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MONOTRON BEAM BREAK UP INSTABILITY ANALYSIS*

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Abstract

New features of monotron beam break up (BBU) instability such as the typing of high order monopole modes (HOMs) in each cavity by two classes, one of them are stable HOMs and other ones are unstable HOMs, HOM effective quality factor depending on average beam current, and normalized invariable threshold current individually characterizes each HOM are investigated in this article in detail.

INTRODUCTION

A DC current beam or bunched one with no resonance repetition frequency harmonics passing through an accelerating structure can excite monopole modes through a longitudinal electric field which always exists for such modes. For a low energy beam, the excitation can be strong enough to oscillate the beam significantly during transit through a single structure into regions of higher longitudinal field and increased coupling.

Therefore, there can be closure of a feedback loop within a single structure: an excited cavity mode oscillates a beam which in turn further excites the mode. Stability comes from the damping of the fields through wall losses and couplers. This effect is known as monotron beam breakup [1], and for a standing wave structure has a threshold current. Above threshold, there is exponential growth of the cavity excitation.

In the article, authors are widely used tracking code ASTRA [2] and EM field calculation code CLANS [3]. We analyze BBU instability in terms of longitudinal beam oscillations with using of the beam current amplitude, phase, and frequency.

BBU INSTABILITY KEY CONSTANT

The BBU energy interchange is the transferring of a part of beam kinetic energy to stored RF mode field energy or vice-versa. Respectively, the beam is potentially unstable in the first case and is stable in the second case. Incidentally, some part of this energy is lost in the cavity wall and in external loads. We recall that this energy transferring process is going on due to beam space-modulations (longitudinal oscillations) with the HOM frequency as a result of the beam interaction with the HOM. Instability occurs if the power transfer from the beam to the cavity HOM becomes higher than the power loss which is the subject of the power balance method.

To be a stable BBU process, the transferred average power (P_{BBU}) from a beam to the HOM must be lower than the dissipated power (P_{loss}), i.e., $P_{BBU} \leq P_{loss}$. Such an approach has been considered by W. K. H. Panofsky and M. Bander in [4] and by another authors of work concentrated on the power balance method.

The common BBU threshold current formula can be derived from the power balance in the system $P_{BBU} \leq P_{loss}$, where $P_{BBU} = -V_{BBU} \cdot I$, and V_{BBU} is an average energy gain (or loss) of beam particles in the HOM that is in units of Volts. I is the average beam current that is assumed always to be a positive value, $P_{loss} = \omega \cdot U/Q$, where ω is angular resonance frequency of the HOM, U is the stored dipole mode energy, and Q is the loaded quality factor of the HOM. By substitution we get

$$I \cdot Q \leq I_{th} Q \equiv -\frac{\omega \cdot U}{V_{BBU}} \equiv I_Q. \quad (1)$$

We enunciate I_Q is a key constant in BBU analysis that characterizes each HOM individually. There are HOMs with $I_Q>0$ which represents the class of unstable HOMs. Eq. (1) directly shows the way to suppress BBU instability via the quality factor damping $I_{th}=I_Q/Q$, where I_{th} is the threshold current that is the function of Q.

If we replace Q by $Q = \omega \cdot U/P_{loss}$ in Eq. (1), we get $I_{th}V_{BBU}=-P_{loss}$, i.e., in accord with definition, the threshold current is those beam current that ensures the value of the BBU energy interchange power to be equal to the losses power (for potentially unstable modes $V_{BBU}<0$). There are the class of stable HOMs usually ignored which have $I_Q<0$.

I_Q is denoting as invariable threshold current.

Fundamentality of Eq. (1)

The I_Q=*constant* assumes that averaged particle energy gain (or loss) V_{BBU} always proportional to the stored energy of the RF mode or to the cavity field squared ($U \sim E^2$). Thus we have from Eq. (1) $V_{BBU}=-(\omega/I_Q) \cdot U$.

