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REGENERATIVE BEAM BREAK UP INSTABILITY ANALYSIS*

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Abstract

New features of regenerative beam break up (BBU) instability such as the typing of all high order dipole modes (HOMs) for each cavity by two classes one of them are stable and other ones are unstable, HOM effective quality factor depending on average beam current, and normalized invariable threshold current individually characterizes each HOM are investigated in this article in detail.

INTRODUCTION

A train of bunches passing through an accelerating structure can excite transverse deflecting modes through a longitudinal electric field which increases linearly off axis for such modes. For a low energy beam, the excitation can be strong enough to oscillate the beam significantly during transit through a single structure.

Therefore, there can be closure of a feedback loop within a single structure: an excited cavity mode oscillates a beam which in turn further excites the mode. Stability comes from the damping of the fields through wall losses and couplers and through transverse beam focusing. This effect is known as regenerative beam breakup [1], and has a threshold current. Above threshold, there is exponential growth of the cavity excitation.

In recent works dedicated to linacs and ERLs, e.g. [2] describing widely used well-known transfer matrix method, there applies simplified methods directly not taken into account regenerative BBU. Hereby, a linac containing different sections and periodic substructures with a given cavity spacing is modelled using a drift-kick approach. Point like bunches are tracked through the complete linac. The particle velocity is assumed constant inside the cavity and the interaction between particle and cavity takes place in the cavity midplane [3]. We will show that in order to take into account regenerative BBU in such works the effective quality factor must be applied.

In the article, authors widely used recent computation capabilities such as tracking code ASTRA [4] and EM field calculation code CLANS2 [5]. So, there is no need modelling of transverse beam momentums for each substructure, so beam current independent Panofsky - Wenzel theorem [6] does not used there. Instead, we made BBU analyze in terms of transverse beam oscillations with using of the beam amplitude, phase, and frequency terms.

BBU INSTABILITY KEY CONSTANT

The BBU energy interchange is the transferring of a part of beam kinetic energy to stored RF mode field energy or vice-versa. Respectively, the beam is potentially unstable in the first case and is stable in the second case. Incidentally, some part of this energy is lost in the cavity wall and in external loads. We recall that this energy transferring process is going on due to beam space-modulations (transversal oscillations) with the HOM frequency. Instability comes if the power transfer from the beam to the cavity mode is higher than the power loss.

To be a stable BBU process, the transferred average power (P_{BBU}) from a beam to the HOM must be lower than the dissipated power (P_{loss}), i.e., $P_{BBU} \leq P_{loss}$. Such an approach has been considered by W. K. H. Panofsky and M. Bander in [7] and another author of works concentrated on the power balance method.

We have to draw attention that only the average transferred power P_{BBU} plays a role in the BBU instability. Each bunch arrives at a different phase of an HOM and so it can either gain or loss energy. If the bunch repetition frequency is not close to any (sub) harmonic of the HOM, P_{BBU} depends on the average current only (or on the DC component of beam current), and not depends on the bunch repetition frequency. Even a DC current beam propagating through a cavity could be unstable [8].

The common BBU threshold current formula can be derived from the power balance in the system $P_{BBU} \leq P_{loss}$, where $P_{BBU} = -V_{BBU} \cdot I$, and V_{BBU} is an average energy gain (or loss) of beam particles in the HOM that is in units of Volts. I is the average beam current that is assumed always to be a positive value, $P_{loss} = \omega \cdot U/Q$, where ω is angular resonance frequency of the HOM, U is the stored dipole mode energy, and Q is the loaded quality factor of the HOM. By substitution we get

$$I \cdot Q \leq I_{th} Q \equiv -\frac{\omega \cdot U}{V_{BBU}} \equiv I_Q. \qquad (1)$$

We enunciate I_Q is the key constant in BBU analysis that characterizes each HOM individually. There are HOMs with $I_Q>0$ which represents the class of unstable HOMs. Equation (1) directly shows the way to suppress BBU instability via the quality factor damping $I_{th} = I_Q/Q$, where I_{th} is the threshold current that is the function of Q. If we replace Q by $Q = \omega \cdot U/P_{loss}$ in Eq. (1), we get $I_{th}V_{BBU} = -P_{loss}$, i.e., in accord with definition, the threshold current is those beam current that ensures the value of the BBU energy interchange power to be equal to the losses power for potentially unstable modes ($V_{BBU}<0$). Also, there are the class of stable HOMs (usually ignored) which have $I_Q<0$.

I_Q is denoting as invariable threshold current.

