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STUDY OF HARMONIC CRAB CAVITY IN EIC BEAM-BEAM SIMULATIONS*

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Abstract

In the Electron-Ion Collider (EIC) design, crab cavities are adopted to compensate the geometric luminosity loss from the crossing angle. From previous studies, higher-order synchro-betatron resonances are excited since the hadron beam is long and the crossing angle is large. To reduce the luminosity degradation rate, different combinations of harmonic crab cavities are studied with both weak-strong and strong-strong simulation methods. The frequency map analysis (FMA) is also used for comparison. This study helps determine the crab cavity parameters for the future EIC.

INTRODUCTION

Although a large crossing angle allows a fast separation to avoid the parasitic collision, there is a geometric luminosity loss. Low order synchro-betatron resonances are also excited. Crab crossing is a good idea to overcome these disadvantages [1]. The crab crossing scheme can be obtained with a rotation in the $x - z$ plane.

Figure 1 shows the crab crossing scheme in EIC. The electron beam and ion beam get a horizontal kick from the upstream crab cavity. With ideally linear crabbing, two beams collide head-on with each other at IP. The geometric luminosity loss is fully compensated. After the collision, an identical crab cavity is used to restore the distribution.

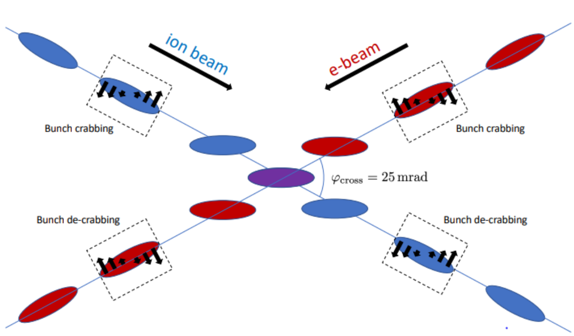

Figure 1: The schematic of EIC crab crossing scheme.

However, the real kick from a crab cavity is a sinusoidal wave,

$$\begin{aligned}\Delta p_x &= -\frac{\theta_c}{k_c\Lambda}\sin(k_c z)\\ \Delta p_z &= -\frac{x\theta_c}{k_c\Lambda}\sin(k_c z)\end{aligned}, \tag{1}$$

where k_c is the wave number of the crab cavity, θ_c half crossing angle, and Λ is determined by the lattice. With the nonlinear kick, each colliding beam will have a transverse offset as a function of the location away from its reference particle z

$$f(z) = -\theta_c\left[\frac{\sin(k_c z)}{k_c} - z\right] \approx \frac{1}{6}k_c^2 z^3 , \tag{2}$$

when $k_c\sigma_z \sim 1$, x couples with z for the particle with a large longitudinal coordinate. Therefore, high-order synchro-betatron resonances will be excited [2].

For EIC, the bunch length of the ion beam is longer than the electron beam. The radiation for ions is negligible. The synchro-betatron resonances in the ion beam are more severe. Therefore, we will focus our study on the ion beam.

The harmonic crab cavity can be used to mitigate the synchro-betatron resonances. This paper will present the strong-strong and weak-strong simulation results with and without the harmonic crab cavity. The frequency map analysis method is also used to show whether the harmonic crab cavity works. The simulation parameters are taken from the EIC Conceptual Design Report (CDR). The detailed parameter table can be found in [3] or [4]. The simulation studies use the strong-strong code BeamBeam3D [5] and a self-written weak-strong code.

DRIVING TERM

When particles are bunched in a small area, the beam-beam potential for an upright bi-Gaussian distribution in crab crossing collision can be truncated

$$\begin{aligned}U\left(x+f, y; \sigma_x, \sigma_y\right) &= -\frac{Nr_0}{\gamma_0}\int_0^\infty \mathrm{d}u\frac{\exp\left[-\frac{(x+f)^2}{2\sigma_x^2+u} - \frac{y^2}{2\sigma_y^2+u}\right]}{\sqrt{2\sigma_x^2+u}\sqrt{2\sigma_y^2+u}}\\ &= \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}(z)\, x^m y^n\end{aligned}, \tag{3}$$

where N is total particle number of the opposite beam, $r_0 = e^2/\left(4\pi\epsilon_0 mc^2\right)$ the classical radius, γ_0 the relativistic factor of the test particle, and $\sigma_{x,y}$ are the RMS bunch sizes of the opposite beam. Notice that $\sigma_{x,y}$ are functions of z. The crabbed offset $f(z)$ is abbreviated as f without confusion.

