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RADIATION OF A CHARGED PARTICLE BUNCH MOVING ALONG A DEEP CORRUGATED SURFACE WITH A SMALL PERIOD

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Abstract

We investigate the electromagnetic radiation of a bunch moving along a corrugated conductive surface. It is assumed that the wavelengths under consideration are much greater than the period of the corrugation. In this case, the corrugated structure can be replaced with a smooth surface on which so-called equivalent boundary conditions (EBC) are fulfilled. Here, we consider the case of deep corrugation, i.e. we assume that the depth of the structure is much greater than its period. Using the EBC we obtain electromagnetic field components in the form of spectral integrals and calculate them numerically. It is shown that the bunch generates surface waves propagating in the plane of the structure, whereas volume radiation is absent at the frequencies under consideration. It is demonstrated that the features of the surface wave can be used for determining the size and the form of the bunch. We also consider the energy of the surface waves. Typical dependences of the energy on corrugation parameters are obtained and analyzed.

INTRODUCTION

This work aims to study electromagnetic radiation from a bunch moving along a deep corrugated conductive surface. We analyze so-called "longwave" radiation when the wavelengths are much greater than the period of the structure. In this case, the analytical solution can be obtained by the use of the equivalent boundary conditions (EBC) [1]. Besides, we assume that the corrugation is deep, i.e. the depth is much greater than the period.

Note that earlier we effectively applied the EBC to solve "longwave" problems [2–4]. In these papers, we considered structures with shallow corrugation, that is, when the wavelengths are much greater than both the period of the structure and its depth. In [2], we studied radiation from a bunch moving in a circular waveguide. In particular, the results of this work contains a comparison between the theory and simulations in the CST Particle Studio. The agreement can be considered good, which justifies the use of the EBC even in situations when the wavelengths are only several times greater than the period. In papers [3, 4], we demonstrated that an ultrarelativistic bunch moving in the presence of a planar shallow corrugated structure excites surface waves. Analysis of these waves showed that their features can be used for determining the bunch properties.

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EQUIVALENT BOUNDARY CONDITIONS

We consider a perfectly conductive planar surface in a vacuum. The surface has rectangular corrugation, as shown in Fig. 1. We assume that the period of the structure d is much less than wavelengths under consideration λ: $d \ll \lambda$. In this case, the corrugated structure can be replaced with a smooth surface on which the EBC are fulfilled [1]. In fact, we deal with an anisotropic surface characterized by a certain matrix impedance.

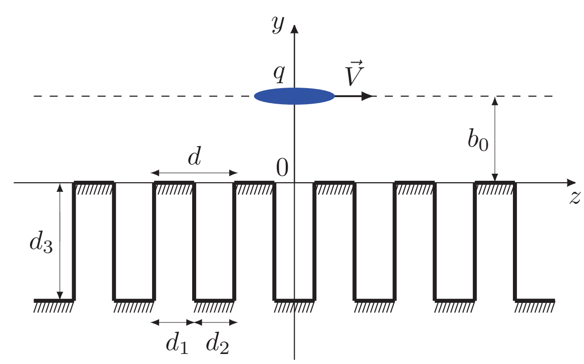

Figure 1: The corrugated structure and the moving bunch.

Unlike previous works [2–4], we study the case of deep corrugation, i.e. it is assumed that the depth of the structure d_3 is much greater than its period: $d \ll d_3$. Then, the EBC have the following form for the Fourier-transforms of electric and magnetic fields [1]:

$$E_{\omega z}\big|_{y=0} = \eta^m H_{\omega x}\big|_{y=0},\ E_{\omega x}\big|_{y=0} = 0. \qquad (1)$$

Here, η^m is an impedance, which is given by the formula [1]

$$\eta^m = i\frac{d_2}{d}\frac{\mathrm{tg}\,(k_0 d_3)}{1 - k_0 d\,\tilde{l}\,\mathrm{tg}\,(k_0 d_3)}, \qquad (2)$$

where d_2 is the width of the structure grooves $k_0 = \omega/c = 2\pi/\lambda$ is the wave number. The parameter of the corrugation $\tilde{l}$ is determined by the expression [1]

$$\tilde{l} = \frac{1}{2\pi}\Big[(2-\xi)^2 \ln(2-\xi) - \xi^2 \ln\xi - 2(1-\xi)\ln 4(1-\xi)\Big], \qquad (3)$$

where $\xi = d_1/d$, $d_1 = d - d_2$. It should be noted that parameter $\tilde{l}$ is positive and small: $0 < \tilde{l} \lesssim 0.082$.