Fundamentality of Eq. (1)

The I_Q = *constant* assumes that averaged particle energy gain (or loss) V_{BBU} always proportional to the stored energy of the RF mode or to the cavity field squared ($U \sim E^2$). Thus, we have from Eq. (1) $V_{BBU} = -(\omega/I_Q) \cdot U$.

RF longitudinal electric field of dipole modes scales approximately linearly on a displacement off-axis (y), i.e.,

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$E_z(z,y) = y \cdot E'_z(z)$, where $E'_z(z) = \partial E_z(z,y)/\partial y$. It is assumed the Y polarization that is independent of z. The averaged energy gain (or loss) V_{BBU} could be obtained from Eq. (2) by integrating on all phases φ while the particle is moved along the trajectory $y = y(z,\varphi)$ defined by the motion differential Eq. (3), where φ is the dipole field initial phase at the coordinate $z = 0$, and $z = \beta ct$.

$$V_{BBU} = \int_0^{2\pi} \int_0^L E'_z(z) \frac{y(z,\varphi)}{2\pi} sin(2\pi z/\beta\lambda + \varphi) dz\, d\varphi\ . \quad (2)$$

$$d(dy/dz)/dz = (e/m\gamma\beta c)\, B_x(z) \cos(2\pi z/\beta\lambda + \varphi), \quad (3)$$

where β is the particle velocity normalized to the velocity of light c, $\lambda = 2\pi c/\omega$, γ is the relativistic factor, $B_x(z)$ is the X axis directed magnetic dipole field distribution. Without loss of generality, it can be assumed that γ be independent of z and $z(t) = \beta ct$ is the linear function of time t.

A number of numerical simulations with ASTRA code made by authors [9, 10] have shown that it is the correct statement for common case where both the $\gamma = \gamma(z)$ is changed due to an accelerating or decelerating main mode and even the polarization of oscillations is changed due to solenoid focusing fields.

By definition, the rf electric field in a cavity (E_z) is proportional to square root of the stored energy and the particle displacement (y) is proportional to the B_x field, i.e., y is proportional to square root of the stored energy. So, the integrating of Eq. (3) gives the proportionality to the stored energy or rather $V_{BBU} \sim (e/m\gamma)U$. These results we have obtained without of exact solution of Eqs. (2) and (3).

So, we have proved the fundamental feature of Eq. (1), i.e., I_Q is the self-sufficing parameter individually characterizing each HOM and completely describing the regenerative BBU instability.

Normalized Invariable Threshold Current

Let us define the normalized invariable threshold current I_{Qn} that is independent of the beam particle properties as it is shown by the previous integration result (we assume $e>0$):

$$I_{Qn} = I_Q \frac{e}{m\gamma}\ . \quad (4)$$

Equation (4) is valid for $\gamma \geq 2$ with sufficient accuracy as it is numerically shown with a number of ASTRA simulations, e.g., made for TESLA-type cavities by authors in Ref. [11]. So, the fundamentality remains valid on the range of all main accelerator facility applications.

Numerical Calculations of I_Q

The numerical value of I_Q in Eq. (1) can be calculated by a number of particle dynamic codes by tracking particles through the HOM field with low stored energy U and the frequency ω to obtain the energy gain (or loss) $\mathcal{E}(\varphi)$. Its first order approximation dependence on initial field phase (φ) is $\mathcal{E}(\varphi) = \mathcal{E}_0 sin(2\varphi + 2\varphi_0) + V_{BBU}$ [9, 10]. To obtain the averaged energy gain (or loss) V_{BBU} it is sufficient to track particles at two different initial field phases φ_0: $\varphi_1 = 0$ *and* $\varphi_2 = \pi/2$. The solution of system of equations $\mathcal{E}_1 = \mathcal{E}(\varphi_1)$ *and* $\mathcal{E}_2 = \mathcal{E}(\varphi_2)$:

$$V_{BBU} = (\mathcal{E}_1 + \mathcal{E}_2)/2\ . \quad (5)$$

According to superposition principle, this solution is correct in a sum of all RF modes including the main accelerating RF mode presented in the cavity. The energy gain (or loss) $\mathcal{E}(\varphi)$ in this case is equal to the beam energy adding component at the cavity exit that comes when the BBU analyzed HOM in the calculation to be appeared.

EFFECTIVE QUALITY FACTOR

To operate with polarized dipole modes let us introduce the new vector parameter F such that $|F| \equiv U^{1/2}$. F vector is directed along the EM force that is perpendicular to the magnetic RF mode field vector B and to the axis.