Relativistic differential equation of longitudinal particle dynamics in an RF monopole mode field $E(z) \cdot \sin(\omega t+\varphi_0)$ is $d\Delta p/dt=eE(z)\sin(\omega t+\varphi_0)$. With assuming $E(z) \to 0$, we can use the approximation $z \approx \beta ct$, where β is the particle velocity normalized to the velocity of light c, and t is the time. So, the equation can be rewritten as follows

$$\frac{d\Delta p}{d\varphi} = \frac{e}{\omega} E\left(\frac{\beta c}{\omega}\varphi\right)[\cos(\varphi)\sin(\varphi_0) + \sin(\varphi)\cos(\varphi_0)], \quad (2)$$

where Δp is a particle momentum gain (or loss), $E(z)$ is the electric RF mode field distribution along the cavity axis, $\varphi=\omega t$ is RF phase, and φ_0 is initial RF phase. The integration of Eq. (2) gives $\Delta p=p_0\cos(\theta+\varphi_0)$, where $p_0=(e/\omega)J$, $J=(J_c^2+J_s^2)^{1/2}$.The integrals J_c and J_s are found to be as follows

$$J_c = \int E\left(\frac{\beta c \varphi}{\omega}\right)\cos(\varphi)\, d\varphi,\ J_s = \int E\left(\frac{\beta c \varphi}{\omega}\right)\sin(\varphi)\, d\varphi,$$
$$\cos(\theta) = J_c/J,\ \sin(\theta) = J_s/J. \quad (3)$$

Therefore, J is proportional to E field in the cavity.

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The well-known relativistic expression for the energy gain is $\Delta W = mc^2[((p+\Delta p)/mc)^2+1]^{1/2} - mc^2[(p/mc)^2+1]^{1/2}$. Averaging ΔW over φ_0 and taking into account that $p_0=(e/\omega)J \to 0$, $V_{BBU}=\int(\Delta W/e)d\varphi_0$, $\gamma=[(p/mc)^2+1]^{1/2}$, and $(1+x)^{1/2}\approx 1+x/2$ if $x\to 0$, we get:

$$V_{BBU} = \frac{mc^2}{e}\int_0^{2\pi}\left[\sqrt{\left(\frac{p+p_0\cos(\varphi_0)}{mc}\right)^2+1}-\sqrt{\left(\frac{p}{mc}\right)^2+1}\right]d\varphi_0 \approx \frac{e}{m\gamma}\left(\frac{J}{\omega}\right)^2. \quad (4)$$

Thus, V_{BBU} is proportional to the stored RF mode energy $V_{BBU}\sim J^2\sim U$ for monopole RF modes as well. So, this proportionality is the fundamental feature of BBU energy interchange processes.

Normalized Invariable Threshold Current

Equation (4) shows, there is a normalized invariable threshold current I_{Qn} directly characterizing longitudinal HOMs which independent of beam properties

$$I_{Qn} = I_Q\frac{e}{m\gamma}. \quad (5)$$

We have used the approximation $E\to 0$ since BBU instability starts from very low fields. A number of numerical ASTRA simulations made by authors [5] shows that it is the correct statement for common case with relatively high E fields as well.

Numerical Calculations of I_Q

In practice, the numerical values of I_Q in Eq. (1) can be calculated by a number of particle dynamic codes by tracking particles through the HOM field with low stored energy U and the frequency ω to obtain the energy gain (or loss) $\mathcal{E}(\varphi)$. Its first order approximated dependence on initial field phase (φ) is $\mathcal{E}(\varphi)=\mathcal{E}_0\sin(\varphi+\varphi_0)+V_{BBU}$. To obtain the averaged energy gain (or loss) V_{BBU} it is sufficient to track particles at two different initial field phases $\varphi_0=\varphi_1=0$ and $\varphi_2=\pi$ for monopole modes. The solution of system of equations $\mathcal{E}_1=\mathcal{E}(\varphi_1)$ and $\mathcal{E}_2=\mathcal{E}(\varphi_2)$:

$$V_{BBU} = (\mathcal{E}_1+\mathcal{E}_2)/2. \quad (6)$$

According to superposition principle, this solution is correct in a sum of all modes including the main accelerating RF mode presented in the cavity. The energy gain (or loss) $\mathcal{E}(\varphi)$ in this case is equal to the beam energy adding component at the cavity exit that comes when the BBU analyzed HOM in the calculation to be appeared.

EFFECTIVE QUALITY FACTOR

Let $I_\sim$ be the oscillating amplitude component of a beam current with the HOM monopole mode frequency. By definition in terms of RLC circuits, the excitation power of monopole HOM is written as $P_{OSC}=I_\sim^2R_s/2$, where R_s is a shunt impedance of the monopole RF mode.

