ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB264

SHIELDING OF CSR WAKE IN A DRIFT*

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Abstract

In this work, we derive formulae for the 1D CSR wake in the drift past the exit from a bend magnet. We assume that the radiation is shielded by two parallel metal plates with the orbit located in the middle plane.

INTRODUCTION

Coherent synchrotron radiation (CSR) wakefield was calculated in Refs. [1, 2] in a 1D model and is used in several computer codes for simulation of relativistic electron beams. It includes transient effects at the entrance and exit from a bending magnet of finite length. In the ultra-relativistic limit, $v = c$, the exit CSR wake decays inversely proportional to the distance from the magnet end. To calculate the total energy loss of the beam one needs to integrate this wake to infinity, but the integral diverges. The physics behind this divergence is the edge radiation at the exit from the magnet that in the limit $\gamma = \infty$ and in the absence of metal walls carries an infinite energy at small angles. Naturally, the integral of the CSR wake that is responsible for the energy balance in this process, takes an infinite value. This means that one has to either drop the assumption $\gamma = \infty$ or take into account the shielding effect of the metal walls in the system in order to get a finite answer. To understand which of these two effects is more important we need to compare the formation length, ℓ_f, of the edge radiation in free space for a bunch with a finite value of γ,

$$\ell_f \sim \gamma^2 \sigma_z, \tag{1}$$

where σ_z if the rms bunch length, with the formation length (sometimes called the "catchup" distance) in a vacuum chamber with transverse dimensions a,

$$\ell_f \sim \frac{a^2}{\sigma_z}. \tag{2}$$

In what follows, we will assume that the length given by Eq. (2) is much shorter than that in Eq. (1); in this case the effect of the shielding dominates and we can keep the assumption $\gamma = \infty$ in our calculations. We will also adopt a model where the vacuum chamber is treated as two parallel metal plates separated by distance a, with the beam orbit situated in the middle plane between the plates, see Fig. 1.

Even taking the shielding effects into account in the drift, we can ignore it inside the magnet if the formation length, $\ell_f \sim (24\rho^2\sigma_z)^{1/3}$, (here ρ is the bending radius), is much smaller than that given by Eq. (2). Dropping numerical factors, we re-write this condition as $a \gg \rho^{1/3}\sigma_z^{2/3}$, which is a well known requirement for the shielding to be ignored in calculations of the CSR wake on a circular orbit [3]. If this condition is satisfied, as we assume in this paper, one can use the free-space equations from [2] for the CSR wakefield inside the magnet and complement them by the equations from the next section for the drift past the magnet.

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.
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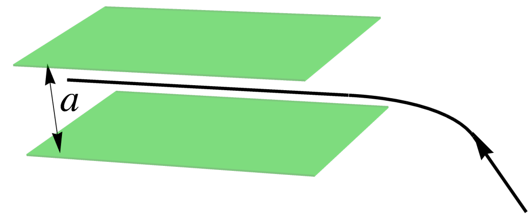

Figure 1: Trajectory of a bunch of particles passing through a bending magnet (the direction of motion is shown by the arrow). In the straight part of the orbit after the exit there are two parallel metal plates (shown by green color).

CSR WAKE IN DRIFT WITHOUT SHIELDING

If we neglect the shielding (which is formally valid in the limit $a \to \infty$), the wake (per unit path length) generated by a bunch in a drift after the exit from the magnet is given by [2],

$$W(z,s) = \frac{4}{\rho}\int_{-\infty}^{z} \frac{1}{\psi(z',z,s) + 2s/\rho}\frac{d\lambda(z')}{dz'}dz'. \tag{3}$$

In this formula, s is the position of the bunch in the drift measured from the exit of the magnet, $\lambda(z)$ is the one-dimensional bunch distribution normalized by unity, and the function $\psi(z',z,s)$ is defined by the equation

$$z - z' = \frac{\rho\psi^3}{24}\frac{\psi + 4s/\rho}{\psi + s/\rho}. \tag{4}$$

Equation (3) is valid for long magnets, when the magnet length L is much longer than the formation length $(24\rho^2\sigma_z)^{1/3}$, which we assume here. If this condition is not satisfied, there are some extra terms in W which are due to the transient effects at the entrance to the magnet.

