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SIMULATION OF INTRA-BEAM SCATTERING IN PyHEADTAIL*

V. Rodin†[1], C. P. Welsch[1], University of Liverpool, Liverpool, UK
A. Oeftiger, GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany
[1]also at The Cockcroft Institute, Daresbury, UK

Abstract

High-intensity beams in low-energy synchrotrons are subject to space charge as well as intra-beam scattering (IBS). Accurate modelling of both effects becomes essential when the transverse emittances and minimum bunch length are determined through heating processes and resonances induced by machine errors. To date, only very few tools available to the general public allow to simultaneously study space charge and IBS in self-consistent simulations. In this contribution, we present our recent development of an IBS module for PyHEADTAIL, an open-source 6D multi-particle tracking tool, which already includes various 2.5D and 3D space-charge models based on the self-consistent particle-in-cell algorithm. A simulation example of high-intensity bunch rotation demonstrates the joint impact of applied heating effects. Our model is based on the Martini and Bjorken-Mitingwa theories. Benchmarks of our implementation against IBS modules provided in the MAD-X, Betacool and JSPEC codes are shown.

INTRODUCTION

The main cornerstones for the advancement of currently operating and future circular hadron accelerators and decelerators are beams of high intensity and small transverse as well as longitudinal beam sizes. Unfortunately, pushing these three quantities requires a trade-off between them. Various collective effects and machine errors act simultaneously on the beam resulting in beam quality and performance degradation – studying and understanding their interplay hence becomes crucial. Space charge and intra-beam scattering (IBS) are among the most important collective effects: space charge refers to the interaction of the bunch particles with the bunch mean-field while IBS denotes the (mostly small-angle) binary Coulomb scattering between bunch particles.

Many research studies observe rms emittance growth to be attributed to either space-charge [1, 2] or IBS [3, 4] effects as the dominating effect and correspondingly neglect the other. However, in some situations both effects can attain similar impact on the beam dynamics, e.g. in the case of the CERN ELENA ring [4] when the bunch is rotated and space charge becomes strong as well. Only recently, simulations of combined physical models for intra-beam scattering and space-charge were demonstrated for the first time by H. Zhao et al., cf. Ref. [5]. In this study, the influence of the individual effects was benchmarked against other codes and results compared to experimental data.

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† volodymyr.rodin@liverpool.ac.uk

In the same manner, to extend future simulation capabilities to predict the evolution of circulating beam as wells as longitudinal bunch manipulations, in this paper, we present the implementation of two IBS models into the multi-particle tracking code PyHEADTAIL [6]. This study will allow us in the future to study the interplay between IBS and space-charge [7]. As an application of the newly developed module we study bunch rotation in the CERN ELENA ring [8, 9].

PyHEADTAIL SIMULATION MODEL

In PyHEADTAIL, the accelerator can be represented as a concatenation of elements where various particle tracking steps are performed (see Fig. 1). The beam is described as a large number of macroparticles that represent a clustered collection of physical particles. It allows to treat large beam intensities in a realistic manner and within limitations of computational power. When space-charge is applied in self-consistent manner large number of macroparticles are required to suppress numerical noise [10, 11].

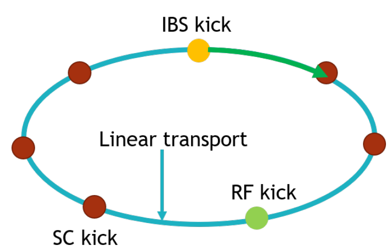

Figure 1: Schematic of PyHEADTAIL simulation structure.

A particle beam itself is transported from one element to another by means of transfer matrices, including detuning effects such as chromaticity and amplitude detuning from octupole components. The machine optics in the transverse planes can be obtained from beam dynamics codes such as MAD-X or BMAD [12, 13]. The tracking of the beam in the longitudinal plane is performed either via linear synchrotron motion or via full non-linear RF kicks supporting also multi-harmonic RF systems. After each tracked segment of the one turn map collective interactions can be modelled via a kick, e.g. space-charge kicks or, like in our case, IBS kicks calculated from an effective (analytical) growth rate model based on the full lattice. More advanced approaches like molecular dynamics or kinetic models [3, 14] may take into account random interactions between particles within the distribution and will be studied in the future.

