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NON-LINEAR VARIATION OF THE BETA-BEATING MEASURED FROM AMPLITUDE

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Abstract

Accelerator physics needs advanced modeling and simulation techniques, for beam stability studies but also for the measurement of beam parameter like the Twiss parameters. A deeper understanding of magnetic fields non-linearities effects will greatly help in the improvement of future circular colliders design, performance and diagnostics. This paper studies the variation of the β-beating with the action of the particle generated by non-linear Resonance Driving Terms, both from a theoretical and an experimental point of view.

INTRODUCTION

In this paper, we study the possibility to use a new beam based observable in order to localize and quantify non-linear magnetic field errors in a circular accelerator. This observable is the variation of the beta-beating with the particle actions when it is measured from amplitude. First, the theory is derived in presence of octupole field errors. Then, a possible machine octupole configuration in the LHC at injection energy, is presented in order to generate a measurable amplitude beta-beating.

In fact, data taken during machine development on the LHC revealed a dependence of the measured beta-beating on the particle action similar to that of the BPM noise (against the oscillation amplitude), hence casting doubt on the real origin of the amplitude-dependent beta-beating observed in Fig. 1. Data correspond to different machine conditions: with uncorrected first order amplitude detuning (called sb4 in Fig. 1); with a quite high value of the dodecapole corrector to generate second order detuning (called sb6); and without dodecapole corrector strength and with corrected first order detuning (called nob). In all the cases, the variation of the RMS beta-beating is of the order of 1-2 % with similar parabolic decrease with the amplitude.

THEORY

In order to express the variation of the measured beta-beating with the action, we use the Normal Forms formalism described in Ref. [1–3]. The general expression of the complex Phase-Space Courant-Snyder variables at a position (b) and after N revolutions, are:

$$h^{(b)}_{x,-} = \sqrt{2I_x}e^{i[\mu_x N+\mu^{(b)}_{x,0}]}\left[1-2i\sum_{jklm} jf^{(b)}_{jklm}2I_x^{\frac{j+k}{2}-1}\times\right.$$
$$\left.\times 2I_y^{\frac{l+m}{2}}e^{i[(k-j)(\mu_x N+\mu^{(b)}_{x,0})+(m-l)(\mu_y N+\mu^{(b)}_{y,0})]}\right], \quad (1)$$

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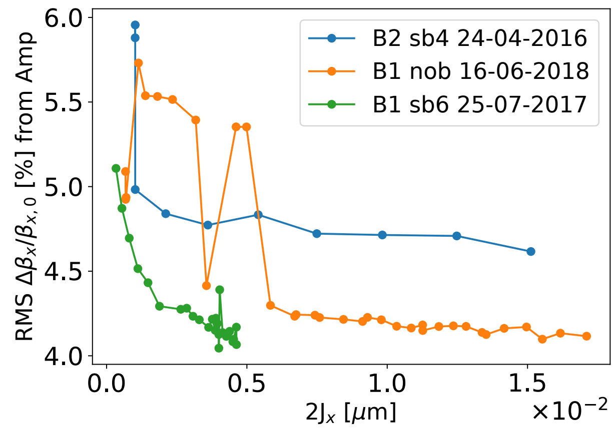

Figure 1: Variation of the Horizontal RMS beta-beating computed from the spectral line amplitude for different actions on horizontal axis, for different LHC Machine Development data.

with the particle action $2I_u$, and $\mu^{(b)}_{u,0}$ the initial phase at the observation position (b) (similarly for the vertical axis). Since the term $h_{x,\pm}$ is a sum of exponential functions, it is transformed into distinct Dirac terms in the Fourier space, i.e. spectral lines. We note $H^{(b)}_{h\pm}(n_x,n_y)$ the Fourier Transform of respectively $h^{(b)}_{x,\pm}$ at a frequency of $n_xQx+n_yQ_y$. Since the particle position in the normalized space is equal to $\tilde{x}=(h^{(b)}_{x,+}+h^{(b)}_{x,-})/2$, and being their Fourier transforms:

