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GENERAL APPROACH TO PHYSICS LIMITS OF ULTIMATE COLLIDERS*

V. Shiltsev†, Fermilab, Batavia, IL, USA

Abstract

This paper presents an attempt to evaluate limits on energy, luminosity and social affordability of the ultimate future colliders - linear and circular, proton, electron positron and muon, based on traditional as well as on advanced accelerator technologies.

INTRODUCTION

Here we present discussion on the ultimate limits of future colliders. We start with general introduction to the issue: define the scope and units, approaches to the limits of on the energy, luminosity, and social cost of the ultimate colliders. Then, take a more detail look into the limits of the circular pp, ee and $\mu\mu$ colliders, linear and plasma-based ee, $\gamma\gamma$, $\mu\mu$ ones, and briefly discuss exotic schemes, such as the crystal muon colliders. The social cost considerations (power consumption, financial costs, carbon footprint and time to construct) are most defined for the machines based on existing core accelerator technologies (RF and magnets), and less so for the emerging or exotic technologies (ERLs, plasma, crystals, etc).

Each type of the ultimate future colliders will be evaluated on base of feasibility of energy E, feasibility of luminosity L, and feasibility of the cost C. For each machine type (technology) we will start with the current state-of-the-art machines – see Ref. [1] – and attempt to make several (1,2,...) orders of magnitude steps in the energy and see how that affects the luminosity and the cost. This study does not include discussion on where are the lower limits on the luminosity or the upper limits of the cost.

UNITS AND LIMITS ON E, L AND C

Everywhere below we will use TeV for the units for E, understood as the c.m.e. equal to twice the beam energy. The units of L are ab^{-1}/yr that is equal, e.g., 10^{35} cm^{-2}s^{-1} over 10^7 sec/yr. For reference, the HL-LHC will deliver 0.3 ab^{-1}/yr. Due to spread of expectations for the machine availability, there might be a factor of ~2 uncertainty in peak luminosity demands for any ab^{-1}/yr value. The units of total facility electric power consumption are TWh/yr and, e.g., at present CERN with operational LHC takes requires P=200 MW of the average power and 1.1-1.3 TWh/yr. The cost is evaluated in "LHC-Units". 1 LHCU is the cost of the LHC construction (≃10B$). For most of the future machines, the cost is estimated using $\alpha\beta\gamma$ model $C = \alpha\sqrt{Length} + \beta\sqrt{Energy} + \gamma\sqrt{Power}$ that is claimed to end up with good estimate within a $O(2)$ range [2]. The $\alpha\beta\gamma$ model still needs to be extended to the advanced technologies (plasma, lasers, crystals, etc).

Synchrotron radiation sets up the first limit of the energy reach if one demands the SR loss per turn to be less than the total beam energy $\Delta E \leq E/2$. That defines the absolute c.m.e. limit for the circular colliders as :

$$E[\text{TeV}] \leq (m/m_e)^{4/3}(R/10[\text{km}])^{1/3}, \tag{1}$$

that is ~1 TeV for electrons, some 1.2 PeV for muons ($m \approx 210 m_e$) and 25 PeV for protons ($m \approx 2000 m_e$), R is the radius of the machine. Beyond these energies, the colliders will have be linear (thus, needing no dipole magnets). Other energy limits are set by the survival of he particles. Indeed, if, for example, an advanced 5 TeV linear collider consist of M = 1000 5 GeV acceleration stages, then the stage-to-stage transfer efficiency must be better than $\eta = 1 - 1/M$. Also, if the particles are unstable with the lifetime at rest τ_0, then to guarantee delivery to the collision point, the minimum accelerator gradient must significantly exceed $G \gg mc/\tau_0$ – that is, e.g., 0.3 MeV/m for muons and 0.3 TeV/m for tau-leptons [3]. Of course, inevitable might be corollary limits as higher E usually demands higher C, P or facility size. For example, the machine of 100 km circumference with $B \leq 16$ T magnets will have $E \lesssim 100$ TeV; or 40,000 km circumference with 1 T magnets will have $E \lesssim 2.6$ PeV; or a linear accelerators with the total length limit of 50 km and gradient $G \leq 0.1$ GV/m will stay under $E \lesssim 5$ TeV; or under $E \lesssim 10$ PeV if the length is 10 km and $G \leq 1$ TV/m.

