ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB250

AXICON-BASED CONCENTRATOR FOR CHERENKOV RADIATION*

S. N. Galyamin†, A. V. Tyukhtin, St. Petersburg State University, St. Petersburg, Russia

Abstract

We propose and discuss a new type of axisymmetric dielectric target – an "axicon-based concentrator" – which effectively concentrates generated Cherenkov radiation (CR) into a small vicinity of a focus point. It consists of two "glued" bodies of revolution: a hollow axicon and a hollow "lens." A theoretical investigation of the radiation field produced by a charge moving through the discussed radiator is performed for the general case where a charge trajectory is shifted with respect to the structure axis. The idea of a dielectric target with a specific profile of the outer surface and suitable analytical methods were presented and developed in our preceding papers. An essential advantage of the current version of the device is that it allows the efficient concentration of CR energy from relativistic particles, making this device extremely prospective for various applications such as beam-driven THz sources and bunch diagnostic systems.

INTRODUCTION

Non-invasive bunch diagnostics based on electromagnetic (EM) radiation emerging during an interaction between charged particle bunches and prolonged dielectric targets of complicated shape is a relatively new modern area of application for Cherenkov radiation (CR) [1]. This scheme possess several advantages compared to traditional ones based on transition or diffraction radiation and requires to calculate CR produced by dielectric target with several boundaries and edges which is marginally possible to do rigorously. To resolve this issue with reliable accuracy we have been developing for several recent years an original combined approach based on certain "etalon" problem, ray-optics laws and Stratton-Chu formulas (see, for example, [2–5]). It is worth noting that this approach has been also approved by direct comparison between its results and results of numerical simulations in COMSOL Multiphysics [6].

In papers [2, 3, 6] we dealt with the axisymmetric dielectric target – "dielectric concentrator for CR" – focusing the majority of generated CR in a small vicinity of a predetermined point (focus) without any additional lenses or mirrors. In the bulk of this target, a CR ray undergoes single refraction before exit to a free space. While possibilities of this "single-refraction concentrator" are rather attractive, an essential disadvantage is that it has more or less convenient dimensions for relatively slow charged particles only. This shortcoming has been eliminated recently with the new type of concentrator, an "axicon-based concentrator" [4], allowing the concentration of CR produced by a relativistic charged particles. In the bulk of this target, a CR ray undergoes single reflection at the cone generatrix and single refraction. In this report, we further discuss this target.

* Work supported by Russian Science Foundation, Grant No. 18-72-10137
† s.galyamin@spbu.ru

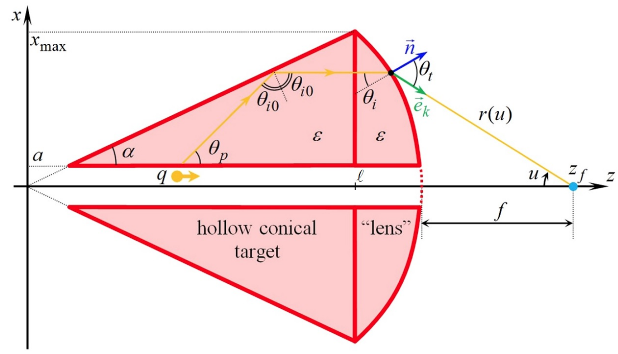

Figure 1: Geometry of the "axicon-based concentrator" and main notations.

ON-AXIS FOCUS CASE

Here we briefly mention main results concerning an "axicon-based concentrator". The geometry is depicted in Fig. 1: a hollow "lens" is attached to the output surface of a hollow conical target (axicon) having its apex facing the incident charged particle bunch q. Both revolution bodies are made from the same dielectric material with permittivity ε. The outer profile of the "lens" is hyperbolic and is determined by the function $r(u)$:

$$r(u) = f(\sqrt{\varepsilon} - 1)\left[\sqrt{\varepsilon}\cos(u) - 1\right]^{-1}, \qquad (1)$$

where f is a "focal" parameter. Cylindrical coordinates of the out surface of the target are

$$\rho_0(u) = r(u)\sin(u), \quad z_0(u) = z_f - r(u)\cos(u),$$

The target is designed so that a beam of CR rays parallel to z-axis (one of them is shown in Fig. 1) converges exactly to the focus $\rho = 0$, $z = z_f$, where $z_f = x_{\max}(\cot\alpha + \cot u_{\max})$ (here $x_{\max}$ is the maximum transverse size of the target, $x_{\max}$ together with channel radius a are predefined and in turn determine $u_{\max}$ and $u_{\min}$). In other words, the focus of the lens is on-axis. The main advantage of this target is a possibility to adjust cone angle α to obtain a parallel beam of CR rays for arbitrary bunch velocity $\beta = V/c = (1-\gamma^{-2})^{-1}$, including ultrarelativistic one $\beta \to 1$. In general case, CR ray is inclined to z-axis on angle $\theta_i' = 2\alpha - \theta_p$, where $\cos\theta_p = (\sqrt{\varepsilon}\beta)^{-1}$, therefore the condition for parallel ray is the following: $\alpha = \theta_p/2$. Moreover, as it was discussed in [4], a charge shift from the z-axis (symmetry axis) disturbs symmetric EM field distributions very weakly for relativistic bunches. Therefore, since relativistic bunches are of most practial importance, it is sufficient to consider the symmetric case $r_0 = 0$.