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After Floquet transformation, the Hamiltonian can be expressed in terms of action angle variables

$$H = \nu_x J_x + \nu_y J_y + h_{00}\left(J_x, J_y; z\right) + \sum_{m,n,l} h_{mn}\left(J_x, J_y; z\right)\cos\left(m\psi_x + n\psi_y + l\theta\right) \quad . \tag{4}$$

The longitudinal coordinate z is simplified as a harmonic oscillator. This can be done based on the truth that the longitudinal dynamics is less affected by the beam-beam distortion.

Further expanding h_{mn} in Eq. (4) about z in Fourier series, the resonance condition turns into

$$m\nu_x + n\nu_y + p\nu_z + l = 0 \quad . \tag{5}$$

The resonances density then increases in the tune space.

The unperturbed working point of the proton beam in EIC CDR is $(0.228, 0.210)$. Two kinds of synchro-betatron resonances can be excited around this working point,

$$\begin{array}{c} 4\nu_x + p\nu_z = 1 \\ 2\nu_x - 2\nu_y + p\nu_z = 0 \end{array} \quad , \tag{6}$$

which are excited by $h_{4,0}$ and $h_{2,-2}$ in Eq. (4) respectively.

When the second-order harmonic crab cavity is used, the crabbed offset turns into

$$f(z) = -\theta_c\left[\frac{1+\alpha}{k_c}\sin(k_c z) - \frac{\alpha}{2k_c}\sin(2k_c z) - z\right], \tag{7}$$

where α is relative strength of the second-order harmonic crab cavity. From [2], $\alpha = 1/3$ is the best combination to mitigate the synchro-betatron resonances.

The driving term strength with or without harmonic crab cavity is present in Fig. 2. Both $h_{4,0}$ and $h_{2,-2}$ curves around IP change slowly. The synchro-betatron resonances with small p in Eq. (6) are dominant for particles with small longitudinal offset. These resonances are not excited due to the choice of the working point. The rapidly changing parts corresponding to large p are pushed to the bunch head or tail when the harmonic crab cavity is used. Therefore, less particles are affected by the resonances in Eq. (6). In one word, the harmonic crab cavity should work from the resonances theory.

SIMULATION RESULTS

Figure 3 shows the mitigation effect of the harmonic crab cavity by strong-strong simulation. In the simulation, $\alpha = 1/3$ when the harmonic crab cavity is used. It seems that there is no benefit with the harmonic crab cavity.

However, the weak-strong simulation gives different result. Figure 4 presents the same simulation by the weak-strong code. The beam size growth rate is largely mitigated when the harmonic crab cavity is used.

To find out whether the harmonic crab cavity really works, FMA is used to explore the beam-beam dynamics. Figure 5 shows the frequency maps tracked by strong-strong simulation. Compared with $\alpha = 0$, the frequency map shrinks

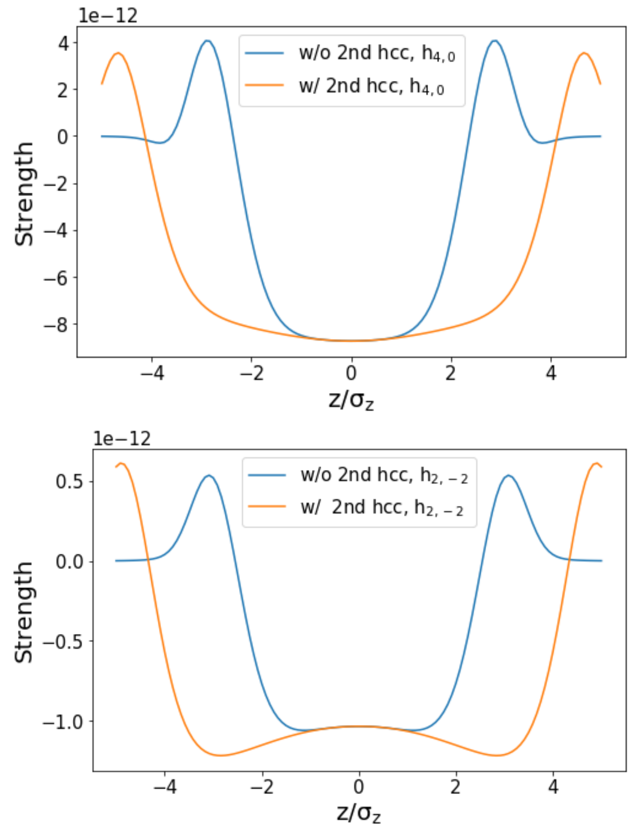

Figure 2: The driving term $h_{4,0}$ and $h_{2,-2}$ with/without harmonic crab cavity.