Performance (luminosity) reach of the ultimate colliders can be limited by a large number of factors and effects – particle production, beamstrahlung, synchrotron radiation power per meter, IR radiation damage, neutrino-radiation dose, beam instabilities, jitter/emittance growth, etc – which are machine specific and will be considered below. But the most fundamental is the limit on the total beam power $P_b = f_0 n_b N\gamma mc^2$. Indeed, from the standard luminosity formula $L = f_0 n_b N^2/4\pi\sigma^2$ one gets:

$$L = P_b^2/(4\pi\gamma n_b \varepsilon \beta^* m^2 c^4) \propto P_b^2/E, \tag{2}$$

see [1] for standard description of the variables. The luminosity scaling with energy $L \propto 1/E$ in Eq. (2) is markedly different from the usual HEP requirement for the luminosity to follow the cross-section scaling $L \propto E^2$.

Of course, there are societal limits on the machine's total cost, total "carbon footprint" and environmental impact. While the total cost C is dependent on the technology (core accelerator technology, civil construction technology, power production, delivery and distribution technology, etc), the probability of (a technically feasible) facility scales down

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† shiltsev@fnal.gov

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with the cost, possibly as $\propto C^2/(1+C^\kappa)$, with $\kappa \approx 4-5$ as for the real estate sales price distributions. Also, to note: i) the costs of civil construction and power systems are mostly driven by larger economy, ii) having an injector complex available (sometimes up to 1/3 of the total cost) results in potential factor of 2 in the energy reach; iii) the collider cost is usually relatively weak function of luminosity (the latest example is the HL-LHC 1B$ project that will increase luminosity of the 10B$ LHC by a factor of 5); iv) so, one can consider starting future machines with high E and relatively low L in anticipation of eventual performance upgrades (e.g., CESR and Tevatron witnessed l increase $O(100)$, LHC by a factor ≥10, etc); v) C is a moderate function of length/circumference; vi) cost is a strong function of E and technology.

Construction time of large accelerator projects to date is usually between 5 and 11 years and approximately scales as $T \propto \sqrt{C}$. It is often limited by the peak annual spending rate, typically in the range 0.2 to 0.5 B$/yr (compare to the world's global HEP budget 4B$) and on the number of available technical experts. Technical commissioning time ("one particle reaches design energy") is $O(1)$ yr – and it is shorter for known technologies and longer for new ones and for larger number of accelerator elements. Progress towards the design (or ultimate) luminosity is dependent on the machine's "complexity" [4] and for the luminosity risk of 100 (ratio of initial to ultimate L) it can take as long as $T \approx ln(100) \cdot 2$=9 yrs.

ULTIMATE COLLIDERS

Below we attempt to explore ultimate limits of various types of future colliders.

Circular pp Colliders

Tevatron (E=2 TeV, B =4.5T, 6.3 km circumference) and 14 TeV LHC (8 T, 27 km) can be used as reference points while discussing future circular *pp* colliders. Also, there are parameter sets available for SCC (40 TeV, 6.6T, 87km), SppC (75 TeV, 12 T, 100 km), FCC-hh (100 TeV, 16 T, 100km), VLHC (175 TeV, 12T, 233km), and Eloisatron (200 TeV, 10 T, 300 km) [1, 5]. Often cited advantages of such colliders are known technology and beam physics and good power efficiency in terms of ab^{-1}/TWh. Their major limitations include i) large size (related to the magnetic field B technological limit), ii) high total facility power; iii) high cost; iv) beam-beam effects, beam burn-off, and instabilities; v) synchrotron radiation power P_{SR} deposition in the SC magnets environment. Considering the beam-beam limit ξ and the P_{SR} per meter to be the major luminosity limitations, one gets $L \propto (\xi/\beta^*)(P_{SR}/2\pi R)(R^2/\gamma^3))$. Figure 1 presents estimates of performance of circular *pp* colliders vs c.m.energy up to E =(3-5) PeV ("Globaltron", ~1 T, 40,000 km). Power consumption of these colliders exceeds 4 TWh (3 times the LHC one) starting at 100 TeV FCC. Cost optimization of these gargantuan machines usually ends up with the estimates exceeding 2 LHCU above E =70-100 TeV. Of course, under continuous exploration are such cost saving ideas as superferric magnets, permanent magnets, better/cheaper conductors (such as, e.g., iron-based SC cables), graphene, etc. It is highly questionable at present whether they can result in a factor of ~5 saving in the magnet cost per (Tm) [6].