Typical EM field distributions over zx-plane for $\alpha = \theta_p/2$, $r_0 = 0$ and $\beta = 0.99$ are shown in Fig. 2. One can see that

12th Int. Particle Acc. Conf. IPAC2021, Campinas, SP, Brazil JACoW Publishing
ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB250

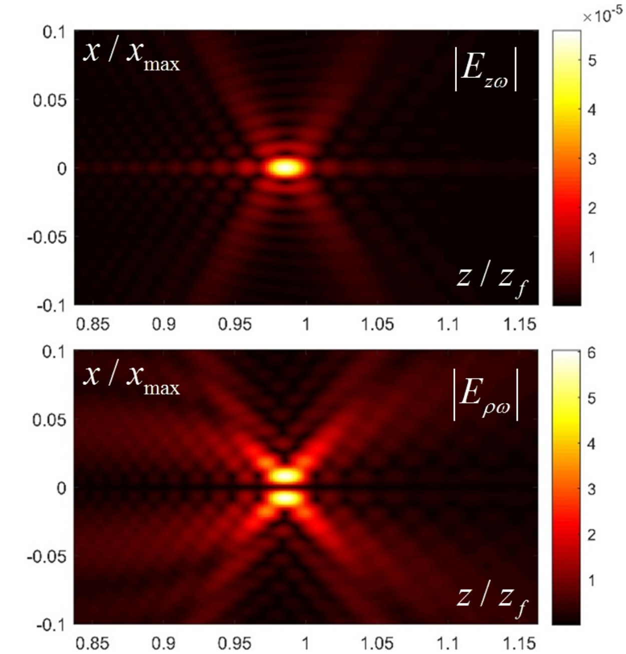

Figure 2: Electric field per unit frequency (in units Vms^{-1}) over zx-plane for symmetrical case $r_0 = 0$ and $\beta = 0.99$, $\varepsilon = 4$, $x_{\max} = f = 23.9$ cm, $a = c/\omega = 0.047$ cm, $\alpha = 30°$, $\omega = 2\pi \cdot 100$ GHz, $q = 1$ nC (point charge).

transverse field $E_{\rho\omega}$ is negligible near z-axis while longitudinal field $E_{z\omega}$ is dominant there. Maximum values of transverse and longitudinal fields are practically equal which is more favorable compared to the "single-refraction" concentrator [3].

The discussed target can be of interest for various applications of relativistic charged particle bunches from modern accelerators. For example, a high-power wide-band longitudinally polarized CR can be obtained with this device, both in visible and terahertz regions.

OFF-AXIS FOCUS CASE

It is of interest to consider a modification of the "lens" with the focus located off the z-axis, Fig. 3. Let us denote θ_i' which corresponds to the "designed" velocity β_0 as u_0, i.e. $u_0 = 2\alpha - \mathrm{acos}(\sqrt{\varepsilon}\beta_0)^{-1}$. The "lens" is still axisymmetric, but in each cross-section the focus is located off-axis: for example, for the cross-section $y = 0$ it is at the point $z = z_f$, $x = \pm h_f$. Resulting focus is the ring $z = z_f$, $\rho = h_f$.

Following calculations can be performed using the scheme from [4]. First, cylindrical coordinates of the out surface of the target are

$$\rho_0(u) = h_f + r(u)\sin(u - u_0),$$
$$z_0(u) = z_f - r(u)\cos(u - u_0),$$

where $r(u)$ is given by Eq. (1). The elementary square of the surface is $d\Sigma = \sqrt{g(u)}\,du\,d\varphi$, where

$$\sqrt{g(u)} = r(u)\frac{h_f + r(u)\sin(u - u_0)}{\sqrt{\varepsilon}\cos u - 1}\sqrt{1 - 2\sqrt{\varepsilon}\cos u + \varepsilon}.$$

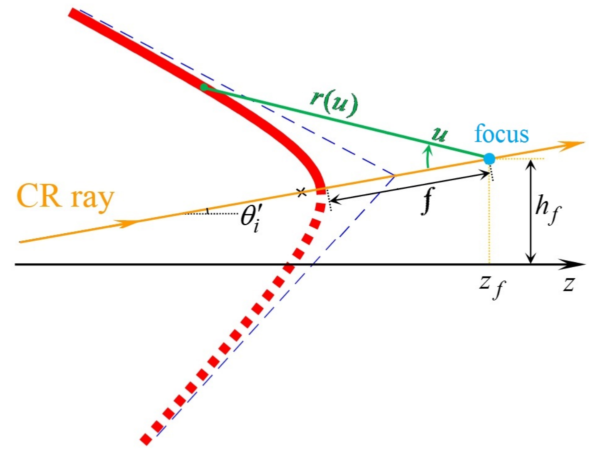

Figure 3: Geometry of the "lens" with shifted focus. Indicated CR ray originates from the most distant point of the axicon with coordinates $x = a$, $z = a\cot\alpha$.

