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TRANSVERSE DENSITY PILEUP AND PATTERN FORMATION IN DENSE ULTRACOLD ELECTRON BEAMLETS UNDER COULOMB EXPANSION

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Abstract

Dynamic Coulomb expansion of dense particle bunches can lead to transverse density shock-like propagation for nonuniform bunch distributions. Furthermore, under favorable circumstances, multiple bunches in close proximity can collide without crossing to form wheel-and-spoke patterns. This process has been observed experimentally for Rubidium ions, but not yet for electrons, where the dynamics occur over far shorter length scales. We simulate the interaction of electron bunches while varying the initial transverse temperature and density profiles to determine the thresholds that characterize this pattern formation. Additionally, we consider the effects of asymmetries and the impact of a low-density halo on the overall process. The simulations are conducted using a novel high-fidelity algorithm for collisional particle dynamics.

INTRODUCTION

Dense ultracold electron beams from arrays with highly localized emission sites have been pursued for applications ranging from electron microscopy and diffraction to injectors for free electron lasers (FELs) and beyond. Typical applications are very concerned with preservation of initial beam quality; in particular with limiting the development of non-uniformities in the beam distribution and energy profiles, which lead to a loss of resolution in imaging techniques and decoherence of the radiation in a FEL. Without large accelerating gradients, dense beams with Gaussian-like distributions will undergo substantive nonlinear Coulomb expansion, which can lead to lasting perturbations in the beam distribution [1].

One such effect was recently experimentally observed during the evolution of cold dense ion beams, generated by multilevel laser excitation of Rubidium atoms cooled in a magneto-optical trap [2]. When arrays of ion beams in close proximity were accelerated, a complex pattern in the transverse profile was observed. This 'wheel and spoke' pattern indicated a hollowing of the beam cores due to Coulomb expansion and a density pileup at the boundaries as the individual beams merged. Similar core hollowing has been observed in the longitudinal component of a high intensity H^- beam [1], and has even been simulated in the transverse case for a pancake electron bunch [3]. Such effects in electron beams are very challenging to measure due to the relevant timescales (on the order of the plasma period) being much shorter for electrons than for heavier ions.

Murphy et al. note that, in addition to the beam hollowing, a low density halo of secondary emission electrons are compressed by the expanding beamlets into the high density boundaries [2]. These cold halo particles not only increase the density, but can also play a role in buffering the Coulombically energized core electrons as the expanding beamlets intersect. In order to better understand the confluence of factors that influence the formation of the wheel-and-spoke patterns in electron beamlets, examples varying the initial temperature and densities were simulated.

SIMULATING THE DYNAMICS OF ELECTRON BEAMLETS

Initial Distribution of Electron Beamlets

Arrays of electron beamlets are generated based on the heavy ion configuration from Murphy et al. [2]. The initial distribution includes nine Gaussian beamlets with a lower density halo of electrons around it (see lower left plot of Fig. 1). The beamlets each have a diameter (full width half max) of 200 µm and are uniformly distributed longitudinally with a length of 5 µm. The halo contains 2×10^4 electrons (3.2 fC) with a uniform density of $n_{halo} = 1.27 \times 10^{15}\ \mathrm{m}^{-3}$, while the beamlets contain 10^4 electrons each (1.6 fC) resulting in a core density (one sigma) of $n_{1\sigma} = 6.15 \times 10^{16}\ \mathrm{m}^{-3}$ when factoring in the halo particles.

The Maxwell-Boltzmann distribution of particle velocity as a function of temperature T is given by

$$f(\mathbf{v}) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m\|\mathbf{v}\|^2}{2k_B T}\right),$$

where k_B is Boltzmann's constant, m is the particle's mass, and $\|\mathbf{v}\|^2$ is the square of the the velocity's Euclidean norm. The particle velocities can be generated using a 3D form of the Box-Muller transform, which projects randomly sampled uniform data to a Gaussian distribution. A set of test values $\mathbf{u} = (u_1, u_2, u_3, u_4)$ is sampled from three uniform distributions taking values within $(0, 1)$. The velocity vector $\mathbf{v}$ is then given by the following relations:

$$v_1 = \sqrt{-\frac{2k_B T \ln u_1}{m}} \cos(2\pi u_2),$$
$$v_2 = \sqrt{-\frac{2k_B T \ln u_1}{m}} \sin(2\pi u_2),$$
$$v_3 = \sqrt{-\frac{2k_B T \ln u_3}{m}} \cos(2\pi u_4).$$

Simulation of Electron Dynamics

The dynamic evolution of electron beamlets was simulated using the collisional N-body code PHAD (particles'