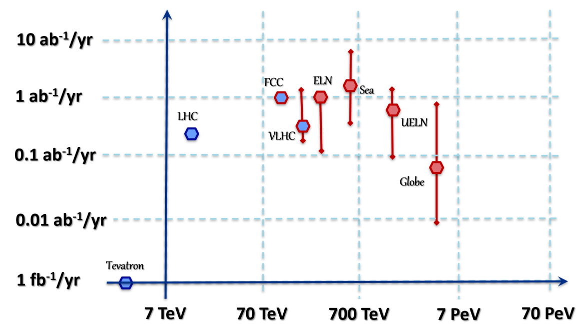

Figure 1: Estimated performance of the circular *pp* colliders vs c.m.energy.

Circular ee Colliders

Due to quickly growing SR power with E, circular *ee* colliders have very limited energy range to expand, even with the use of the ERL technologies [7]. For example, a E ~0.5 TeV machine will be need to be big (~, 200-300 km circumference), low luminosity $O(10\ \mathrm{fb}^{-1}$/yr) and require a lot of expensive RF acceleration, that would drive its cost above 1.5-2 LHCU.

Circular $\mu\mu$ Colliders

There are parameter sets available for 1.5, 3, 6, 10, 14 TeV circular $\mu\mu$ colliders [1]. Their major advantages are thought to be [8]: i) factor of ×7 in equivalent E reach compared to *pp* colliders; ii) arguably the best power efficiency in terms of ab^{-1}/TWh and iii) traditional core technologies. Major limitations include efficient muon production, fast muon cooling and potential neutrino radiation hazard.

For the muon colliders $L \propto B$ and grows with the average particle production rate $dN/dt = f_r N$. At some energy, neutrino radiation dose $D \propto (dN/dt)E^3/\Phi$ sets the limit and the ultimate luminosity depends on suppression "neutrino flux dilution factor Φ, which some believe can be as high as 10-100:

$$L \propto B \frac{D\Phi}{E^2} \frac{N}{4\pi\epsilon_n\beta^*} . \quad (3)$$

That results in a scaling with energy as $L \propto 1/E^k$, where k=1...2 depending on whether the beta-function at the IP can be reduced as $\beta^* \propto 1/E$ – see Fig. 2. Above approximately 14-30 TeV, the power consumption of the muon colliders exceeds 2 TWh/yr and the construction cost estimates goes over 2 LHCU. The LEMMA scheme and the Gamma-Factory concept can in principle offer higher performance (due to smaller emittances and/or higher brightness N/ϵ_n, if not limited by beam-beam interaction [9]) but both require quite expensive additional e^+ or p^+ machines.

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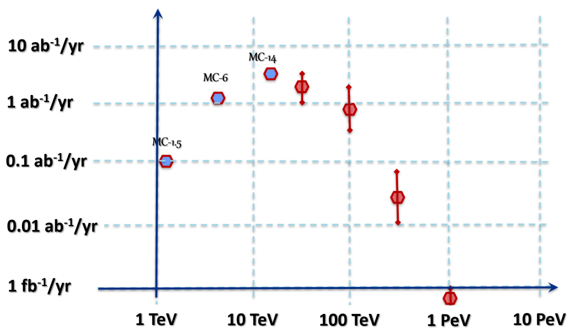

Figure 2: Estimated performance of the circular $\mu\mu$ colliders.

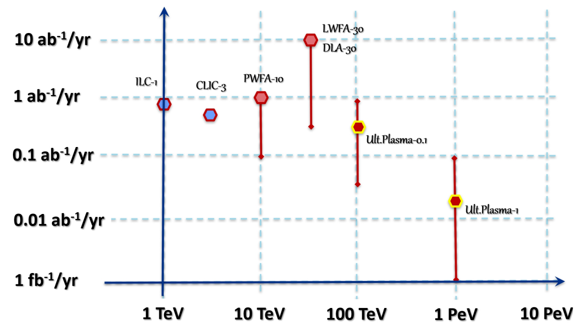

Figure 3: Estimated performance of the linear lepton colliders.