The power balance in the system of the monopole HOM with the beam is written as

$$-V_{BBU}I - P_{OSC} = P_{loss} + \Delta U/\Delta t. \quad (7)$$

Substituting Eq. (1), P_{OSC}, and P_{loss} in Eq. (7), we get:

$$-\frac{\omega}{Q_{eff}}U - \frac{\omega}{Q_{eff}}\frac{I_\sim^2R_sQ_{eff}}{2\omega} = \frac{\Delta U}{\Delta t}. \quad (8)$$

The solution of this differential equation is

$$U = U_0\cdot e^{-\frac{\omega t}{Q_{eff}}} + I_\sim^2R_s\cdot\frac{Q_{eff}}{2\omega}\left(1-e^{-\frac{\omega t}{Q_{eff}}}\right), \quad (9)$$

$$Q_{eff}(I) = Q/\left(1-I\cdot Q/I_Q\right). \quad (10)$$

Equation (9) describes the monotron BBU instability process. Without of the beam ($I=0$), Eq. (9) describes usual process of decaying field oscillations in a cavity and for $I\neq 0$ it is the usual excitation process of the cavity by a current I. If the dipole mode is potentially instable ($I_Q>0$) and the beam current is large than the threshold one ($I>I_{th}=I_Q/Q$) then the dipole mode field growth exponentially in time. In the last case the effective quality factor described by Eq. (10) becomes negative one.

So, we have in Eq. (10) the unique expression of the quality factor dependency on the average beam current.

APPLICATIONS

The conventional exercise for monopole BBU instability is "Monotron" whose clear description and review can be found in Ref. [6]. It is a klystron based on a single gap cavity, where a DC current electron beam propagates along the cavity axis and excites RF monopole mode fields due to the monotron BBU instability effect.

The simple case of the "Monotron" numerically analyzed by authors in Ref. [5] is shown in Fig. 1. In the cavity, the main monopole mode with the frequency of 2856 MHz becomes to be instable with $I\cdot Q\geq I_Q$=80 kA, at the beam energy of about 250-400 keV, and at the cavity length of about 70-120 mm. The beam becomes periodically stable or instable as the length is elongated.

Draw attention that higher order monopole modes (HOMs) in this cavity also become periodically instable or stable depending on the cavity length. And there is unique cavity length (~90 mm) at which only the main mode is unstable but all other HOMs with frequencies lower the cut-off frequency (there are 5 HOMs in the cavity) are stable ones, i.e., only main mode can be excited here and for all other HOMs these are excluded.

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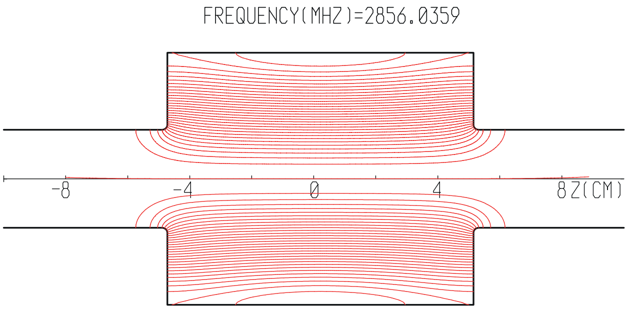

Figure 1: "Monotron" cavity exercise with electric force lines that considered in [5]. The Figure is the upgraded CLANS data output.

In high-power klystron amplifiers, HOMs and main mode monotron oscillations are the parasitic effect considered as absolute instability, which can destroy the RF power generation [7]. So, there is easier for klystrons to analyze its work stability with the help of HOMs effective quality factors. This has to do not only for monopole HOMs but both with dipole HOMs effective quality factors detail analyzed in [8].

EXPERIMENTS

Monopole BBU instability is frequently observed in superconducting cavities during cold tests in liquid helium bath at 2 K. This effect is described for TESLA-type cavities in Ref. [9].

The spontaneous excitation of parasitic modes with resonance frequencies of the main frequency band are frequently observed, especially the 7/9 PI mode. The 7/9 PI mode with a frequency of 1297 MHz is excited and it grows exponentially with a time constant that depends on the quality factor. It has a high-quality factor (Q of order 10^{10}) due to a weak external coupling.

We suppose [10], the grows rate is appeared through the monotron HOMs BBU instability if the product of field emission dark currents in the microampere range on a high (10^{10}) quality factor becomes $IQ>I_Q\approx 10^{-6}\cdot 10^{10}=10^4$. Direct numerical calculations of the BBU effect by tracking field emitted particles in 9 cell TESLA type cavities launched under the main mode field have shown the I_Q values for the first frequency bend HOMs, really, are in the range of 10^4 [10].

CONCLUSION

- Quality factors of all monopole HOMs identically depends on a beam current. We have in mind only average beam current or its DC component.
- This dependency includes a key constant parameter for each monopole HOMs individually found for each considered cavity and named as invariable threshold current I_Q.
- All monopole HOMs in a cavity are divided by two classes one of them are stable HOMs with $I_Q<0$ and another one is potentially unstable HOMs with $I_Q>0$.

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