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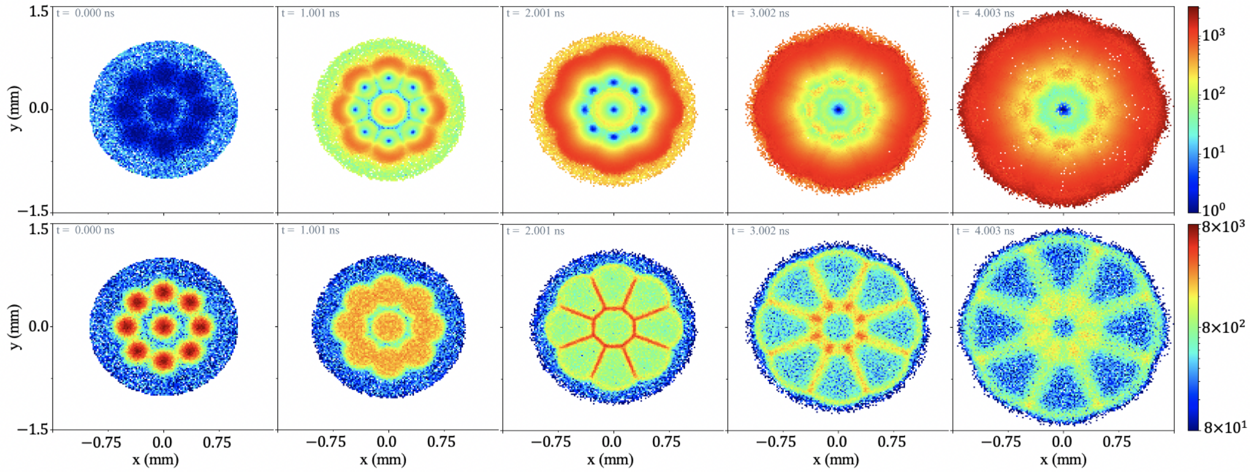

Figure 1: Time evolution of nine beamlets initially at 1 K with an electron halo initially at 10 K. Plotted is the electron temperature (K) above and charge density (μC/m^3) below at time t=0 ns, 1 ns, 2 ns, 3 ns, and 4 ns from left to right.

high-order adaptive dynamics) [4]. PHAD is characterized by the use of Strang splitting to divide the fundamental forces on each particle due to other particles and external fields into near and far components. The far forces are efficiently computed via an adaptive Fast Multipole Method [5], while the near interactions and dynamics are accurately captured by the collisional high-order adaptive Simò integrator [6].

PHAD is written for COSY Infinity, a Fortran-based general purpose nonlinear-dynamics scripting language which boasts a language level differential algebra implementation [7]. PHAD, along with a script for generating an electron beamlets example similar to those in this work, is available for download with documentation from [8]. All of the simulations are performed on the Gaea Cluster maintained by the Center for Research Computing and Data at Northern Illinois University [9].

TEMPERATURE AND DENSITY DEPENDENCIES

The initial trial consists of the beamlet profile described above with beamlet temperature set to 1 K and halo temperature set to 10 K; the secondary/alternate emission processes that lead to halo formation could yield higher temperature electrons. Figure 1 illustrates the dynamics of the beamlets as they merge to form a single beam over a period of 4 ns. Once allowed to evolve, the initial Gaussian profile leads to a Coulomb explosion as the dense core electrons gain transverse energy and expand. As the beamlets impinge on their neighbors, high density regions are formed at the boundaries after about 2 ns from the combined effect of the expanding beamlet cores and the halo electrons pushed together. A less distinct dense outer ring begins to form at this time, and the full wheel-and-spokes pattern, similar to [2], is observed around 4 ns.

The upper row of Fig. 1 illustrates the transverse temperature

$$T = \frac{|\mathbf{p}_\perp|^2}{2m_e k_b} \tag{1}$$

(where m_e is the electron mass and k_b Boltzmann's constant) evolution throughout the process. For the remainder of this work, it is assumed, unless referring to the initial simulation parameters, that temperature is defined as in Eq. (1). Notably from the second plot, the proximity of adjacent beamlets produces a damping effect on the temperature gain, with the hottest electrons located in the outer ring. While the halo perticles may play a small role in moderating the temperature increase, the predominant damping factor comes from the neighboring beamlets. This effect persists as the beam continues to evolve, with a core temperature that is orders of magnitude cooler than the extremity electrons after 4 ns.

Role of Initial Electron Temperatures

Density pileup at the boundaries of merging beamlets is fundamentally a phenomenon of ultracold particle dynamics driven by Coulombic interactions. The nature and coherence of the pattern formation is functionally dependent on the initial temperature of the beamlets and, to a much lesser degree, the temperature of the halo particles. Figure 2 shows the merged beam after 3 ns for increasing initial temperatures. As the temperature increased, the pattern blurs until it is barely visible in Fig. 2(d). For beamlet temperatures above 100 K (~ 1 keV), the pattern formation is completely swamped by the inherent thermal noise.

