

ELECTRON-ION LUMINOSITY MAXIMIZATION IN THE EIC*

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Abstract

The electron-ion luminosity in the EIC has a number of limits, including the ion intensity available from the injectors, the total ion beam current, the electron bunch intensity, the total electron current, the synchrotron radiation power, the beam-beam effect, the achievable beta functions at the Interaction Point, the maximum angular spreads at the Interaction Point, the ion emittances reachable with Strong Hadron Cooling, the ratio of horizontal to vertical emittance, and space charge effects. We map the e-A luminosity over the center-of-mass energy range for some ions ranging from deuterons to uranium ions. For e-Au collisions the present design provides for electron-nucleon (e-Au) peak luminosities of up to $4.8 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ with Strong Hadron Cooling.

INTRODUCTION

The Electron-Ion Collider EIC design has estimated luminosities for electron-proton (e-p) collisions with Strong Hadron Cooling (SHC) [1, 2]. For electron-ion operation in the EIC [3] we map the e-A luminosity over the operational energy range for a number of ions, ranging from deuterons to uranium ions (Table 1).

Table 1: Ion beam parameters for the EIC. The proton parameters for the maximized e-p luminosity with $E_A = 275 \text{ GeV}$ are listed for reference [1].

Quantity	Unit	p [†]	d [†]	h [†]	Cu	Au	U
charge number Z	...	1	1	2	29	79	92
mass number A	...	1	2	3	63	197	238
min energy $E_{A,\text{min}}$	GeV/n	41	50	50	50	50	50
max energy $E_{A,\text{max}}$	GeV/n	275	138	183	127	110	106
max bunch intensity N_A							
for $k_b = 290$	10^{10}	19.1	10	10	0.54	0.20	0.045
for $k_b = 1160$	10^{10}	6.9	2.5	2.5	0.14	0.05	0.011
spin polarization	%	70	70	70	-	-	-

Ions can also be cooled with Strong Hadron Cooling, and stochastic cooling is available as a backup for medium and heavy ions [4–6]. Here we only consider the case of strongly cooled ion beams. The cooling strength and intrabeam scattering (IBS) will determine the equilibrium emittances for a given bunch intensity N_A . With cooling the average store luminosity is close to the peak luminosity as it is already the case in RHIC [7]. The RHIC ion bunch intensity changes through burn-off, which is not a significant effect in the EIC.

In the EIC, the number of colliding bunches can be $k_b = 290, 1160$ bunches, and the horizontal and vertical

beam size of both beams are always matched at the Interaction Point. The full crossing angle is fixed at $\theta = 25 \text{ mrad}$, and the effective crossing angle is close to zero due to a crab crossing scheme. The luminosity constraints can be grouped into three groups: (i) geometric, (ii) intensity and (iii) crossing angle and longitudinal constraints (Table 2).

