes $m = 0$ and $m = -1$ still couple. However, compared to the threshold obtained by dipolar impedance (plus a superimposed tune shift generated by quadrupolar impedance according to the old picture) the threshold is larger (Fig. 8).

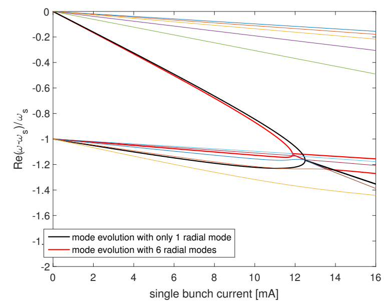

Figure 7: Vertical ALBA impedance model's mode evolution: one radial mode vs. six radial modes.

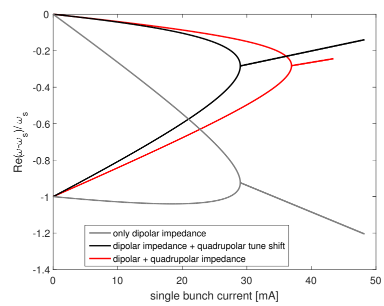

Figure 8: Horizontal ALBA impedance model's mode evolution compared to the pure dipolar impedance model + tune shift from the quadrupolar impedance.

CONCLUSIONS

Special BBR-settings and the consideration of many radial modes are opportunities the code LINDE offers for the reduction of the vertical TMCI-threshold thereby contributing a bit to close down the gap between the measured and the predicted one – the latter is notoriously higher than the measured one [14]. On the horizontal plane under the effect of a realistic impedance model the modes still couple albeit at higher current. At ALBA a check of horizontal TMCI-threshold is not possible because its value is beyond critical heatload risks. By the way the same effect also leads to an increase of the computed horizontal TMCI-threshold at the old (status 2005) ESRF ring [16] which rectifies the at that time found disparity between vertical and horizontal TMCI-threshold. Upgrades of LINDE allowing for variable bunchlength and spectra are planned. In particular an iterative procedure with an increasing number of radial modes is planned in order to find the detuning slopes. A larger paper discussing the whole topic in more details is foreseen.

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