Advanced and Exotic Linear ee or $\mu\mu$ Colliders

In principle, linear colliders (LC) can operate in e^+e^-/e^-e^- and $\gamma\gamma$ regimes (muons are possible, but their sources are expensive and of limited production rate; protons are possible, too, but *pp* collisions lose factor of 7 ineffective c.m. energy reach w.r.t. leptons) and be based on the NC RF, SC RF, plasma, wakefields, etc. Major advantages of such machines are: i) no SR power losses; ii) RF acceleration is a well developed technology. Their major limitations include: i) luminosity scales with total beam power as $L \propto (P/E)(N_\gamma/\sigma_y)$, ii) the last factor (N_γ/σ_y) determines the beamstrahlung energy spread while small beam size - often used to compensate for the loss of luminosity with E - makes jitter tolerances extremely challenging [10]; iii) plasma and wakefield acceleration is not fully matured acceleration technique yet (there are many unknowns such as the energy staging, production and acceleration of e^+, power efficiency of large facilities, cost, etc). Of course, there are some appealing alternatives under study: positron production and acceleration in plasma can be avoided by switching to *ee* operation and conversion into $\gamma\gamma$ at the IP, the beamstrahlung issues can be solved by colliding ultra-short bunches or switching to $\gamma\gamma$ or $\mu\mu$, etc. But in general, there are always some unavoidable challenges and limits, such as instabilities in the RF structures or plasma cells, jitter/emittance control problems that grow with the number of cells and elements, smaller and smaller beam sizes are required at the IP (approaching the limit of 1 A) [11].

Figure 3 presents estimated luminosities of very high energy linear lepton colliders, starting with the 1 TeV ILC (40 km) and 3 TeV CLIC (50 km). The cost of he latter one is already 2.5 LHCU and P is about 3 TWh/yr. Higher energy 10-30 TeV LCs based on beam-plasma, laser-plasma and dielectric plasma wakefield acceleration – see Ref. [12, 13]), not speaking of 100 TeV and 1 PeV options, are extremely power hungry and costly beyond any reasonable limits on P and C.

An interesting opportunity of acceleration of muons in structured solid media, e.g., CNTs or crystals [14], promises extreme gradients 1-10 TV/m, continuous focusing and acceleration (no cells, one long channel, particles get strongly cooled betatron radiation), small facility size (10 km for 10 TeV) - and, therefore, promise of low cost - but very low luminosity 0.001-0.1 ab^{-1}/yr at best. Of course, such exotic technique is still under study [15] and awaits the proof-of-principle E336 experiment at the FACET-II.

CONCLUSION

The above considerations of ultimate high energy colliders for particle physics indicate that their major thrust is attainment of the highest possible energy E, while accelerator design challenge is high luminosity L and the major limit is the cost C. The cost is critically dependent on core acceleration technology. Employment of already existing injectors and infrastructure can greatly help to reduce C. For most collider types we found the the pursue of high energy typically results in low(er) luminosity. So, e.g., one should not expect more than 0.1-1 ab^{-1}/yr at $E \geq 30$ TeV to 1 PeV. In the luminosity calculations, one might also assume the total facility (and, therefore, the beam) annual power consumption should better be limited to 1-3 TWh/yr.

For the considered collider types we found that : i) for circular *pp* colliders the overall $E - L - C$ feasibility limit is close or below 100 TeV (~14 TeV cme for constituents); ii) for circular *ee* colliders the limit is at ~0.5 TeV; iii) for circular $\mu\mu$ colliders the limit is about 30 TeV; iv) for linear RF-based lepton colliders as well as for plasma $ee/\gamma\gamma$ colliders the limit is between 3 and 10 TeV; v) there are exotic schemes, such as crystal channeling muon colliders, which have promise of 0.1-1 PeV c.m.e. thought at low Luminosity. All in all, muons seems to be the particles of choice the future ultimate HEP colliders [16].

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