

# INVESTIGATION OF DAMPING EFFECTS OF THE CRAB CAVITY NOISE INDUCED EMITTANCE GROWTH\*

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## Abstract

Crab cavities will be installed at the two main interaction points (IP1 and IP5) of the High Luminosity LHC (HL-LHC) in order to restore the reduction of the luminosity caused by the crossing angle. Two prototype crab cavities have been installed in the SPS machine and were tested with a proton beam in 2018, to study the expected emittance growth induced by noise in their RF control system. The measured emittance growth was found to be a factor 2-3 lower than predicted from the available analytical and computational models. Damping effects from the transverse impedance, which are not included in the current theories, are studied as a possible explanation for the observed discrepancy.

## INTRODUCTION

The High Luminosity LHC (HL-LHC) project is the designed upgrade of the LHC machine. It aims to increase the instantaneous and integrated luminosity by a factor of five and ten, respectively, beyond the present LHC operational values for proton beams. Crab cavities (CCs) are a key component of the upgrade, as they will be employed to restore the geometric overlap of bunches at the large crossing angles required with the small  $\beta^*$  values anticipated [1]. In particular, the CCs will rotate the bunches in the crossing plane, restoring the head-on collisions in the two main experiments, ATLAS and CMS.

Noise in the Low-Level Radio Frequency (LLRF) system of the CCs is expected to induce transverse emittance growth resulting in a reduced luminosity lifetime. A theoretical relationship for the transverse emittance growth caused by amplitude and phase noise in a CC has been derived for a single bunch in a hadron machine and validated through HEADTAIL simulations [2]. For a uniform noise spectrum across the betatron tune distribution, the emittance growth resulting from amplitude noise can be estimated from:

$$\frac{d\epsilon}{dt} = \beta_{CC} \left( \frac{eV_{CC}f_{rev}}{2E_b} \right)^2 C_{\Delta A}(\sigma_\phi) \sum_{k=-\infty}^{+\infty} S_{\Delta A}[(k \pm \bar{\nu}_b \pm \bar{\nu}_s)f_{rev}]. \quad (1)$$

For phase noise, the emittance growth can be estimated from:

$$\frac{d\epsilon}{dt} = \beta_{CC} \left( \frac{eV_{CC}f_{rev}}{2E_b} \right)^2 C_{\Delta\phi}(\sigma_\phi) \sum_{k=-\infty}^{+\infty} S_{\Delta\phi}[(k \pm \bar{\nu}_b)f_{rev}]. \quad (2)$$

In these formulae,  $\beta_{CC}$  is the beta function at the location of the CC,  $V_{CC}$  the CC voltage,  $f_{rev}$  the revolution frequency of the beam,  $E_b$  the beam energy, and  $\bar{\nu}_b$  and  $\bar{\nu}_s$  the mean of the betatron and synchrotron tune distribution.  $S_{\Delta A}$  and  $S_{\Delta\phi}$  are the power spectral densities (PSD) [3] of the noise at all the betatron and synchrotron (for the amplitude noise case) sidebands.  $C_{\Delta A}$  and  $C_{\Delta\phi}$  are correction terms to account for the bunch length:

$$C_{\Delta A}(\sigma_\phi) = e^{-\sigma_\phi^2} \sum_{l=0}^{+\infty} I_{2l+1}(\sigma_\phi^2), \quad (3)$$

$$C_{\Delta\phi}(\sigma_\phi) = e^{-\sigma_\phi^2} \left[ I_0(\sigma_\phi^2) + 2 \sum_{l=1}^{+\infty} I_{2l}(\sigma_\phi^2) \right], \quad (4)$$

with  $\sigma_\phi$  the rms bunch length (in radians) with respect to the CC frequency  $f_{CC}$ , and  $I_n(x)$  the modified Bessel function of the first kind.

To benchmark the theoretical model with experimental data, a series of machine development (MD) studies with CCs was carried out in the SPS in 2018.