Table 2: Parameter Constraints for the EIC Luminosity Maximization

No	Quantity	Unit	Limit for p	Limit for A	Derived from
geometric constraints					
1	IP β -function $\beta_{e,x,\text{min}}^*$	m	> 0.35		DA
2	IP β -function $\beta_{e,y,\text{min}}^*$	m	> 0.05		DA
3	IP β -function $\beta_{A,x,\text{min}}^*$	m	> 0.90		DA
4	IP β -function $\beta_{A,y,\text{min}}^*$	m	> 0.04		DA
5	rms emittance $\epsilon_{e,x}$				
	$E_e = 5 \text{ GeV}$	nm	= 20		lattice
	$E_e = 10 \text{ GeV}$	nm	= 20		lattice
	$E_e = 18 \text{ GeV}$	nm	= 24		lattice
6	norm. rms emitt. $\epsilon_{e,n,y,\text{min}}$	μm	> 20		source, accel.
7	norm. rms emitt. $\epsilon_{A,n,x,y,\text{min}}$			Ref. [1]	IBS+cooling
8	ratio $(\epsilon_{A,y}/\epsilon_{A,x})_{\text{min}}$...	> 0.05		coupling
9	IP rms spread $\sigma'_{e,x,\text{max}}$	μrad	< 220		synch. rad.
10	IP rms spread $\sigma'_{e,y,\text{max}}$	μrad	< 220		synch. rad.
11	IP rms spread $\sigma'_{A,x,\text{max}}$				
	$\text{HA}^\ddagger, E_{p,A} = 41 \text{ GeV/n}$	μrad	< 220	< 45*	Roman pots
	$\text{HA}^\ddagger, E_{p,A} = 100 \text{ GeV/n}$	μrad	< 180	< 45*	Roman pots
	$\text{HA}^\ddagger, E_p = 275 \text{ GeV}$	μrad	< 65	NA*	Roman pots
	$\text{HD}^\ddagger, E_{p,A} = 41 \text{ GeV/n}$	μrad	< 220	275*	Roman pots
	$\text{HD}^\ddagger, E_{p,A} = 100 \text{ GeV/n}$	μrad	< 220	220*	Roman pots
	$\text{HD}^\ddagger, E_p = 275 \text{ GeV}$	μrad	< 150	NA*	Roman pots
12	IP rms spread $\sigma'_{A,y,\text{max}}$				
	HA^\ddagger	μrad	< 380	< 45*	Roman pots
	HD^\ddagger	μrad	< 380	< 380*	Roman pots
intensity constraints					
13	bunch intensity $N_{e,\text{max}}$	10^{10}	< 17.2		source, instab.
14	bunch intensity $N_{A,\text{max}}$		Table 1		source, accel.
15	total beam current $I_{e,\text{max}}$	A	< 2.5		B-factories
16	total beam current $I_{A,\text{max}}$	A	< 1.0		e-cloud, cryo
17	synch. rad. power $P_{e,\text{max}}$	MW	< 10		admin. limit
18	beam-beam $\xi_{e,x,y,\text{max}}$...	< 0.1		e^+e^- colliders
19	beam-beam $\xi_{A,x,y,\text{max}}$...	< 0.015		RHIC
20	space charge $\Delta Q_{sc,A,x,y,\text{max}}$...	< 0.05		experience
21	IBS emitt. growth times, all 3 dimensions, minimum				
	without cooling	h	> 8	NA	operation
	with cooling	h	> 2	NA	operation
crossing angle and longitudinal constraints					
22	full crossing angle θ	mrad	= 25		bb, synch. rad.
23	RF voltages $V_{\text{rf,max}}$				
	24.6 MHz ($h = 315$)	MV	< 0.6		hardware
	197 MHz ($h = 8 \times 315$)	MV	< 6		hardware
	591 MHz ($h = 24 \times 315$)	MV	< 20		hardware
24	L reduction factor H_{min}	...	> 0.8	NA	beam-beam

*HA = High Acceptance, [‡]HD = High Divergence, *under review, changes likely

FORMULAS

For matched horizontal and vertical rms beam sizes at the IP ($\sigma_{e,x,y} = \sigma_{A,x,y} = \sigma_{x,y}$) the electron-nucleon luminosity is

$$L_{eN} = A \frac{N_e N_A f_{\text{rev}} k_b}{4\pi \sigma_x \sigma_y} H(\sigma_{e,A,s}, \beta_{e,A,x,y}^*, \theta, f_{\text{crab}}), \quad (1)$$

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where A is the ion mass number, $N_{e,A}$ are the electron and ion bunch intensities, f_{rev} is the revolution frequency, and $H \leq 1$ a geometric factor that accounts for the crossing angle θ , the crabbing scheme with cavities of frequency f_{crab} and the hourglass effect. $\beta_{x,y}^*$ are the lattice functions at the IP and σ_s the rms bunch length. The beam sizes $\sigma_{x,y}$ and angular spreads $\sigma'_{x,y}$ at the IP are

$$\sigma_{x,y} = \sqrt{\epsilon_{x,y} \beta_{x,y}^*}, \quad (2)$$

$$\sigma'_{x,y} = \sqrt{\frac{\epsilon_{x,y}}{\beta_{x,y}^*}} = \frac{\epsilon_{x,y}}{\sigma_{x,y}} = \frac{\sigma_{x,y}}{\beta_{x,y}^*}, \quad (3)$$

where $\epsilon_{e,A,x}$ are the unnormalized rms emittances. Without crabbing and with crossing in the horizontal plane the reduction factor H is given by [8]

$$H(\theta, \beta_{e,A,x,y}^*, \sigma_{e,A,s}) = 2\sqrt{\frac{2}{\pi}} \cos \frac{\theta}{2} \frac{\sigma_x^2 \sigma_y^2}{\sqrt{\sigma_{e,s}^2 + \sigma_{A,s}^2}} \times \int_{-\infty}^{+\infty} ds \frac{\exp\left\{s^2 \left(-\frac{1+\cos\theta}{\sigma_{e,s}^2 + \sigma_{A,s}^2} + \frac{\cos\theta-1}{S_{e,x}^2 + S_{A,x}^2}\right)\right\}}{\sqrt{S_{e,x} + S_{A,x}} \sqrt{S_{e,y} + S_{A,y}}} \quad (4)$$

with

$$S_{e,A,x,y} = \frac{s^2 \cos^2\left(\frac{\theta}{2}\right) + \beta_{e,A,x,y}^{*2}}{\beta_{e,A,x,y}^{*2}} \sigma_{e,A,x,y}^2. \quad (5)$$

With perfect crabbing $\theta = 0$, but crab cavities with imperfect non-linearity compensation will also reduce H [1].

