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STUDY OF THE THIRD-ORDER PARAMETRIC RESONANCE INDUCED BY RF MODULATION

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Abstract

There were both theoretical studies and simulations on the effects of RF modulation on bunch lengthening in electron storage rings. Nevertheless, the increase of bunch energy spread would happen in the meantime, which may limit the potential applications of the RF modulation technique. We believe that the comprehensive studies of the parametric resonance induced by RF modulation are necessary for understanding the physics picture better and seeking new applications of this technique. The studies on the third-integer RF phase modulation would be presented here. We derived the longitudinal modulated Hamiltonian and various parameters, such as the fixed points, island tune, which could be confirmed by simulations. Furthermore, the dependence of the bunch parameters, such as energy spread and bunch length, on the modulation settings was also discussed in this paper.

INTRODUCTION

Obtaining higher density bunches with smaller emittances has been one of the goals pursued with the ongoing development of synchrotron radiation light sources. Lengthening the bunch length is an effective method to reach the goal [1, 2]. And there are studies to prove that the RF modulation is a valid way to lengthen the bunch theoretically and verified by simulations [3–8]. To date, there were some experimental studies carried out to research the influence of RF modulation on the beam in a number of synchrotron facilities, such as LNLS [9, 10], KEK-PF [4, 5], SRRC [11, 12] and PLS [13].

Diffraction-limited storage rings (DLSRs), as the frontier of the synchrotron light source, has meet many challenges, one of which is the pursuit of small emittances, which results from stronger focusing. However, stronger focusing usually corresponds to stronger nonlinearity in the lattice, which can make dynamic aperture (DA) optimization very challenging [14]. The on-axis swap-out injection scheme [15, 16] is considered to meet the constraint of DA in HEPS [17]. When injecting the full-charge bunches, the injection efficiency will reduce due to the injection transient instability [18, 19]. There is paper suggesting that applying RF modulation to booster was useful to suppress the instability [1,2,17]. Although the injection efficiency increased with second-integer modulation, the improvement of efficiency was unstable. The third-integer RF modulation is valuable to further study.

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Table 1: Parameters of Lattice

Parameters	Value	Units
Circumference	454.0665	m
Energy	6	GeV
Total RF Voltage	8	MV
Momentum Compaction Factor	3.6803e-3	-
Synchronous Phase Angle	149.8359	degree
Radiation Damping Time τ_x, τ_y, τ_z	4.52,4.52, 2.26	ms
Primary RF Frequency	499.8	MHz
Harmonic Number	757	-

The paper mainly focused on the beam dynamics while applying the third-integer RF phase modulation. The used lattice parameters were shown in Table 1. The paper was organized as follows. In Section 2, the modulated synchrotron Hamiltonian was derived. In Section 3, the expressions of fixed points and island tune were given analytically. In Section 4 the parameters were obtained based on the expressions in Section 3 and verified with simulation results. And the dependence of bunch length on modulation parameters was also studied. The conclusion was given in Section 5.

LONGITUDINAL HAMILTONIAN

According to the knowledge in "Accelerator Physics" written by S. Y. Lee [20], the longitudinal equations of motion were given by

$$\dot{\phi} = \omega_s \frac{\eta}{|\eta|}\delta^*, \qquad \dot{\delta}^* = \omega_s(\sin\phi - \sin\phi_s), \tag{1}$$

where ν_s and ω_s were the synchronous tune and angular frequency, and η was the phase-slip factor, and $\delta^* = \frac{h|\eta|}{\nu_s}\frac{(p-p_0)}{p_0}$, and h was the harmonic number, and ϕ_s was the synchronous phase angle.

When applying RF phase modulation, the perturbed synchrotron equations of motion with $\eta > 0$ could be written as

$$\dot{\phi} = \omega_s\delta^*, \qquad \dot{\delta}^* = \omega_s[\sin(\phi + A_m \sin\omega_m t) - \sin\phi_s]. \tag{2}$$

And the modulated Hamiltonian is

$$H_0 = \frac{\omega_s}{2}\delta^{*2} + \omega_s[\cos(\phi + A_m\sin\omega_m t) + (\phi - \phi_s)\sin\phi_s]. \tag{3}$$