## EXPERIMENTAL RESULTS

Measurements in the SPS were performed with four bunches at 270 GeV/c with low intensity ( $3 \cdot 10^{10}$  ppb). Only one CC was used, providing a vertical kick to the beam. The linear chromaticity,  $Q'$ , of the machine was corrected to small positive values ( $\sim 1-2$ ) in both transverse planes to minimise emittance growth from other sources [4]. The Landau octupoles were switched off; however, a residual non-linearity was present in the machine mainly due to multipole components in the dipole magnets [5, 6]. Some of the relevant SPS parameters during the experiment are listed in Table 1.

### Injected RF Noise

To characterize the CC noise induced emittance growth, noise was injected into the CC LLRF system and the bunch evolution was recorded for about 20-40 minutes. The RF noise was a mixture of phase and amplitude noise covering a frequency range up to 10 kHz, primarily exciting the first betatron sideband ( $\sim 8$  kHz). The phase noise was always

\* Work supported by the HL-LHC project

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Table 1: SPS Parameters During the 2018 MD Studies

Parameters	Values
$E_b$	270 GeV
$f_{\text{rev}}$	43.375 kHz
$\nu_x, \nu_y$	26.13, 26.18
$\nu_s$	0.0051
$V_{\text{RF}}, f_{\text{RF}}$	5 MV, 200 MHz
$\beta_{x,\text{CC}}, \beta_{y,\text{CC}}$	30.31 m, 73.82 m
$V_{\text{CC}}, f_{\text{CC}}$	1 MV, 400 MHz

dominant. In order to make a comparison between different levels of noise, the concept of effective phase noise is introduced: this is the phase noise level that would lead to the same emittance growth as that from both amplitude and phase noise. The noise levels mentioned in the following correspond to the calculated effective phase noise.

Emittance growth measurements were performed with six different noise levels, listed in Table 2. Noise levels were measured with a spectrum analyser E5052B [7]: this provides a single sideband measurement (SSB), expressed as  $10 \log_{10} \mathcal{L}(f)$  [dBc/Hz]. The PSDs in Eq. (1) and Eq. (2) are given by  $S_{\Delta} = 2\mathcal{L}(f)$  [8], with  $S_{\Delta A}$  in 1/Hz and  $S_{\Delta\phi}$  in  $\text{rad}^2/\text{Hz}$ .

### Summary of the Measurements

Due to the vertical CC kick, the emittance growth is expected in the vertical plane. However, the CC noise was also observed to induce growth in the horizontal emittance as a result of residual coupling in the machine. Thus, the total emittance growth given by  $d\epsilon_y/dt + d\epsilon_x/dt$  should be considered here, as was confirmed by PyHEADTAIL simulations [9] in the presence of transverse coupling.

Figure 1 shows the measured emittance growth rates for each of the four bunches with each of the noise levels applied in the MD studies. The measured emittance growth was observed to be different for the different bunches. During the analysis it was found that bunches 2, 3 and 4 were

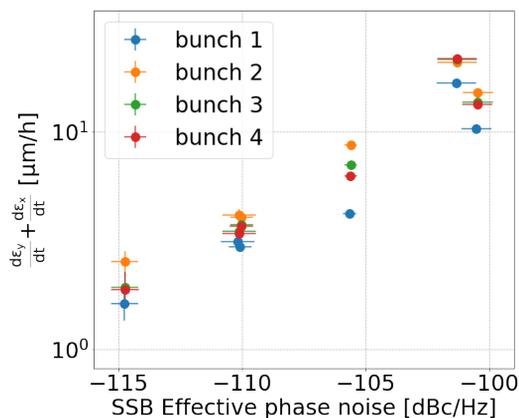


Figure 1: Measured normalised emittance growth rates for different CC LLRF noise levels.

longitudinally unstable. This is believed to be due to the fact that the phase loop was sampling only the first bunch because of the large bunch spacing of 525 ns [10]. For this reason, the following analysis is focused only on bunch 1, which was not affected by the instability.