SOME EFFECTIVE IBS THEORIES

Commonly employed methods of IBS analysis in storage rings are based on a "Gaussian description" of the beam,

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which is assuming that distribution profiles are of Gaussian type for all degrees of freedom. It was shown that a time evolution of the Gaussian beam is described by a system of three differential equations for the rms emittances of the beam. Numerically, the IBS effects can be described by two fundamental models: Eq. (1), Martini [15] and Eq. (2), Bjorken–Mtingwa [16] theory. For the PyHEADTAIL module, we follow the fast computational implementation described by Ref. [17].

Martini Method

Also referred to as the modified Piwinski model, the Martini model describes the longitudinal and transverse emittance growth rates τ_i for bunched beam averaged over the ring circumference $\langle\cdot\rangle$ by [15, Eq. (35)],

$$\begin{aligned}\frac{1}{\tau_p} &= \left\langle A_M \left(1-d^2\right) f_z \right\rangle, \\ \frac{1}{\tau_{x'}} &= \left\langle A_M \left[f_x + \left(d^2+\tilde{d}^2\right) f_z\right] \right\rangle, \\ \frac{1}{\tau_{y'}} &= \left\langle A_M f_y \right\rangle,\end{aligned} \tag{1}$$

$$A_M = \frac{c r_i^2 N_b}{64\pi^2 \beta^3 \gamma^4 \epsilon_x \epsilon_y \sigma_p \sigma_z} \sqrt{\left(1+\alpha_x^2\right)\left(1+\alpha_y^2\right)}, \tag{2}$$

where $d, \tilde{d}$ are dispersion components, r_i the classical particle radius, c the speed of light, N_b the bunch population, β particle velocity over c, γ the Lorentz energy factor, σ_z the bunch length, $\alpha_{x,y}$ the correlation Twiss functions, $\epsilon_{x,y}$ the geometric (unnormalised) transverse rms emittances, p the longitudinal momentum and x', y' the transverse conjugate particle momenta. The standard rms values of the longitudinal beam size and momentum spread are correspondingly denoted σ_z and σ_p. Piwinski's scattering functions $f_{x,y,z}$ [18, Eq. (52)] depend on the lattice and are calculated via numerical integration for each dimension of the distribution.

Bjorken-Mtingwa Method

As Martini writes [19], this approach of IBS theory is based on the scattering matrix (S-matrix) formalism related to quantum electrodynamics (QED), which relates transitions from an initial state to the final state of a quantum field system. The formalism develops the Fermi scattering "Golden Rule" and employs it to compute the low energy scattering amplitudes between particles. The growth rates according to Bjorken-Mtingwa [16, Eq. (3.4)],

$$\frac{1}{\tau_i} = A_{BM} C_{log} \Bigg\langle \int_0^\infty \mathrm{d}\lambda \frac{\sqrt{\lambda}}{\sqrt{\det(L+\lambda I)}} \left\{\mathrm{Tr}(L^{(i)})\mathrm{Tr}\left[(L+\lambda I)^{-1}\right] - 3\mathrm{Tr}\left[L^{(i)}(L+\lambda I)^{-1}\right]\right\} \Bigg\rangle, \tag{3}$$

where (i) represents p, x or y. The Coulomb logarithm C_{log} and auxiliary matrices $L^{(i)}$ are defined according to Ref. [16]. With the bunched version of [16, Eq. (4.12)] (i.e. times $\sqrt{2}$), the Bjorken-Mtingwa scattering constant A_{BM} reads

$$A_{BM} = \frac{c r_i^2 N_b}{8\pi \beta^3 \gamma^4 \epsilon_x \epsilon_y \sigma_p \sigma_z}. \tag{4}$$

BENCHMARK OF GROWTH RATES

In analogy to Ref. [20], we choose the CERN ELENA ring and antiproton beam parameters at the low-energy extraction plateau to compare different calculations of IBS growth rates in MAD-X, Betacool [21] and JSPEC against the Python implementation in PyHEADTAIL. The latter two codes are expected to be in close agreement as their implementation is equivalent. Table 1 summarises all important values that were used for the simulation input. MAD-X is used to compute the machine optics. The initial growth rates from all codes are given in Table 2.