In the limit $s \to \infty$, the integral in Eq. (3) can be simplified,

$$W(z,s) \approx \frac{2}{s}\lambda(z). \tag{5}$$

As was pointed out in the Introduction, this wake decays as $1/s$, and if integrated to $s = \infty$ gives an infinite energy loss of the beam. In the next section, we will see how this unphysical result is modified when the shielding is taken into account.

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CSR IMPEDANCE IN DRIFT WITH SHIELDING

The CSR wake for a bend of finite length with the shielding represented by two parallel metal plates was studied in Ref. [4]. A general expression was derived in the ultra-relativistic limit, $v = c$, for the longitudinal impedance, $Z(k)$, related to the wake of a point charge, $w(z)$, by the equation

$$Z(k) = \frac{1}{c}\int_{-\infty}^{\infty} w(z)e^{-ikz}dz, \tag{6}$$

with the wake $w(z)$ defined so that the energy change ΔE of a particle at coordinate z in the bunch is

$$\Delta E(z) = -eQ\int_{-\infty}^{\infty} \lambda(z')w(z-z')dz', \tag{7}$$

where Q is the charge of the bunch and $z = s - ct$. For a bunch of the rms length of σ_z the characteristic value of the wavenumber is $k \sim 1/\sigma_z$.

The CSR wake in the drift after a bend magnet was calculated in Appendix B.4 of Ref. [4] and was denoted by Z_4 (in this paper, we drop the subscript "4", and use the notation Z for the impedance generated by the drift only). A small bending angle was assumed, which means that $L \ll \rho$, where L is the bend length. It was also assumed that $ka \gg 1$ and $k\rho \gg 1$ (which means that the bunch length σ_z is much smaller than the gap between the plates, as well as the bending radius). The expression for Z is given by Eq. (B15) in Ref. [4],

$$\begin{aligned} Z(k) \approx{}& \frac{Z_0}{4\pi}(i-1)\frac{\sqrt{\pi k}}{a\rho^2}\exp\left(\frac{1}{3}ik\theta_0^2 L\right)\sum_{p=0}^{\infty}\int_0^L (L-s')^2 ds' \\ &\times \int_L^{\infty}\frac{ds}{\sqrt{\zeta}}\exp\left[i\left(\frac{ks'^3}{6\rho^2} - \frac{1}{2}k\theta_0^2 s + \frac{k\theta_0^2}{2\zeta}\left(s - \frac{1}{2}L - \frac{s'^2}{2L}\right)^2\right.\right. \\ &\left.\left. - \zeta\frac{(2p+1)^2\pi^2}{2ka^2}\right)\right], \end{aligned} \tag{8}$$

where $Z_0 = 377$ Ohm, $\theta_0 = L/\rho$ and $\zeta = s - s'$, with the entrance to the bend corresponding to $s' = s = 0$[1]. The integration over s in the second integral extends from the exit from the bend ($s = L$) to infinity, assuming an infinitely long drift. The lower zero limit in the integral over s' corresponds to the entrance to the bend; if the length of the bend much longer then the formation length, $L \gg (24\rho^2\sigma_z)^{1/3}$, which we assume here, then this limit can be replaced by $-\infty$. This means that the transient effects at the entrance to the bend do not interfere with the wake after the exit (as was also assumed for Eq. (3)).