By using the action-angle variables (J, ψ) defined as

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$$\delta^* = -\sqrt{2J}\sin\psi, \qquad \Delta\phi = \phi - \phi_s = \sqrt{2J}\cos\psi,$$

the perturbed Hamiltonian is given by

$$\begin{aligned} H(J,\psi) &= \omega_s J \sin^2\psi \\ &+ \omega_s[\cos\phi_s \cos(\sqrt{2J}\cos\psi + A_m \sin\omega_m t) \\ &- \sin\phi_s \sin(\sqrt{2J}\cos\psi + A_m\sin\omega_m t) \\ &+ \sqrt{2J}\cos\psi \sin\phi_s]. \end{aligned} \tag{4}$$

With the help of the Bessel expansion , under the assumption of $\phi_s \approx \pi$, the $H(J,\psi)$ can be expanded to

$$\begin{aligned} H(J,\psi) &= \omega_s J\sin^2\psi \\ &- \omega_s[J_0(\sqrt{2J}) + \sum_{k=1}^{\infty}(-1)^k J_{2k}(\sqrt{2J})\cos(2k\psi) \\ &- A_m \sum_{k=0}^{\infty}(-1)^k J_{2k+1}(\sqrt{2J})\sin(\omega_m t \pm (2k+1)\psi)]. \end{aligned} \tag{5}$$

Considering the third-integer resonance (k=1), and using approximate expression $J_0(\sqrt{2J}) \approx 1 - J/2 + J^2/16$, the modulated Hamiltonian became

$$\begin{aligned} H(J,\psi) &= \omega_s J\sin^2\psi - \omega_s(1 - \frac{J}{2} + \frac{J^2}{16}) \\ &+ \omega_s \sum_{k=1}^{\infty}(-1)^k J_{2k}(\sqrt{2J})\cos(2k\psi) \\ &- \omega_s A_m J_3(\sqrt{2J})\sin(\omega_m t - 3\psi). \end{aligned} \tag{6}$$

Then a new canonical transformation was obtained by the generating function [7] $F_2(\tilde{J},\tilde{\psi}) = (\psi - \frac{\omega_m t}{3} - \frac{\pi}{2})\tilde{J}$, where $\tilde{J} = J, \tilde{\psi} = \psi - \frac{\omega_m t}{3} - \frac{\pi}{2}$.
So the time-averaged Hamiltonian was

$$\begin{aligned} K_t(\tilde{J},\tilde{\psi}) &= \omega_s\tilde{J} - \frac{\omega_m\tilde{J}}{3} - \frac{\omega_s\tilde{J}^2}{16} \\ &- \omega_s A_m \frac{(2\tilde{J})^{\frac{3}{2}}}{48}\cos(3\tilde{\psi}) - \omega_s. \end{aligned} \tag{7}$$

PHYSICAL PARAMETERS ANALYSES

Fixed Points

The fixed points can be obtained by following conditions:

$$\frac{d\tilde{J}}{dt} = -\frac{\partial K_t}{\partial\tilde{\psi}} = 0, \qquad \frac{d\tilde{\psi}}{dt} = \frac{\partial K_t}{\partial\tilde{J}} = 0. \tag{8}$$

According to the $\frac{\partial K_t}{\partial\tilde{J}} = 0$, by solving a quadratic equation with respect to $\sqrt{2\tilde{J}}$, we can get

$$\sqrt{2\tilde{J}} = -\frac{A_m\cos(3\tilde{\psi})}{2} \pm \frac{|A_m|}{2}\sqrt{1 + \frac{64}{A_m^2}(1 - \frac{\omega_m}{\omega_s})}. \tag{9}$$

According to the $\frac{\partial K_t}{\partial\tilde{\psi}} = 0$, we can get $\sin(3\tilde{\psi}) = 0$, $\tilde{\psi} = n\pi/3$, where $n = 0, 1, 2, \cdots$. Because K_t goes to infinitesimal as $\sqrt{2\tilde{J}}$ goes to infinity, $\tilde{\psi} = \pi/3, \pi, 5\pi/3$, when $\cos(3\tilde{\psi}) = -1$ to make K_t have a maximum. So we can get that

$$\sqrt{2\tilde{J}} = \frac{A_m}{2} + \frac{A_m}{2}\sqrt{1 + \frac{64}{A_m^2}(1 - \frac{\omega_m}{\omega_s})}, \tag{10}$$

where $A_m > 0$. Finally, the three stable fixed points are (for $\tilde{\psi} = \pi/3, \pi, 5\pi/3$)

$$\begin{aligned} \delta^* &= \frac{A_m}{2}(1+R), \quad \phi = 0, \\ \delta^* &= -\frac{A_m}{4}(1+R), \quad \phi = -\frac{\sqrt{3}A_m}{4}(1+R), \\ \delta^* &= -\frac{A_m}{4}(1+R), \quad \phi = \frac{\sqrt{3}A_m}{4}(1+R), \end{aligned} \tag{11}$$

where the $R = \sqrt{1 + \frac{64}{A_m^2}(1 - \frac{\omega_m}{\omega_s})}$, and A_m is the modulation amplitude. The unstable fixed points can also be calculated in the same method.