Components of the external unit normal $\vec{n}$

$$n_\rho(u) = \frac{\sqrt{\varepsilon}\sin u_0 + \sin(u - u_0)}{\sqrt{1 + \sqrt{\varepsilon}\cos u + \varepsilon}},$$
$$n_z(u) = \frac{\sqrt{\varepsilon}\cos u_0 - \cos(u - u_0)}{\sqrt{1 + \sqrt{\varepsilon}\cos u + \varepsilon}}$$

define the unit vectors of the refracted CR ray:

$$e_{k\rho}(u) = n_\rho(u)\cos\theta_t(u) - n_z(u)\sin\theta_t(u),$$
$$e_{kz}(u) = n_\rho(u)\sin\theta_t(u) + n_z(u)\cos\theta_t(u),$$

where

$$\sin\theta_t(u) = \sqrt{\varepsilon}\frac{\sqrt{\varepsilon}\sin(w - u_0) - \sin(w + u - u_0)}{\sqrt{1 - 2\sqrt{\varepsilon}\cos u + \varepsilon}},$$

w is a θ_i' corresponding to the actual β (β can differ from β_0), i.e. $w = 2\alpha - \mathrm{acos}(\sqrt{\varepsilon}\beta)^{-1}$.

We define the axicon length (including imaginary "nose") l, therefore $x_{\max} = l\tan\alpha - a$. Lower limit $u_{\min} = 0$ while the upper limit is determined from the relation

$$r(u_{\max})\sin u_{\max} = l\frac{\sin(\alpha - u_0)}{\cos\alpha} - a\frac{\sin(\alpha + u_0)}{\sin\alpha},$$

where $z_f = l + r(u_{\max})\cos(u_{\max} - u_0)$,

$$h_f = a + \tan u_0(z_f - a\cot\alpha).$$

Now all is ready for the numerical calculations, corresponding results are presented below.

Top plot in Fig. 4 shows a family of rays in the area outside the target (in the xz-plane). One can see that the rays converge into two exact "focuses" (these points are the projections of the ring focus to zx-plane) highlighted by two blue circles. Moreover, the rays are concentrated in a certain area near z-axis, this area is highlighted by yellow rectangle. Two other plots in Fig. 4 show field distribution calculated