There are limits for $N_{e,A}$ from the injectors, and limits on the total beam currents

$$I_{e,A} = f_{rev} k_b Z e N_{e,A} \quad (6)$$

($Z = 1$ for I_e). The beam-beam parameters are

$$\xi_{e,A,x,y} = \frac{r_{e,p} Z}{2\pi A} \frac{N_{A,e} \beta_{e,A,x,y}^*}{\gamma_{e,A} \sigma_{x,y} (\sigma_x + \sigma_y)} \quad (7)$$

($A = 1$ for ξ_e), where $r_{e,p}$ is the classical electron, proton radius, $\gamma_{e,A}$ are the Lorentz factors. There is an administrative limit on the synchrotron radiation power

$$P_e = f_{rev} k_b N_e \frac{C_\gamma}{2\pi} E_e^4 \mathcal{I}_2, \quad (8)$$

with $C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3}$, where r_e is the classical electron radius, c the speed of light, m_e , E_e the electron mass and energy respectively and $\mathcal{I}_2 = \oint \left(\frac{1}{\rho_x^2} + \frac{1}{\rho_y^2}\right) ds$ the second radiation integral. $\rho_{x,y}$ are the s -dependent bending radii. The space-charge parameters for ions are

$$\Delta Q_{sc,A,x,y} = -\frac{Z^2 r_p}{A} \frac{r_p}{4\pi} \frac{N_A}{\beta_A \gamma_A^2 \epsilon_{A,n,x,y}} \frac{C}{\sqrt{2\pi} \sigma_s}, \quad (9)$$

where β_A is the ion speed divided by c , and $\epsilon_{A,n,x,y} = (\beta_A \gamma_A) \epsilon_{A,x,y}$ the normalized rms emittance.

LUMINOSITY MAXIMIZATION

With Eq. (1) luminosity maximization amounts to minimizing $\sigma_{x,y}$, and maximizing $N_{e,A}$ and H . With the constraints in Table 2 this can be maximized in three steps:

1. Geometric constraints determine (σ_x, σ_y) .
2. Intensity constraints determine $(\beta_{x,y}^*, \epsilon_{x,y})$ and $N_{e,A}$.
3. Crossing angle and longitudinal constraints determine k_b and H .

Step 1. Using Eq. (2) the minimum σ_x for a species is found. This configuration may, however, yield an angular spread σ'_x that is too large, and an adjustment is needed:

$$\sigma_x = \begin{cases} \max\left\{\frac{\epsilon_{x,\min}}{\sigma'_{x,\max}}, \beta_{x,\min} \sigma'_{x,\max}\right\}, & \text{if } \sqrt{\frac{\epsilon_{x,\min}}{\beta_{x,\min}^*}} > \sigma'_{x,\max} \\ \sqrt{\epsilon_{x,\min} \beta_{x,\min}^*}, & \text{otherwise.} \end{cases} \quad (10)$$

The minimum horizontal beam size is then given by $\sigma_x = \max\{\sigma_{e,x}, \sigma_{A,x}\}$, and the same is done for σ_y .

Step 2. For given (σ_x, σ_y) it is favorable to minimize β^* and maximize ϵ_x for the intensity dependent constraints (Eqs. (7) and (9)). We can therefore determine ϵ_x, β_x^* in the following way

$$\epsilon_x = \begin{cases} \sigma_x \sigma'_{x,\max}, & \text{if } \frac{\sigma_x}{\beta_{x,\min}^*} > \sigma'_{x,\max} \\ \frac{\sigma_x^2}{\beta_{x,\min}^*}, & \text{otherwise,} \end{cases} \quad (11)$$

for both e, A . For the ion vertical plane we also need to enforce constraint no. 8 in Table 2

$$\text{If } \epsilon_{A,y} < \epsilon_{A,x} \left(\frac{\epsilon_{A,y}}{\epsilon_{A,x}}\right)_{\min} \text{ then } \epsilon_{A,y} = \epsilon_{A,x} \left(\frac{\epsilon_{A,y}}{\epsilon_{A,x}}\right)_{\min}. \quad (12)$$