GENERAL SOLUTION

We assume that a charged particle bunch moves with constant velocity $\vec{V} = c\beta\vec{e}_z$ at distance b_0 from the surface

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(Fig. 1). The bunch has an negligible transversal size and longitudinal charge distribution $\varkappa(z-Vt)$, i.e. the charge and current densities are $\rho = q\delta(x)\delta(y-b_0)\varkappa(z-Vt)$ and $j = j_z = \rho V$.

We use Hertz potential $\vec{\Pi}$ and present it as a sum of "forced" potential $\vec{\Pi}^{(i)}$ and "free" potential $\vec{\Pi}^{(r)}$. The "forced" field is the well-known Coulomb field of a charge moving in an unbounded vacuum, whereas the "free" field is connected with the influence of the corrugated structure.

One can show that the "free" field is described by the Hertz potential with two non-zero components. The Fourier-transforms of these components are

$$\begin{Bmatrix} \Pi_{\omega x}^{(r)} \\ \Pi_{\omega z}^{(r)} \end{Bmatrix} = -\frac{q\tilde{\varkappa}}{ck_0} e^{i\frac{k_0}{\beta}z} \int_{-\infty}^{+\infty} dk_x \begin{Bmatrix} R_x \\ R_z \end{Bmatrix} \frac{e^{ik_x x + ik_{y0}(y+b_0)}}{k_{y0}}, \quad (4)$$

where $k_{y0} = i\sqrt{k_x^2 + k_0^2\frac{1-\beta^2}{\beta^2}}$ ($\mathrm{Im}\, k_{y0} > 0$) and $\tilde{\varkappa}$ is the Fourier-transform of the bunch profile:

$$\tilde{\varkappa} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\zeta \varkappa(\zeta) e^{-i\frac{k_0}{\beta}\zeta}, \quad \zeta = z - Vt. \quad (5)$$

Relations between the Fourier-transforms of electromagnetic field and the Hertz potential are given by the formulas $\vec{E}_\omega = \vec{\nabla}\,\mathrm{div}\,\vec{\Pi}_\omega + k_0^2\vec{\Pi}_\omega$, $\vec{H}_\omega = -ik_0\,\mathrm{rot}\,\vec{\Pi}_\omega$. Substituting the electromagnetic field into (1) and solving the system of equations we obtain

$$R_x = \frac{2k_0 k_x k_{y0}\beta\eta^m}{k_0^3 + \beta^2\left(k_x^2 - k_0^2\right)\left(k_0 - k_{y0}\eta^m\right)}, \quad (6)$$

$$R_z = -\frac{k_0^3 + \beta^2\left(k_x^2 - k_0^2\right)\left(k_0 + k_{y0}\eta^m\right)}{k_0^3 + \beta^2\left(k_x^2 - k_0^2\right)\left(k_0 - k_{y0}\eta^m\right)}. \quad (7)$$

SURFACE WAVES

Further, we will investigate the asymptotical behaviour of "free" Hertz potential [Eq. (4)]. Note that the integrands in these formulas have several singularities. In particular, coefficients R_x and R_z have poles

$$k_x = \pm k_{x0} = \pm k_0\sqrt{1 - \frac{\mathrm{sgn}\left(\eta_0^m\right)}{2\eta_0^{m2}}\left(\sqrt{1+\frac{4\eta_0^{m2}}{\beta^2}} - \mathrm{sgn}\left(\eta_0^m\right)\right)}, \quad (8)$$

where $\eta_0^m = \mathrm{Im}\,\eta^m$. Poles [Eq. (8)] are real under condition $\beta > \left(1+\eta_0^{m2}\right)^{-1/2}$. If this inequality is satisfied, the contributions of the poles are surface waves.

The asymptotical investigation of integrals in Eq. (4) shows that the initial integration path can be transformed to the steepest descent path with separation of the contributions of the pole $k_x = +k_{x0}$ for $x > 0$ and $k_x = -k_{x0}$ for $x < 0$. These contributions exist necessarily if the observation point is close to the corrugated structure. As for the contribution of the saddle point, it decreases exponentially with increasing in $k_0\beta^{-1}\sqrt{1-\beta^2}\sqrt{x^2+(y+b_0)^2}$, and we will neglect it. Emphasize that exponential decreasing the contribution of the saddle point means the absence of volume radiation.