The comparison between the predictions of the theoretical model and simulation results would be discussed in section 4.

Island Tune

The island tune is defined by f_{island}/f_0, which is a important property for a rotate system. Here the f_{island} is the frequency that a particle revolves around a SFP in the resonant precessing frame. In order to get the island tune, we need to expand the phase space coordinates around a fixed point of the Hamiltonian. That's

$$\begin{aligned} \delta' &= \sqrt{2\tilde{J}}\cos\tilde{\psi} - \sqrt{2\tilde{J}_{SFP}}\cos\tilde{\psi}_{SFP}, \\ \phi' &= -\sqrt{2\tilde{J}}\sin\tilde{\psi} + \sqrt{2\tilde{J}_{SFP}}\sin\tilde{\psi}_{SFP}. \end{aligned} \tag{12}$$

Substituting Eq.(12) into $K_t(\tilde{J},\tilde{\psi})$, the new Hamiltonian becomes

$$K_t(\delta',\phi') = \frac{A}{2}\delta'^2 + \frac{B}{2}\phi'^2 + \text{High-order terms}, \tag{13}$$

where

$$\begin{aligned} A &= \omega_s - \frac{\omega_m}{3} - \frac{3\tilde{J}_{SFP}}{8}\omega_s\cos^2\tilde{\psi}_{SFP} \\ &- \frac{\tilde{J}_{SFP}}{8}\omega_s\sin^2\tilde{\psi}_{SFP} - \frac{\sqrt{2\tilde{J}_{SFP}}}{8}A_m\omega_s\cos\tilde{\psi}_{SFP}, \\ B &= \omega_s - \frac{\omega_m}{3} - \frac{3\tilde{J}_{SFP}}{8}\omega_s\sin^2\tilde{\psi}_{SFP} \\ &- \frac{\tilde{J}_{SFP}}{8}\omega_s\cos^2\tilde{\psi}_{SFP} + \frac{\sqrt{2\tilde{J}_{SFP}}}{8}A_m\omega_s\cos\tilde{\psi}_{SFP}. \end{aligned} \tag{14}$$

So the small amplitude island tune was $Q_{island} = \frac{\sqrt{AB}}{\omega_0}$.

SIMULATIONS

In this section we present a comparison between the predictions of the theoretical model and simulation results. In order to get a better insight of the behavior of the bunches under phase modulation, a longitudinal dynamics simulation code was developed.

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Cross Check with SFP and Q_{island}

Using the pelegant [21] simulation code, in the condition of Table 1, Under the condition that the modulation amplitude is 40°, modulation frequency 2.75 ω_s, the phase-space distribution of the particles in a single bunch of 10000 macroparticles is shown in Fig. 1, which is in good agreement with the ones calculated using the Hamiltonian K_t.

In particular, the positions of the resonant island centers are corresponded to the stable fixed points, and the conformity can be reflected in the Fig. 1.

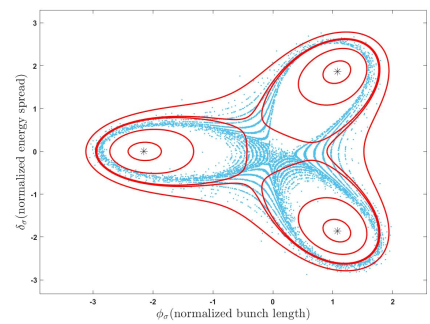

Figure 1: Comparison between particles distribution and K-constant contours. The blue points represent particles in normalized phase space, and the rad curves are the K-constant contours in different Hamiltonian, and the asterisks represent the results of SFPs calculated by theoretical model.

Under the condition that the modulation amplitude is 35°, modulation frequency 2.75 ω_s, the island tune $Q_{island} = 0.09127 Q_s$.