Introducing the new integration variables

$$\tau = (L-s')^3\frac{k}{\rho^2}, \qquad \xi = \frac{s-s'}{ka^2}, \tag{9}$$

[1] Note that here the coordinate s is measured from the entrance to the magnet, while in Eq. (3) it is measured from the exit.

after some transformations Eq. (8) can be written in the following form

$$\begin{aligned} Z(k) \approx{}& (i-1)\frac{\sqrt{\pi}}{3}\frac{Z_0}{4\pi}\sum_{p=0}^{\infty}\int_0^{\infty} d\tau \exp\left(-\frac{1}{6}i\tau\right) \\ &\times F\left(\frac{1}{8}\frac{\tau^{4/3}}{\varkappa^{4/3}}, \frac{(2p+1)^2\pi^2}{2}, \frac{\tau^{1/3}}{\varkappa^{4/3}}\right), \end{aligned} \tag{10}$$

where $\varkappa = ka^{3/2}/\rho^{1/2}$ is the normalized wavenumber and the function F is defined by the integral

$$F(a,b,c) = \int_c^{\infty}\frac{d\xi}{\sqrt{\xi}}\exp\left(i\frac{a}{\xi} - ib\xi\right). \tag{11}$$

Function F can be expressed in terms of the error function of a complex argument, see [4]. It has the following scaling property,

$$F(a,b,c) = \sqrt{t}F\left(\frac{a}{t}, bt, \frac{c}{t}\right), \tag{12}$$

so that one of the arguments can be effectively converted into a scaling factor (e.g., choosing $t = a$).

Mathematical analysis of Eq. (10) gives the following formula for the impedance in the limit of large wave numbers, $\varkappa \gg 1$, (or, equivalently, large frequencies $\omega = ck$),

$$Z(k) = \frac{Z_0}{4\pi}\left[1.33\ln(0.97\varkappa) - 1.05i\right]. \tag{13}$$

We see that the real part of the impedance grows as logarithm of the wavenumber while the imaginary part tends to a finite value at infinity.

We emphasize here that the impedance, Eq. (10), corresponds to the wake integrated in the drift from the exit from the magnet to infinity. If the drift has a finite length l, then the upper limit in the integral over s in Eq. (8) is equal to $L + l$. In this case the corresponding impedance is given by the following formula,

$$\begin{aligned} Z(k,l) \approx{}& (i-1)\frac{\sqrt{\pi}}{3}\frac{Z_0}{4\pi}\sum_{p=0}^{\infty}\int_0^{\infty} d\tau \exp\left(-\frac{1}{6}i\tau\right) \\ &\times\left[F\left(\frac{1}{8}\frac{\tau^{4/3}}{\varkappa^{4/3}}, \frac{(2p+1)^2\pi^2}{2}, \frac{\tau^{1/3}}{\varkappa^{4/3}}\right)\right. \\ &\left. - F\left(\frac{1}{8}\frac{\tau^{4/3}}{\varkappa^{4/3}}, \frac{(2p+1)^2\pi^2}{2}, \frac{\tau^{1/3}}{\varkappa^{4/3}} + \frac{l}{\varkappa\rho^{1/2}a^{1/2}}\right)\right]. \end{aligned} \tag{14}$$

NUMERICAL CALCULATIONS

We numerically computed the impedance, Eq. (10), for an infinitely long drift, see Fig. 2. Note that the real part of Z is negative in the region $\varkappa < 4$ which seems to contradict to the requirement that the real part of any impedance be always positive. One has to remember, however, that here we only calculate the contribution to the impedance from a part of the beam trajectory—the impedance for the full trajectory that also includes the circular part of the orbit inside the bend will have a positive real part for all frequencies.

In Fig. 3 we show plots of the impedance for a finite length l of the drift calculated with Eq. (14) for two values

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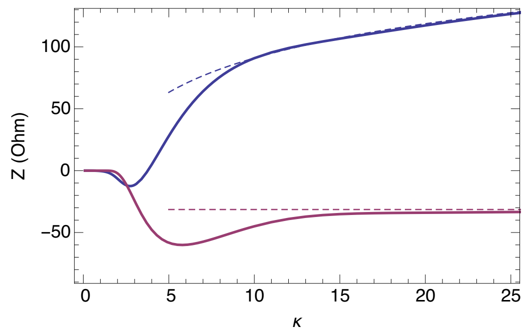

Figure 2: Plot of the real (blue) and imaginary (magenta) parts of the impedance Z (in Ohms) given by Eq. (10) as a function of the dimensionless wavenumber $\varkappa$. The dashed lines show the high-frequency approximation Eq. (13).