### COMPARISON WITH THE THEORY

Figure 2 compares the measured and the theoretically calculated emittance growth rates of bunch 1 for the different noise levels. In making the comparison, the natural emittance growth ( $d\epsilon_x/dt = 0.55 \text{ m/h}$  and  $d\epsilon_y/dt = 0.45 \text{ m/h}$ ) from other sources (observed without CC) is subtracted from the measured values. A bunch length increase of about 10% per hour was observed: this is the usual rate in the SPS for similar machine conditions.

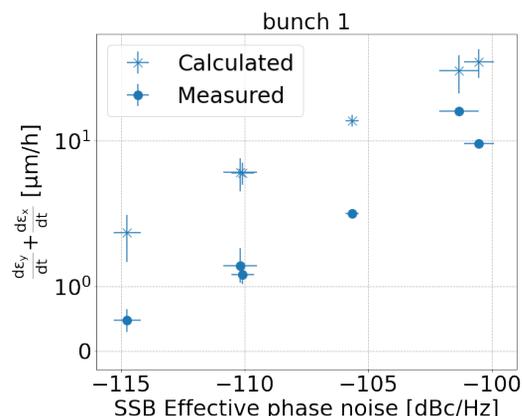


Figure 2: Measured and theoretically calculated emittance growth rates for bunch 1. On average the theory overestimates the measurements by a factor of around 4.

The expected emittance growth due to the CC noise was estimated for all noise settings using Eq. (2). For each setting, the machine characteristics from Table 1, the measured noise PSDs and the average bunch length over each observation window were used in the calculation. It is evident that the theory systematically overestimates the measured growth rates. The average discrepancy over all noise levels is a factor of around 4: numerical values are given in Table 2.

Table 2: Comparison Between the Measured and the Calculated Emittance Growth Rates for Bunch 1 for the Different Noise Levels, and Average Bunch Length for Each Case

$10 \log_{10} \mathcal{L}(f)$ [dBc/Hz]	Growth rate [ $\mu\text{m/h}$ ]		$\langle \sigma_{\phi} \rangle$ [rad]
	Measured	Calculated	
-114.8	0.47	1.83	1.05
-110.2	1.32	5.33	1.03
-110.1	1.18	5.29	1.10
-105.7	2.36	14.88	1.06
-101.3	18.08	40.04	1.08
-100.5	9.37	48.01	1.09

## EFFECT OF IMPEDANCE

Among many studies performed to explain the discrepancy, it was recently found that the beam coupling impedance can change the decoherence properties in the conditions of the experiment. Simulations with PyHEADTAIL including the most updated SPS transverse impedance model [11] revealed a clear suppression of emittance growth induced by the CC phase noise.

### PyHEADTAIL Simulations

The lattice parameters shown in Table 1 were used to simulate the emittance growth in the SPS in the conditions of the 2018 MD studies. The linear chromaticity in both horizontal and vertical planes was set to  $Q'_{x,y} = 1$ . The initial bunch was generated with Gaussian distributions in transverse and longitudinal planes with transverse normalised emittances of  $\epsilon_{x,y} = 2.3 \mu\text{m}$  and a bunch length of  $\sigma_\phi = 1.07 \text{ rad}$ . The bunch charge of  $3 \cdot 10^{10}$  particles was represented by  $5 \cdot 10^5$  macroparticles in the simulation.

For computational simplicity, the effect of the crab cavity phase noise was modeled as a cosine kick on each individual particle according to the expression derived in [2]. It was verified in PyHEADTAIL simulations that the result from this simplified implementation is equivalent to the simulation with the true CC kick including phase noise.