Let us consider an initially displace modulated electron beam such that particles propagate parallel to the axis of a cavity with a dipole HOM. The averaged energy gain (or loss) V_{OSC} could be described by Eq. (2) as this beam oscillation component is represented by $y(z,\varphi) = A \cdot sin(\varphi + \theta)$, where θ is the phase difference between the beam displacement oscillation and B field phase of the dipole HOM, and A is the off axis maximal (or amplitude) displacement.

Using Eq. (2) and the trigonometric identity $sin(x+y) = sin(x)cos(y) + cos(x)sin(y)$, we obtain $V_{OSC} = V \cdot cos(\Phi)/4\pi$, were $V = (\mathcal{E}_c^2 + \mathcal{E}_s^2)^{1/2}$ is the maximal energy gain of a single bunch propagating through the cavity. Here $\cos(\Phi) = [1 + Q^2(\omega/\omega_0 - \omega_0/\omega)^2]^{-1/2}$, where Φ is interpreted as the oscillation phase shift relative to the on-crest acceleration phase that appears due to a difference between the beam oscillation frequency and the mode resonance frequency ω_0; $\mathcal{E}_c = A \int_0^L E'_z(z) cos(2\pi z/\beta\lambda) dz$, $\mathcal{E}_s = A \int_0^L E'_z(z) sin(2\pi z/\beta\lambda) dz$, were L is the cavity length.

By definition, $V = A \cdot [2\omega(R_{II}/Q)U]^{1/2}$, where R_{II}/Q is the conventional longitudinal coupling impedance of the dipole mode in terms of RLC circuits and in units of Ohm/m^2. It follows that

$$V_{OSC} = -(\bar{A} \cdot \bar{F})\sqrt{\omega(R_{II}/Q)} \cdot cos(\Phi)/4\pi, \quad (6)$$

where the polarization and the field direction are taken into account by the scalar product of vectors A and F.

There must be a limitation on V_{OSC} values to ensure a high accuracy of Eq. (5) since Eq. (1) was derived without taking into account of V_{OSC}. For the results of Eq. (5) to be accurate we must require $|V_{OSC}| \ll |V_{BBU}|$. Using Eqs. (1) and (6), we get

$$A \ll \frac{4\pi}{|I_Q|}\sqrt{\frac{\omega U}{R_{II}/Q}}\ . \quad (7)$$

On the basis of power balance, the differential equation: $-V_{BBU} \cdot I - V_{OSC} \cdot I = P_{loss} + \Delta U/\Delta t$ be the case. Insert-

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ing expressions $P_{loss} = \omega U/Q$, Eqs. (1) and (6), and integrating the mentioned balance equation, we get:

$$F(t) = F_0 \cdot exp\frac{-\omega t}{2Q_{eff}} + \frac{A_0}{4\pi} I Q_{eff}\sqrt{(R_{II}/Q)/\omega} \times \left(1 - exp\frac{-\omega t}{2Q_{eff}}\right), \quad (8)$$

$$Q_{eff}(I) = Q/\left(1 - I \cdot Q/I_Q\right), \quad (9)$$

where F_o and A_o are the vector values of initial dipole mode field and initial beam oscillation respectively.

Here we consider $\Phi=0$, i.e., oscillations are in resonance with the mode. We have introduced here the new parameter Q_{eff} presented in Eq. (9) that naturally describes this energy interchange process. If Q_{eff} is diminished while the beam current is increasing then the mode is stable and if $1/Q_{eff}$ is diminished then it indicates potentially instable modes that become absolutely instable at $1/Q_{eff}<0$.

Equation (8) describes the regenerative BBU instability process. Without of the beam ($I = 0$), Eq. (8) describes usual process of decaying field oscillations in a cavity and for $I\neq 0$ it is the usual excitation process of the cavity by a current I. If the dipole mode is potentially instable ($I_Q>0$) and the beam current is large than the threshold one ($I>I_{th} = I_Q/Q$) then the dipole mode field growth exponentially in time.

EXPERIMENTS

By using Eq. (8) we have obtained [12] the equation of effective quality factor for ERL machine cavities as

$$Q_{eff}^{ERL} = Q/\left(1 - \frac{IQ}{I_Q^{ERL}} - \frac{IQ}{I_Q^{REG}}\right), \quad (10)$$

where the invariable threshold current for ERL machines I_Q^{ERL} is obtained from conventional one given by [2]

$$I_Q^{ERL} = I_{th}^{ERL} \cdot Q = \frac{2V_b}{k(R_\perp/Q)\cdot M^{*}\cdot sin(\omega T)}, \quad (11)$$

where V_b is the beam voltage, $k = \omega/c$, $R\perp/Q$ is the transverse impedance in units of Ohms, M^* is full 4x4 transfer matrix that has been used to take into account coupled transverse motion, T is the period of the ERL machine. We denote all conventional ERL values by ERL index and regenerative threshold in the ERL cavity by REG index.