Table 1: Simulation Machine and Beam Parameters for Antiprotons in the ELENA Ring at the Low-energy Plateau

Parameter	**Value**
Coulomb logarithm C_{log}	12.5
Bunch length rms $\sigma_{z,t}$ (m, ns)	0.3282, 75
Relative momentum spread $\Delta P/P_0$	1e-4
Reference momentum P_0 (MeV/c)	13.7
Hor./vert. rms emittance $\epsilon_{x,y}$ (µmrad)	2.5, 2.5
Nominal working point $Q_{x,y}$	2.454, 1.416
Bunch intensity N_b	4.5e6
Maximum $\beta_{x,y}$ (m)	14.1, 4.5
RF voltage V_{rf} (Volt)	0 and 100
RF frequency (kHz)	144

Table 2: Calculated Growth Rates

Sim. code, (model)	$\boldsymbol{\tau_x}$, $\mathbf{s^{-1}}$	$\boldsymbol{\tau_y}$, $\mathbf{s^{-1}}$	$\boldsymbol{\tau_z}$, $\mathbf{s^{-1}}$
PyHEADTAIL (Martini)	0.2239	-0.3074	51.64
JSPEC (Martini)	0.224	-0.3074	51.65
Betacool (Martini)	0.216	-0.301	51.71
PyHEADTAIL (BM)	0.2208	-0.3061	51.69
JSPEC (BM)	0.221	-0.3062	51.69
MAD-X (Mod. BM)	0.2109	-0.3034	52.48

When the growth rates are known, it is possible to calculate the mean square of the scattering angle θ as an extra addition to the initial momentum components of the particle. After random scattering of all particles from this distribution, e.g. the updated horizontal emittance can be found from

$$\epsilon_{x,new} = \sqrt{\left\langle (x_i - \langle x \rangle)^2 \right\rangle \left\langle (x_i' - \langle x' \rangle + \theta)^2 \right\rangle}, \tag{5}$$

with $\langle\theta^2\rangle$ determined by the growth rates and time step dt,

$$\langle\theta_i^2\rangle = 2\frac{\epsilon_i dt}{\beta_i \tau_i}. \tag{6}$$

The horizontal and vertical emittance growth in tracking simulations with JSPEC and PyHEADTAIL based on the computed growth rates are demonstrated in Fig. 2.

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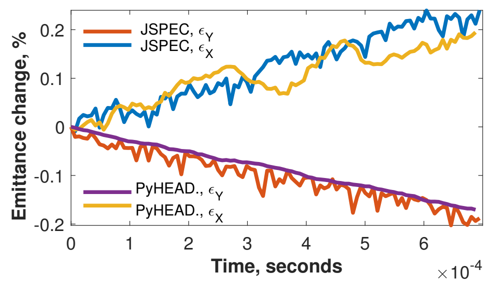

Figure 2: Comparison of transverse rms emittance evolution between JSPEC and PyHEADTAIL.

Both codes predict the same evolution: increasing horizontal emittance due to positive growth rates and decreasing vertical emittance due to negative growth rates. The jitter in the horizontal plane can be attributed to the random distribution of the applied scattering angle θ in the IBS kick. While PyHEADTAIL continuously tracks the same distribution self-consistently, JSPEC assigns a random phase advance for each particle after the given time step dt (which allows faster tracking and avoids numerical resonance artifacts but inherently leads to jitter). The residual small discrepancy in the vertical plane can thus be attributed to the difference in the particle propagation in PyHEADTAIL and JSPEC.