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB250

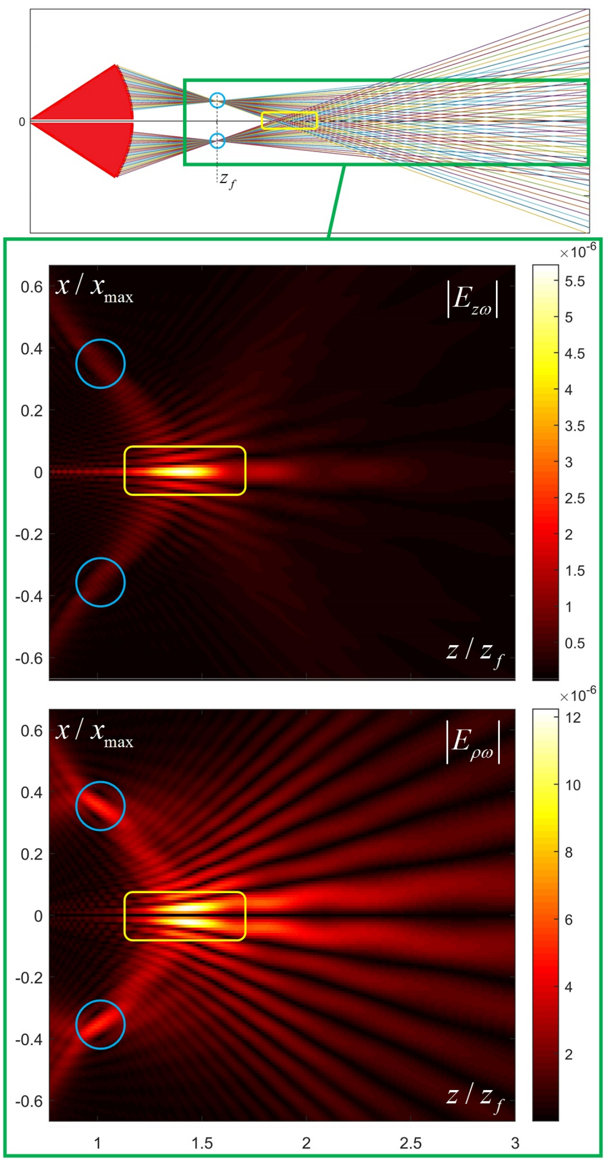

Figure 4: Rays (top plot) and field distribution (middle and bottom plots) over zx-plane for the concentrator with shifted focus. Electric field is in units Vms^{-1} (electric field per unit frequency). Calculation parameters: $r_0 = 0$, $\beta = \beta_0 = 0.999$, $\varepsilon = 4$, $x_{max} = 15.4$ cm, $f = 23.8$ cm, $a = c/\omega = 0.047$ cm, $\omega = 2\pi \cdot 100$ GHz, $q = 1$ nC (point charge), $\alpha = 33°$, $u_0 = 6°$, $h_f = 5.5$ cm.

using Stratton-Chu formulas from [4] and additional formulas presented above. One can see that there is certain field concentration on the focus ring, but for $E_{z\omega}$ component it is much weaker compared to $E_{\rho\omega}$ component. Moreover, it is clearly seen that main field concentration occurs beyond the focus ring in the area of rays intersection near the symmetry axis. For $E_{\rho\omega}$ component, field magnitude in this area as several times larger compared to that on the focus ring, for $E_{z\omega}$ the field near z-axis is of order of magnitude larger than that on the focus ring.

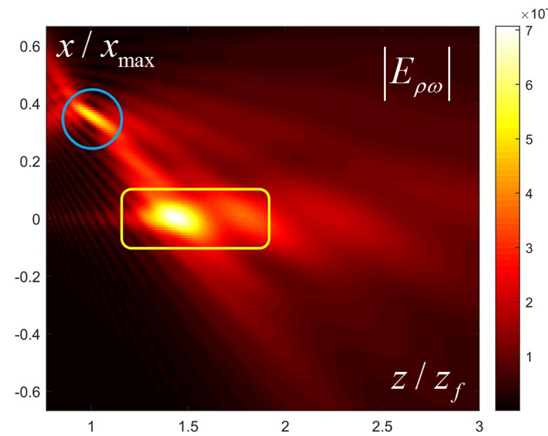

Figure 5: $|E_{\rho\omega}|$ over zx-plane for the quarter of concentrator with shifted focus ($\varphi \in [-\pi/4, \pi/4]$). Other parameters are the same as in Fig. 4.

If we consider a slice of the initial axisymmetric target then typical ratio between field magnitudes on the focus ring and in the area of rays intersection near the z-axis can be enhanced. For example, for a quarter of the target ($\varphi \in [-\pi/4, \pi/4]$), $E_{\rho\omega}$ component is of comparable magnitude in these two regions (see Fig. 5), while for $E_{z\omega}$ component this is not the case. Moreover, field distribution is asymmetric and $E_{\rho\omega}$ not equals zero for $\rho = 0$.

REFERENCES

[1] A. Curcio *et al.*, "Noninvasive bunch length measurements exploiting Cherenkov diffraction radiation", *Phys. Rev. Accel. Beams*, vol. 23, p. 022802, 2020. doi:10.1103/PhysRevAccelBeams.23.022802

[2] S. N. Galyamin and A. V. Tyukhtin, "Dielectric Concentrator for Cherenkov Radiation", *Phys. Rev. Lett.*, vol. 113, p. 064802, 2014. doi:10.1103/PhysRevLett.113.064802

[3] S. N. Galyamin *et al.*, "Radiation of a charge in dielectric concentrator for Cherenkov radiation: Off-axis charge motion", *Phys. Rev. Accel. Beams*, vol. 22, p. 083001, 2019. doi:10.1103/PhysRevAccelBeams.22.083001

[4] S. N. Galyamin and A. V. Tyukhtin, "Cherenkov radiation of a charge in axicon-based dielectric concentrator", *Phys. Rev. Accel. Beams*, vol. 23, p. 113001, 2020. doi:10.1103/PhysRevAccelBeams.23.113001

[5] A.V. Tyukhtin *et al.*, "Cherenkov radiation of a charge flying through the inverted conical target", *Phys. Rev. A*, vol. 102, p. 053514, 2020. doi:10.1103/PhysRevA.102.053514

[6] S. N. Galyamin, A. Tyukhtin, and V. Vorobev, "Focusing the Cherenkov radiation using dielectric concentrator: simulations and comparison with theory", *J. Instrumentation*, vol. 13, p. C02029, 2018. doi:10.1088/1748-0221/13/02/C02029