Calculating the contributions of poles [Eq. (8)] we obtain the following expressions for the Fourier-transforms of the electromagnetic field:

$$\begin{Bmatrix} E_{\omega x}^{(s)} \\ E_{\omega y}^{(s)} \\ E_{\omega z}^{(s)} \end{Bmatrix} = \frac{2\pi q\tilde{\varkappa}k_0}{c\sqrt{\beta^2-g^2}} \begin{Bmatrix} 0 \\ -i\eta_0^m \\ \beta\left(g^2-1\right)g^{-2} \end{Bmatrix} \exp\left(ik_0\beta^{-1}z\right)$$
$$\times \exp\left[ik_0\beta^{-1}\sqrt{\beta^2-g^2}|x| - k_0|\eta_0^m|g^2\beta^{-2}\left(y+b_0\right)\right], \quad (9)$$

$$\begin{Bmatrix} H_{\omega x}^{(s)} \\ H_{\omega y}^{(s)} \\ H_{\omega z}^{(s)} \end{Bmatrix} = \frac{2\pi q\tilde{\varkappa}k_0}{c} \begin{Bmatrix} i|\eta_0^m|g^2\left(\beta^2-g^2\right)^{-\frac{1}{2}} \\ -\mathrm{sgn}(x)\left(g^2-1\right)g^{-2} \\ -i\,\mathrm{sgn}(x)\,\eta_0^m\beta^{-1} \end{Bmatrix} \exp\left(ik_0\beta^{-1}z\right)$$
$$\times \exp\left[ik_0\beta^{-1}\sqrt{\beta^2-g^2}|x| - k_0|\eta_0^m|g^2\beta^{-2}\left(y+b_0\right)\right]. \quad (10)$$

Function g^2 in Eqs. (9) and (10) is given by the formula

$$g^2 = \frac{\beta^2}{2\eta_0^{m2}}\left(\sqrt{1+\frac{4\eta_0^{m2}}{\beta^2}} - 1\right). \quad (11)$$

Note that Fourier-transforms [Eqs. (9) and (10)] decrease exponentially with increasing y, that is these components describe the surface wave propagating along the structure and diminishing rapidly with a distance from it.

Further, we present some results of numerical calculating Fourier-integrals $F(\vec{r},t) = \int_{-\infty}^{+\infty} d\omega F_\omega e^{-i\omega t}$ where Fourier-transforms are determined by Eqs. (9) and (10). We carry out the computation for the Gaussian bunch with the following charge distribution $\varkappa_{gaus}$ and its Fourier-transform $\tilde{\varkappa}_{gaus}$ (see Eq. (5)):

$$\varkappa_{gaus}(\zeta) = \frac{e^{-\zeta^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma}, \quad \tilde{\varkappa}_{gaus} = \frac{e^{-k_0^2\sigma^2/(2\beta^2)}}{2\pi}, \quad (12)$$

where σ is a half of the bunch length. Figure 2 shows the dependences of components $H_y^{(s)}$ and $H_z^{(s)}$ on coordinate z. As follows from the plots, the field magnitude increase with decreasing the length of the bunch 2σ and increasing its velocity β. The dependences also allow determining the bunch length which is equal to the distance between the extremums of $H_z^{(s)}$.

In addition, we analyse the energy losses of the bunch which can be obtained by the calculation of the energy flow through two parallel half-planes $x = \pm x_0$, $y > 0$. This way results in the following expression for the energy losses per the unit of the path length:

$$\frac{dW}{dz_0} = \frac{1}{V}\frac{dW}{dt} = \frac{2}{c\beta}\int_{-\infty}^{+\infty} dz \int_0^{+\infty} dy S_x\Big|_{x=x_0>0}, \quad (13)$$

where $S_x = \frac{c}{4\pi}\left(E_y^{(s)}H_z^{(s)} - E_z^{(s)}H_y^{(s)}\right)$ is the x-component of Poynting vector. Equation (13) can be transformed to the