In the simulations the beam was tracked for  $10^5$  turns, which corresponds to about 2.5 s in the SPS. For a noise level of  $-100.8 \text{ dBc/Hz}$  chosen here (for illustration), a growth rate of about 26 nm/s is expected (exciting sidebands at  $\pm 7.8 \text{ kHz}$ ). The emittance growth rate was computed by performing a linear fit to the emittance values obtained from the tracking. Multiple simulation runs were performed to reduce the uncertainty of the results. The sequence of noise kicks was regenerated randomly for 20 runs. For each of those 20 runs the initial bunch distribution was also regenerated randomly 3 times. The mean and the standard deviation (including the uncertainty on the slope of the fit) were computed over all the trials. As the machine non-linearities were not explicitly characterised during the experiment, the dependence on the detuning coefficient in the vertical plane,  $\alpha_{yy}$ , was studied. The horizontal detuning coefficient and the cross-term were left at zero, i.e.  $\alpha_{xx} = \alpha_{xy} = 0$ .

### Emittance Growth Suppression

Simulations were performed with and without the SPS impedance model to study its impact on the emittance growth induced by CC noise. Figure 3 shows the dependence of the growth rates on the amplitude detuning coefficient,  $\alpha_{yy}$ . The secondary horizontal axis shows the resulting rms tune spread. It can be seen that when the wakefield kicks are not applied on the beam the emittance growth rate agrees very well with the value predicted by Eq. (2) and (within the reproducibility of the simulation) is independent of the tune spread value. It should be noted that the theoretical model is not valid for zero tune spread, and that the observed

emittance growth rate for  $\alpha_{yy} = 0$  is a result of the geometric distortion of the beam caused by the CC kick.

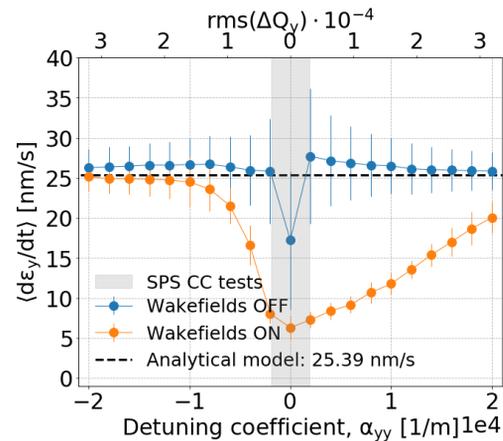


Figure 3: Transverse emittance growth without (blue) and with (orange) the impedance effects.

Figure 3 also shows a clear suppression of the transverse emittance growth when the wakefield kicks are included. The suppression depends on the tune spread and is asymmetric for positive and negative values of the detuning coefficient. Over a realistic range of tune spread values (estimated with MAD-X [12] including the non-linearities of SPS [5,6] and  $Q'_{x,y} = 1$ , and shown by the grey shaded area in Fig. 3) the suppression reaches up to a factor 4-5. This suppression is very close to that observed in the experiments and suggests that the impedance effects might explain the discrepancy between the measured and theoretically estimated emittance growth rates.

## CONCLUSIONS AND FUTURE PLANS

In the SPS CC tests of 2018, a discrepancy of a factor of around 4 on average was observed between the measured CC RF noise induced emittance growth rates and the predictions from the available theoretical models. Simulations performed with PyHEADTAIL including the accurate SPS impedance model show a strong suppression of the transverse emittance growth, similar to the experimental observations. Thus, effects from the beam coupling impedance is an important factor to be considered for such studies.

The mechanism behind the emittance growth suppression and the reason for the asymmetric dependence on the amplitude detuning are presently under study and will be presented in a later publication. It is suspected that mitigation of the decoherence arises from the existence of a discrete coherent betatron mode outside of the incoherent spectrum [13]. The development of a theoretical formalism to describe the emittance suppression mechanism is in progress. Future plans also include applying the understanding developed from these studies to the HL-LHC, in order to characterize the long term emittance evolution following the LHC upgrade, and to define limits on the acceptable noise levels for the CCs.

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