Based on the parameters in Table 1, the synchrotron tune $Q_s = \sqrt{\frac{H \alpha e V cos\phi_s}{2\pi\beta^2 E}} = 0.02261$. That's to say

$$Q_{island} = 0.09127 Q_s = 2.0634 \times 10^{-3}. \quad (15)$$

After analyzing the simulation results obtained by Elegant program with naff program, the undisturbed synchrotron tune Q_s is equal to 0.02256. Under the modulation above, the disturbed synchrotron tune $Q_{disturbed}$ is equal to 0.02069. So the difference between Q_s and $Q_{disturbed}$ is

$$\Delta Q_s = Q_s - Q_{disturbed} = 1.8706 \times 10^{-3}. \quad (16)$$

The relative difference between Q_{island} and ΔQ_s is 9.344%.

Dependence of Bunch Length on Modulation Conditions

We also simulate the dependence of the beam length and energy spread to the modulation frequency in different modulation amplitude while sweeping the modulation frequency slowly.

Figure 2 shows how the bunch length changes with modulation frequency in different modulation amplitude. The figure indicates that the beam length increases first and then decreases, as the modulation frequency increases. Furthermore, When the modulation amplitude was raising, the modulation frequency corresponding to the maximum beam length decreases gradually. For example, when the modulation amplitude are 24 deg, 28 deg and 36 deg, the modulation frequency for maximum beam length are 2.86 ω_s, 2.82 ω_s and 2.72 ω_s.

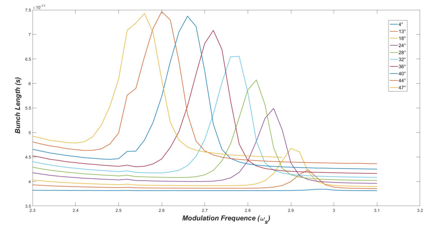

Figure 2: Bunch length vs. the modulation frequency. The difference color curves represent the different modulation amplitude.

As for the energy spread, it has the similar change rules with bunch length, which shown in Fig. 3.

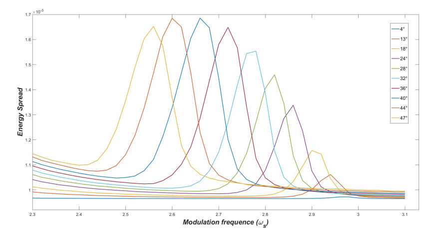

Figure 3: Energy spread vs. the modulation frequency. The difference color curves represent the different modulation amplitude.

CONCLUSION

In the paper, we have shown theoretical and simulation results, when the RF phase modulation on the third harmonic of the synchrotron frequency is used. And simulations indicate that the phase modulation can lengthen the bunch length successfully. However, the energy spread is also increased, which will result in the strong beam collective effects.

REFERENCES

[1] C. J. Bocchetta *et al.*, "Beam Density Manipulations in the ELETTRA Storage Ring", in *Proc. 17th Particle Accelerator Conf. (PAC'97)*, Vancouver, Canada, May 1997, paper 6V024, pp. 829–831.

[2] P. Kuske, "Emittance Manipulations at BESSY I", in *Proc. 6th European Particle Accelerator Conf. (EPAC'98)*, Stockholm, Sweden, Jun. 1998, paper THP14G, pp. 1297–1299.

[3] D. Li *et al.*, "Effects of rf voltage modulation on particle motion", *Nucl. Instrum. Methods Phys. Res. A*, vol. 364, no. 2, p. 205, 1995. doi:10.1016/0168-9002(95)00440-8

[4] S. Sakanaka, M. Izawa, T. Mitsuhashi, and T. Takahashi, "Improvement in the beam lifetime by means of an rf phase modulation at the KEK Phton Factory storage ring", *Phys. Rev.*

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ST Accel. Beams, vol. 3, p. 050701, 2000. doi:10.1103/PhysRevSTAB.3.050701

[5] S. Sakanaka and T. Obina, "Observation of Longitudinal Quadrupolemode Oscillations of a Bunch which were Induced by RF Phase Modulation in the Electron Storage Ring", *Jpn. J. Appl. Phys.*, vol. 40, no. 4R, pp. 2465–2474, 2001. doi:10.1143/JJAP.40.2465

[6] N. P. Abreu, R. H. A. Farias, and P. F. Tavares, "Longitudinal dynamics with rf phase modulation in the Brazilian electron storage ring", *Phys. Rev. ST Accel. Beams*, vol. 9, p. 124401, 2006. doi:10.1103/PhysRevSTAB.9.124401