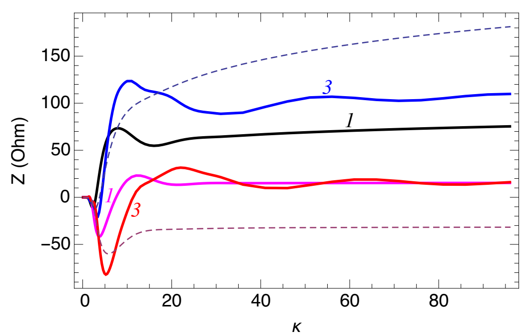

Figure 3: Plot of the impedance for $l/\sqrt{a\rho} = 1$ ($\Re Z$ - black, $\Im Z$ - magenta) and for $l/\sqrt{a\rho} = 3$ ($\Re Z$ - blue, $\Im Z$ - red) calculated with Eq. (14). The dashed lines show the real and imaginary parts of Z from Fig. 1 corresponding to $\ell = \infty$.

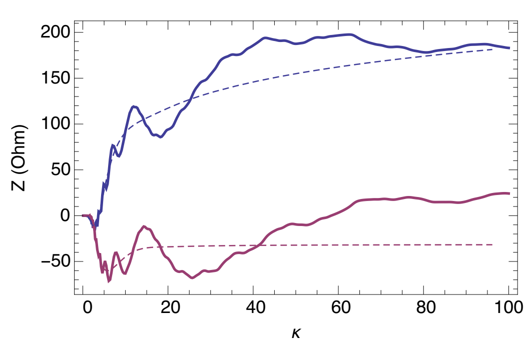

Figure 4: Plot of the real (blue) and imaginary (magenta) parts of the impedance Z for $\ell = 20$. The dashed lines show the real and imaginary parts of Z from Fig. 1 corresponding to $\ell = \infty$.

of the parameter $\ell = l/\sqrt{a\rho}$ (the values of this parameter are indicated by the numbers near the curves). Figure 4 shows the impedance for a long drift, $l/\sqrt{a\rho} = 20$. Note that a non-smooth character of the solid lines in this figure is not an artifact of calculation errors—it reflects the fact that the impedance curves approach the limit $\ell = \infty$ only on average, executing rapid oscillations around the limiting values of the impedance. A similar behavior was observed in the past for the impedance of collimators and tapers in a perfectly conducting pipe [5].

Using Eqs. (6) and (7) one can calculate the average energy loss per particle of the beam due to the CSR impedance in the drift. The energy loss is given by the following equation,

$$\langle \Delta E \rangle = \frac{Qe}{\pi} c \int_0^\infty dk\, \Re Z(k) |\hat{\lambda}(k)|^2, \qquad (15)$$

where $\hat{\lambda}(k)$ is the Fourier transform of the distribution function $\hat{\lambda}(k) = \int_{-\infty}^{\infty} dz \lambda(z) e^{ikz}$. For a Gaussian function with the rms bunch length σ_z we have $\hat{\lambda}(k) = e^{-k^2\sigma_z^2/2}$.

For a numerical example we take the same parameters of the beam and the bending magnet as in Ref. [2]: $Q = 1$ nC, $\rho = 1.5$ m, $\sigma_z = 50\,\mu$m, and assume the gap between the plates $a = 2$ cm. For an infinitely long drift we find $\langle \Delta E \rangle = 0.18$ MeV. Calculating the wake of the bunch, we can also find the rms energy spread induced by the wake in the beam, which turns out to be 0.10 MeV.

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