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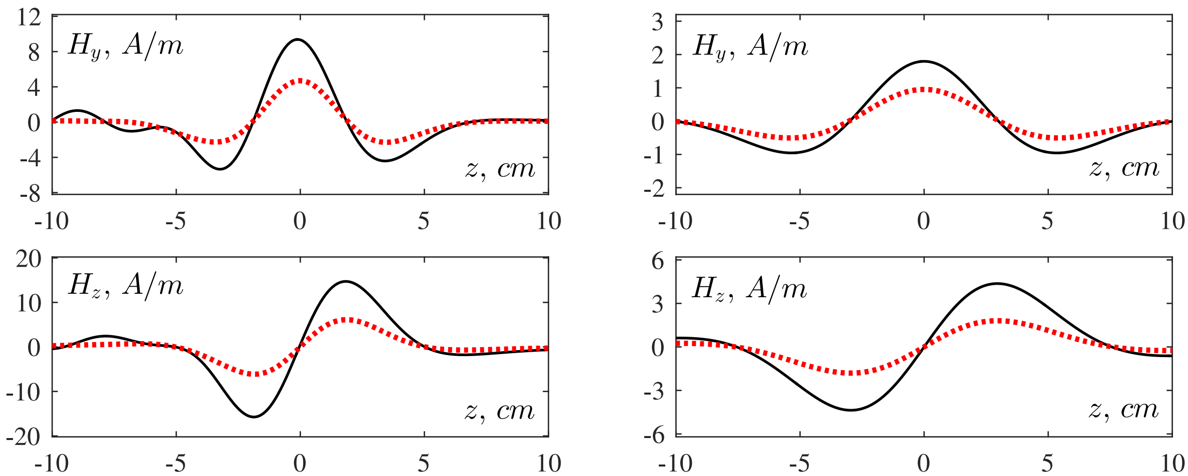

Figure 2: The components of the surface wave $H_y^{(s)}$ (top row) and $H_z^{(s)}$ (bottom row) depending on coordinate z for the Gaussian bunch with $q = 1$ nC. The bunch velocity is $\beta = 1$ (solid black curves) and $\beta = 0.75$ (dotted red curves). The bunch length is $2\sigma = 3$ cm (left column) and $2\sigma = 6$ cm (right column). The parameters: $d = 0.05$ cm, $d_2 = 0.04$ cm, $d_3 = 1$ cm, $b_0 = 3$ cm, $x = y = 0$, $t = 0$.

expression

$$\frac{dW}{dz_0} = 2c^2 \iint_0^{+\infty} dk_0 dy \,\mathrm{Re}\left(E_{\omega y}^{(s)} H_{\omega z}^{(s)*} - E_{\omega z}^{(s)} H_{\omega y}^{(s)*}\right). \quad (14)$$

Substituting Eqs. (9) and (10) into Eq. (14) we obtain

$$\frac{dW^{(s)}}{dz_0} = 4\pi^2 q^2 \beta \int_0^{+\infty} dk_0 k_0 |\tilde{\kappa}|^2$$
$$\times \frac{\eta_0^{m2} + \beta^2 g^{-4} \left(g^2 - \mathrm{sgn}\left(\eta_0^m\right)\right)^2}{|\eta_0^m| g^2 \sqrt{\beta^2 - \mathrm{sgn}\left(\eta_0^m\right) g^2}} e^{-2k_0 \beta^{-2} |\eta_0^m| g^2 b_0}. \quad (15)$$

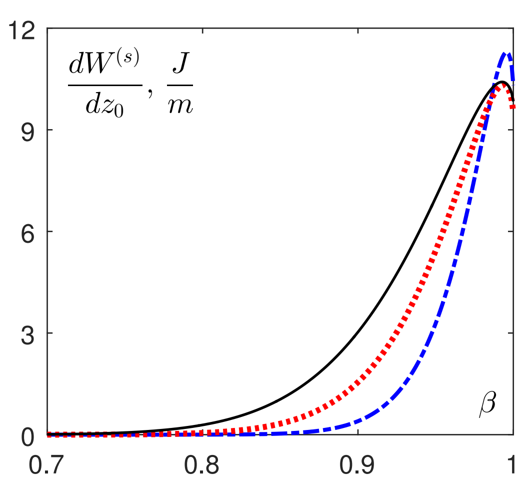

Figure 3: The energy of the surface wave $dW^{(s)}/dz_0$ depending on bunch velocity β for the Gaussian bunch with $q = 1$ nC. The depth is $d_3 = 1$ cm (solid black curves), $d_3 = 0.8$ cm (dotted red curves) and $d_3 = 0.6$ cm (dashed-dotted blue curves). The parameters: $2\sigma = 3$ cm, $d = 0.05$ cm, $d_2 = 0.04$ cm, $b_0 = 3$ cm.

Figure 3 shows the dependences of energy losses $dW^{(s)}/dz_0$ on bunch velocity β for different values of structure depth d_3. As we can see, the energy losses increase with increasing the velocity and have a maximum when the velocity has a certain value close to c. Note also that the range of the velocities, at which the surface waves are generated, increases with increasing the depth. This is a significant advantage over the case of the shallow corrugation, when the radiation is generated by only ultrarelativistic bunches [3].

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