[7] F. Orsini and A. Mosnier, "Effectiveness of rf phase modulation for increasing bunch length in electron storage rings", *Phys. Rev. E*, vol. 61, p. 4431, 2000. doi:10.1103/PhysRevE.61.4431

[8] D. Quartullo, E. Shaposhnikova, and H. Timko, "Effectiveness of rf phase modulation for increasing bunch length in electron storage ring", *J. Phys.: Conf. Ser.*, vol. 874, p. 012066, 2017. doi:10.1088/17426596/874/1/012066

[9] N. P. Abreu, R. H. A. Farias, and P. F. Tavares, "RF Phase Modulation at the LNLS Electron Storage Ring", in *Proc. 21st Particle Accelerator Conf. (PAC'05)*, Knoxville, TN, USA, May 2005, paper MPPP020, pp. 1686-1688.

[10] A. N. Pham, S. Y. Lee, and K. Y. Ng, "Method of phase space beam dilution utilizing bounded chaos generated by rf phase modulation", *Phys. Rev. ST Accel. Beams*, vol. 18, no. 12, pp. 124001–124005, 2015. doi:10.1103/PhysRevSTAB.18.124001

[11] L. H. Chang, K. T. Hsu, C. C. Kuo, W. K. Lau, and C. S. Hsue, "Effect of RF Phase Modulation on the Longitudinal Beam Dynamics", in *Proc. 17th Particle Accelerator Conf. (PAC'97)*, Vancouver, Canada, May 1997, paper 2V045, pp. 1487–1489.

[12] M. H. Wang *et al.*, "Experiment of RF Voltage Modulation at SRRC", in *Proc. 18th Particle Accelerator Conf. (PAC'99)*, New York, NY, USA, Mar. 1999, paper THA106, pp. 2837–2839.

[13] Yujong Kim *et al.*, "Nonlinear RF phase modulation in PLS storage ring", in *Proc. 2nd Asian Particle Accelerator Conf. (APAC'01)*, Beijing, China, Sep. 2001, paper WEP047, pp. 418–420.

[14] R. O. Hettel, "Challenges in the Design of Diffraction-limited Storage Rings", in *Proc. 5th Int. Particle Accelerator Conf. (IPAC'14)*, Dresden, Germany, Jun. 2014, pp. 7–11. doi:10.18429/JACoW-IPAC2014-MOXBA01

[15] R. Abela, W. Joho, P. Marchand, S. V. Milton, and L. Z. Rivkin, "Design Considerations for a Swiss Light Source (SLS)", in *Proc. 3rd European Particle Accelerator Conf. (EPAC'92)*, Berlin, Germany, Mar. 1992, pp. 486–489.

[16] L. Emery and M. Borland, "Possible Long-Term Improvements to the Advanced Photon Source", in *Proc. 20th Particle Accelerator Conf. (PAC'03)*, Portland, OR, USA, May 2003, paper TOPA014, pp. 256–258.

[17] Haisheng Xu, Zhe Duan, Na Wang, and Gang Xu, "Mitigation of collective instability in the transient process after swap-out injection by RF modulation", *Nucl. Instrum. Methods Phys. Res. A*, vol. 986, p. 164658, 2021. doi:10.1016/j.nima.2020.164658

[18] M. Borland, T. G. Berenc, L. Emery, and R. R. Lindberg, "Simultaneous Simulation of Multi-particle and Multi-bunch Collective Effects for the Aps Ultra-low Emittance Upgrade", in *Proc. 12th Int. Computational Accelerator Physics Conf. (ICAP'15)*, Shanghai, China, Oct. 2015, pp. 61–65. doi:10.18429/JACoW-ICAP2015-WEBJI1

[19] R. R. Lindberg, M. Borland, and A. Blednykh, "Collective Effects at Injection for the APS-U MBA Lattice", in *Proc. North American Particle Accelerator Conf. (NAPAC'16)*, Chicago, IL, USA, Oct. 2016, pp. 901–903. doi:10.18429/JACoW-NAPAC2016-WEPOB08

[20] S. Y. Lee, *Accelerator Physics 3rd ed.*, Singapore: World Scientific, 2012.

[21] M. Borland, "elegant: A flexible SDDS-compliant code for accelerator simulation", Argonne National Laboratory, Illinois, United States, Rep. LS-287, Sep. 2000.