BUNCH ROTATION WITH IBS INCLUDED

As an applied benchmark of our module, the dynamic process of bunch rotation has been investigated employing full 6D tracking. The PyHEADTAIL model of the ring assumes smooth approximation with mean values of the optics calculated from

$$\beta_i = \frac{R}{Q_i}, \; D_i = \frac{\beta_i}{Q_i}, \tag{7}$$

with R the machine radius and Q_i the betatron tunes of the machine. The resulting evolution of the bunch length and longitudinal growth rates are shown in Fig. 3.

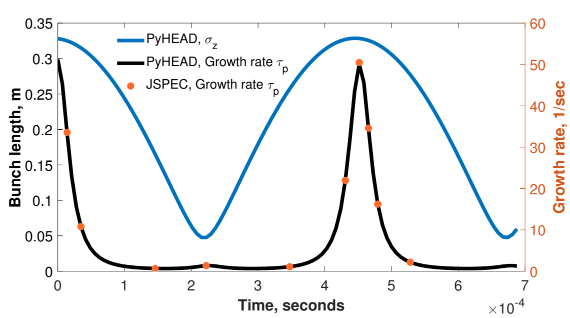

Figure 3: Bunch length and longitudinal growth rate evolution during bunch rotation process.

The maximum longitudinal growth rate is observed when the bunch is fully extended and momentum spread is the smallest, in line with the theoretical scaling to first order in Eqs. (1) and (2): $1/\tau_p \propto \sigma_p$. The overall growth rate value has slightly decreased after one full rotation due to an increase of momentum spread from the IBS heating. The instantaneous growth rates in black compare well to the corresponding statically computed JSPEC values in red. The top panel in Fig. 4 depicts the horizontal growth rate $1/\tau_x$ during the bunch rotation, again with agreeing PyHEADTAIL and static JSPEC predictions. The lower panel displays the evolving horizontal rms emittance ϵ_x: each time the bunch is maximally compressed and $1/\tau_x$ peaks, ϵ_x exhibits a fast increase.

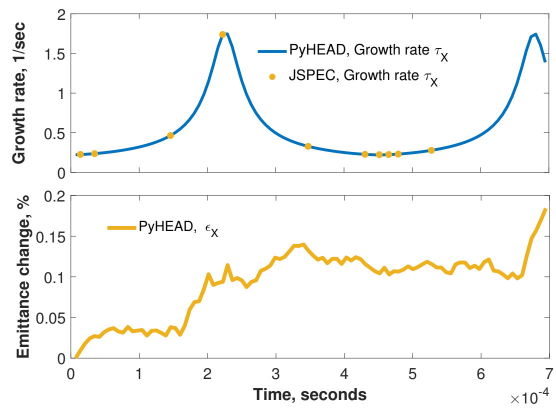

Figure 4: (Top) horizontal growth rate τ_x and (bottom) horizontal rms emittance ϵ_x evolution during bunch rotation.

CONCLUSIONS AND OUTLOOK

An effective algorithm based on computed growth rates for estimation of intra-beam scattering was successfully implemented into PyHEADTAIL. The calculations of the growth rates are based on Martini as well as Bjorken-Mtingwa theories, which both consider the Gaussian approximation for all 3 dimensions of the beam. A benchmark of growth rates has been presented using the CERN ELENA ring, showing excellent agreement between all used codes, viz. PyHEADTAIL, JSPEC, Betacool, and MAD-X.

As an application of the new module, the bunch rotation process of a longitudinally unmatched bunch has been investigated in the ELENA ring. Longitudinal growth rates have been analysed and shown to match the prediction computed by JSPEC. Future plans include simulations, where both space-charge and IBS effects are applied to the bunch rotation process and other similarly challenging situations.

Furthermore, the implementation of a kinetic version of the IBS kick is foreseen in close analogy to the proposals by previous studies in Refs. [5, 14].

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