Variations in Initial Beamlet Density

Varying the initial density of the beamlets will also impact the nature of the beamlet merger and the form of patterns that arise. Decreasing the initial beamlet radius by a factor of two (quadrupling the density) leads to a more rapid Coulomb expansion causing the beamlets to merge around

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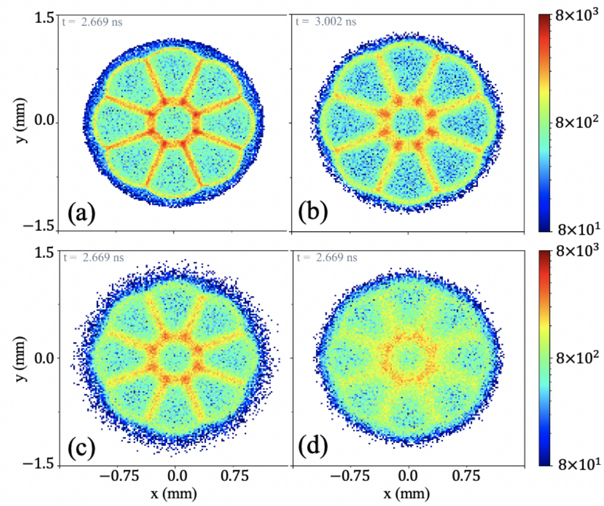

Figure 2: Charge density (μC/m^3) after 3 ns for variations of initial beamlet and halo temperatures: $T_b = T_h = 0.2$ mK (a), $T_b = 1$ K and $T_h = 10$ K (b), $T_b = 10$ K and $T_h = 100$ K (c), $T_b = 100$ K and $T_h = 10$ K (d).

1.5 ns. Unlike previous cases, Fig. 3(c) shows that the dense exterior of the beamlets cross each other due to higher energy from the Coulomb explosion rather than forming the density pileup; Fig. 3(d) shows that while some structure remains, it does not resemble the familiar wheel-and-spokes pattern. Additionally, the beam thermal profile is much hotter than the corresponding cases with lower initial density.

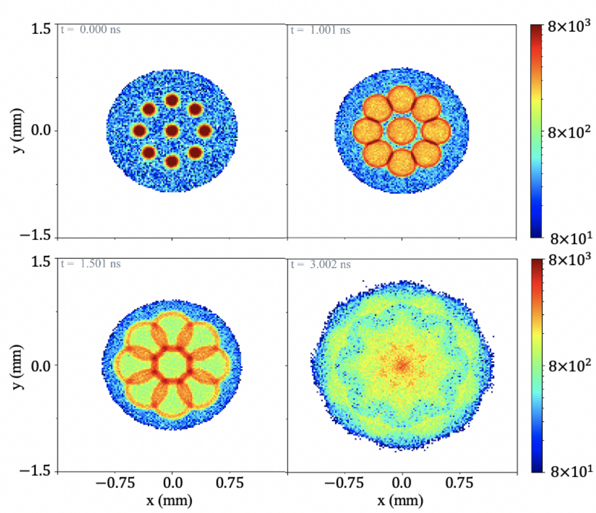

Figure 3: Time evolution of nine beamlets with the beamlet radius halved. Plotted is the charge density (μC/m^3) at time t=0 ns (a), 1 ns (b), 1.5 ns (c) and 3 ns (d).

Another scenario involves decreasing the radius of the outer beamlets commensurately with the decrease in beamlet radius. These results, shown in Fig. 4, reveal that the wheel-and-spoke pattern again forms (this time within 1 ns). However the rapid Coulomb expansion injects enough energy into the system that the pattern breaks down over time.

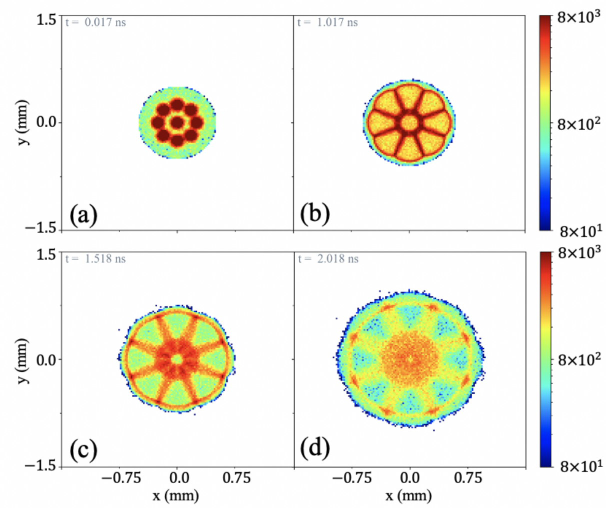

Figure 4: Time evolution of nine beamlets spatially scaled down by 50%. Plotted is the charge density (μC/m^3) at time t=0 ns (a), 1 ns (b), 1.5 ns (c) and 3 ns (d).

CONCLUSION

When generating beamlets in the ultracold regime, the specific geometric arrangement and density of the initial beamlets along with the initial temperature play a substantial role in the way that they combine. This leads to the possibility of utilizing this phenomenon as a downstream method for characterizing the initial emittance or temperature profile for such an electron source.

Based on thermal plots of the results, the core beam temperature will be limited if geometric parameters are optimized. Since high energy electrons are concentrated at the extremity of the beamlet array, they can be selectively removed (ie. via collimation), efficiently cooling the beam with a minimal loss of particles. This would be especially effective for sources that generate a large array (ie. thousands) of beamlets if this phenomenon persists on that scale.

ACKNOWLEDGMENTS

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