If Q is a constant in experiments, Eq. (10) simplifies to

$$Q_{eff}^{ERL} = Q/\left(1 - \frac{I}{I_{th}^{ERL}} - \frac{I}{I_{th}^{REG}}\right). \quad (12)$$

This assumption of Eq. (12) without of the last term is derived and proved experimentally [2]. But unfortunately, in these experiments the BBU effect in separate cavities does not taken into account (is assumed to be $I_{th}^{REG} \to \infty$).

CONCLUSION

- Quality factors of all dipole HOMs identically depends on a beam current. We have in mind only average beam current or its DC component.
- This dependency includes a key constant parameter for each dipole HOMs individually found for each considered cavity and named as invariable threshold current I_Q.
- All dipole HOMs are divided by two classes one of them are stable HOMs with $I_Q<0$ and another one is potentially unstable HOMs with $I_Q>0$.

REFERENCES

[1] J. J. Bisognano, "Superconducting RF and Beam-Cavity Interactions", in *Proc. of The Third Workshop on RF Superconductivity*, Argonne National Laboratory, Illinois, USA, Sep. 1987, paper SRF87C01, pp. 237-248.

[2] C. D. Tennant, "Studies of Energy Recovery Linacs at Jefferson Laboratory: 1 GeV Demonstration of Energy Recovery at CEBAF and Studies of the Multibunch, Multipass Beam Breakup Instability in the 10 kW FEL", Ph.D. thesis, The College of William & Mary, Williamsburg, Virginia, USA, 2006.

[3] M. Schuh, F. Gerigk, J. Tuckmantel, and C. P. Welsch, "Influence of Higher Order Modes on the Beam Stability in the High-Power Superconducting Proton Linac", *Phys. Rev. ST Accel. Beams*, vol. 14, p. 051001, 2011. doi:10.1103/PhysRevSTAB.14.051001

[4] *ASTRA User's Manual*, DESY, Hamburg, Germany, Mar. 2017. http://www.desy.de/~mpyflo/Astra_dokumentation/.

[5] D. G. Myakishev and V. P. Yakovlev, "CLANS2 - A Code for Calculation of Multipole Modes in Axisymmetric Cavities with Absorber Ferrites", in *Proc. PAC'99*, New York, NY, USA, Mar. 1999, paper THA76, pp. 2775-2777.

[6] W. K. H. Panofsky and W. A. Wenzel, "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", *Rev. Sci. Instrum.*, vol. 27, no. 11, pp. 967–967, Nov. 1956. doi:10.1063/1.1715427

[7] W. K. H. Panofsky and M. Bander, "Asymptotic Theory of Beam Break-Up in Linear Accelerators", *Rev. Sci. Instrum.*, vol. 39, pp. 206-212, 1968. doi:10.1063/1.1683315

[8] P. B. Wilson, "Excitation of Longitudinal and Transverse Mode Oscillations in a Superconducting Accelerator Structure by a Low Energy DC Beam", High Energy Physics Laboratory, Stanford University, Stanford, USA, Rep. HEPL-TN-67-6, Jul. 1967.

[9] V. Volkov, "Prediction and suppression of beam breakup instability in multicell superconducting cavities", *Phys. Rev. Spec. Top. Accel. Beams*, vol. 12, no. 1, 2009. doi:10.1103/physrevstab.12.011301

[10] V. Volkov, J. Knobloch, and A. Matveenko, "Beam breakup instability suppression in multicell superconducting rf guns", *Phys. Rev. Spec. Top. Accel. Beams*, vol. 14, no. 5, 2011. doi:10.1103/physrevstab.14.054202

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-WEPAB151

[11] V. Volkov and V. M. Petrov, "Numerical Estimation of Beam Break-Up Instability in TESLA Cavities", in *Proc. SRF'19*, Dresden, Germany, Jun.-Jul. 2019, pp. 1178-1181. doi:10.18429/JACoW-SRF2019-THP104

[12] V. Volkov and V. M. Petrov, "Regenerative and Monotron Beam Break up Instabilities", unpublished.