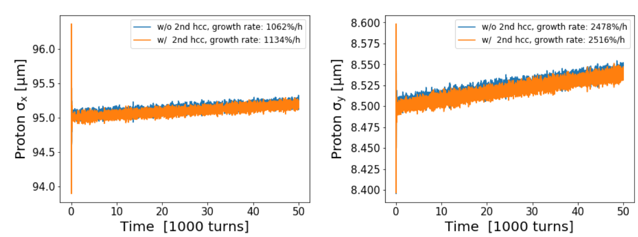

Figure 3: Proton beam size evolution with and without harmonic crab cavity by strong-strong simulation.

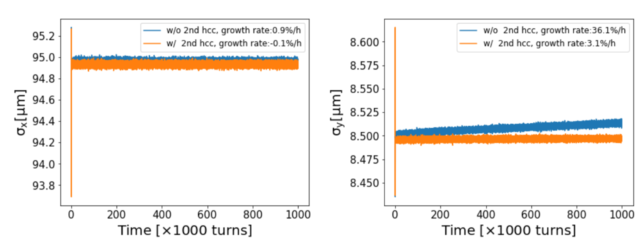

Figure 4: Proton beam size evolution with and without harmonic crab cavity by weak-strong simulation.

when $\alpha = 1/3$. There are no particle labeled by red color in the last subfigure. The frequency maps prove that the harmonic crab cavity works in the strong-strong simulation. But the benefit may be buried by the numerical noise.

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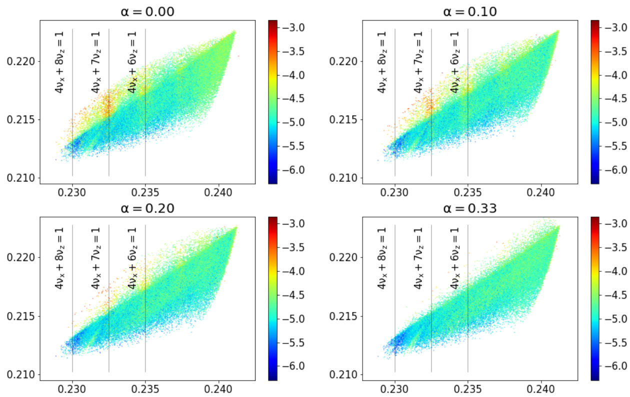

Figure 5: Frequency map tracked by strong-strong simulation. α is the relative strength of the second-order harmonic crab cavity.

Figure 6 presents the frequency maps tracked by weak-strong simulation. There is less numberical noise in the weak-strong model, and more resonance structure appears in the frequency map. Similar to the strong-strong frequency map, the footprint shrinks when $\alpha = 1/3$. In the first subfigure, a strong resonance line $2\nu_x - 2\nu_y - 3\nu_z = 0$ is excited, and all particles close to it are labeled by the red color. However, when the harmonic crab cavity is set to the optimum value, the red block is eliminated from the footprint.

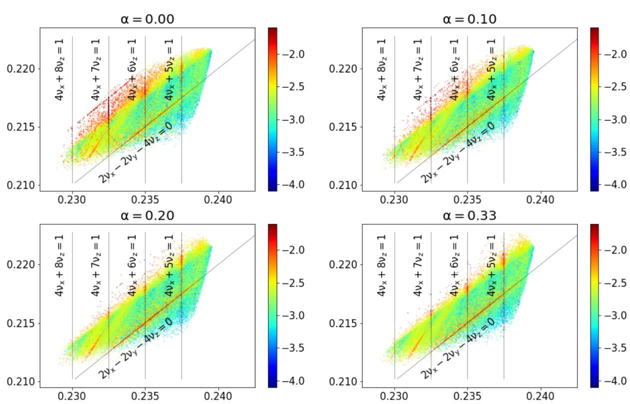

Figure 6: Frequency map tracked by weak-strong simulation. α is the relative strength of the second-order harmonic crab cavity.

SUMMARY

In this paper, we present the strong-strong and weak-strong simulation results with the second-order harmonic crab cavity. The harmonic crab cavity helps to reduce the footprint in the tune space. Less resonance are excited. The proton beam size growth is mitigated in weak-strong simulation. However, the growth rate is not reduced in strong-strong simulation. The benefit may be buried by the numerical noise. More noise study can be found in [3].

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