The beta functions are then

$$\beta_{x,y}^* = \sigma_{x,y}^2 / \epsilon_{x,y}, \quad (13)$$

also for both e, A . After this is done the maximum $N_{e,A}$ are determined from Eqs. (6), (7), (8) and (9) and Table 2 as

$$N_{e1,A1} = N_{e,A,\max} \text{ [Table 2]}, \quad (14)$$

$$N_{e2,A2} = N_{e2,A2}(I_{e,A,\max}) \text{ [Eq. (6)]}, \quad (15)$$

$$N_{e3,A3} = N_{e3,A3}(\xi_{e,A,x}) \text{ [Eq. (7)]}, \quad (16)$$

$$N_{e4,A4} = N_{e4,A4}(\xi_{e,A,y}) \text{ [Eq. (7)]}, \quad (17)$$

$$N_{e5} = N_{e5}(P_{e,\max}) \text{ [Eq. (8)]}, \quad (18)$$

$$N_{A5,A6} = N_{A5,A6}(\Delta Q_{sc,A,x,y,\max}) \text{ [Eq. (9)]}, \quad (19)$$

and the maximum bunch intensities $N_{e,A}$ are

$$N_e = \min\{N_{e1}, \dots, N_{e5}\}, \quad N_A = \min\{N_{A1}, \dots, N_{A6}\}. \quad (20)$$

Step 3. The above maximization is calculated for the number of bunches under consideration $k_b = 290, 1160$ and the highest luminosity L_{eN} is retained. For different k_b the bunch intensity N_A , minimum emittances $\epsilon_{A,n,x,y}$, rms bunch length $\sigma_{s,A}$, and luminosity reduction factor H change.

The emittance maximization algorithm, with the boundary conditions listed in Tables 1 and 2 has been implemented in Mathematica [9].

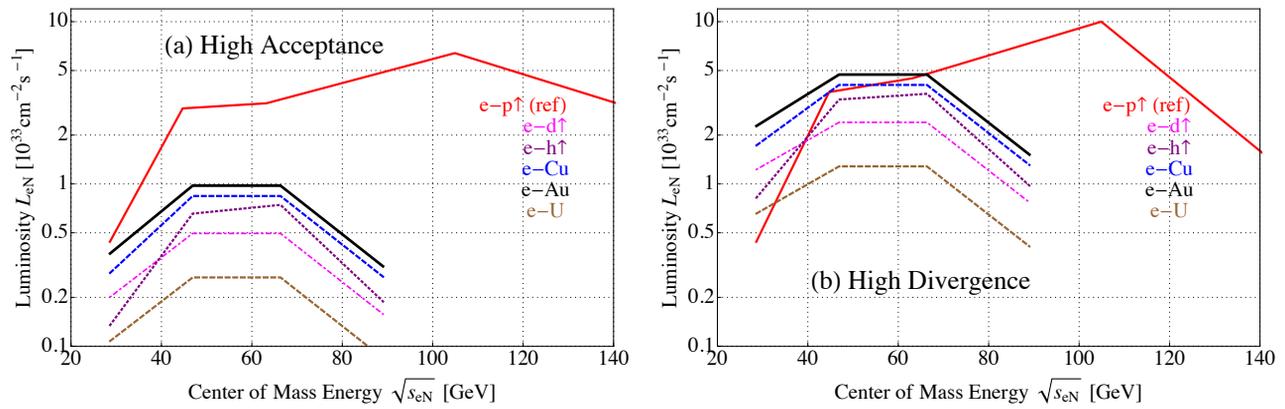


Figure 1: EIC e-A luminosity L_{eN} for $N = p, d, h, \text{Cu}, \text{Au}$ and U beams, all with Strong Hadron Cooling. Part (a) shows the luminosities for the High Acceptance limits imposed on the angular beam spreads $\sigma'_{A,x,y,\text{max}}$ at the Interaction Point (Table 2), and part (b) for the High Divergence limits. With Strong Hadron Cooling the peak and average store luminosity are close. The e-p reference case data are taken from Ref. [1].

RESULTS

In electron-ion collisions it is critical to detect all possible particles, and especially those with small transverse momentum p_t . The dominant operating mode will therefore be the High Acceptance mode, and the limits for the angular rms spread at the IP $\sigma_{A,x,y,\text{max}}$ will constrain the maximum luminosity. There likely will also be cases where the High Divergence mode is required, with which more luminosity can be delivered.

With Roman pots at $s = 25$ m from the IP, a distance of the Roman pots of $10 \sigma_{x,y}$ from the beam center, and the ability of detecting particles with p_t as low as 50 MeV, the HA limits shown in Table 2 result. The $\sigma'_{A,x,y,\text{max}}$ limits, and correspondingly the luminosity, may be increased with a different Interaction Region (IR) optics with a secondary focus.

Figure 1 shows the maximized peak luminosity over the full EIC energy range for the ion species listed in Table 1 together with the e-p case for reference [1]. With Strong Hadron Cooling the average store luminosity will be close to the peak luminosity. Part (a) shows the luminosities for the range of ion species for the main e-A mode with HA, and part (b) for the HD mode with higher luminosities. In the HA mode the luminosity is constrained by 8 limits simultaneously, and in the HD mode by 9 limits (Table 2).

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