$$H^{(b)}_{h-}(1,0) = \sqrt{2I_x}e^{i\mu^{(b)}_{x,0}} \quad (2)$$

$$\begin{aligned} H^{(b)}_{h-}(-1,0) &= -2i\sqrt{2I_x}e^{-i\mu^{(b)}_{x,0}}\sum_{j,l\geqslant 0}(j+2)f^{(b)}_{(j+2)jll}2I_x^j 2I_y^l \\ &\approx -2i\sqrt{2I_x}e^{-i\mu^{(b)}_{x,0}}\left[3f^{(b)}_{3100}2I_x+2f^{(b)}_{2011}2I_y\right], \end{aligned} \quad (3)$$

it follows that:

$$\begin{aligned} 2H^{(b)}_{\tilde{x}}(1,0) &= H^{(b)}_{h-}(1,0)+\overline{H^{(b)}_{h-}(-1,0)} \\ &= \sqrt{2I_x}e^{i\mu_x(b)}\left[1+6i\overline{f^{(b)}_{3100}}2I_x+4i\overline{f^{(b)}_{2011}}2I_y\right]. \end{aligned} \quad (4)$$

In order to measure the beta-beating, one of the methods consists in using the amplitude of the spectral line $H^{(b)}_{\tilde{x}}(1,0)$ and dividing it by the action $2J_x$, previously estimated using another method. For linear optics $2J_x$ is equivalent to $2I_x$, in presence of non-linear perturbation this might not necessarily be the case. Suppose $2J_x=2I_x$ and let's decompose the Resonance driving terms (RDT) $f^{(b)}_{jklm}$ into polar coordinates, with $q^{(b)}_{jklm}$ its phase and $a^{(b)}_{jklm}$ its amplitude, in order to find the expression of the amplitude of the spectral line. Using

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$r = jklm$, it can be demonstrated that:

$$\begin{aligned}\left|1 + 2i\sum_r a_r e^{-iq_r}\right|^2 &= 1 + 4\sum_r a_r \sin(q_r) + 4\sum_r a_r^2 \\ &\quad +8\sum_{r<r'} a_r a_{r'} \cos(q_r - q_{r'}) \qquad (5)\\ &= 1 + 4\sum_r \Im\{f_r\} + 4\sum_r a_r^2 \\ &\quad +8\sum_{r<r'} \Re\{f_r \overline{f_{r'}}\}. \qquad (6)\end{aligned}$$

Thus, applied to Eq. (4), the Octupolar contributions to the spectral line amplitude in the normalised space follows:

$$\begin{aligned}|2H_{\tilde{x}}^{(b)}(1,0)|^2 &= 2I_x\Big[1 + 12a_{3100}^{(b)}\sin(q_{3100}^{(b)})(2I_x) \\ &\quad +8a_{2011}^{(b)}\sin(q_{2011}^{(b)})(2I_y) + 36a_{3100}^{(b)2}(2I_x)^2 \\ &\quad +16a_{2011}^{(b)2}(2I_y)^2 + 48a_{3100}^{(b)}a_{2011}^{(b)}\times \\ &\quad \times\cos(q_{3100}^{(b)} - q_{2011}^{(b)})(2I_x)(2I_y)\Big] \qquad (7)\\ &= 2I_x\Xi_x^{(b)}(2I_x, 2I_y) \qquad (8)\\ |2V_{\tilde{y}}^{(b)}(0,1)|^2 &= 2I_y\Big[1 + 12a_{0031}^{(b)}\sin(q_{0031}^{(b)})(2I_y) \\ &\quad +8a_{1120}^{(b)}\sin(q_{1120}^{(b)})(2I_x) + 36a_{0031}^{(b)2}(2I_y)^2 \\ &\quad +16a_{1120}^{(b)2}(2I_x)^2 + 48a_{0031}^{(b)}a_{1120}^{(b)}\times \\ &\quad \times\cos(q_{0031}^{(b)} - q_{1120}^{(b)})(2I_x)(2I_y)\Big] \qquad (9)\\ &= 2I_y\Xi_y^{(b)}(2I_x, 2I_y), \qquad (10)\end{aligned}$$

where $\Xi_x^{(b)}(2I_x, 2I_y)$ and $\Xi_y^{(b)}(2I_x, 2I_y)$ are the new expressions of the beta-beating, which depend on the particle actions. Obviously, the expression is not enough in this form. Octupoles also excite higher Resonance Driving terms, such as dodecapole one, and the impact of the action-independent beta-beating term from the b_2 is not considered in this equation [4, 5].

MACHINE CONFIGURATION

Here, the aim is to increase on purpose the measured amplitude beta-beating (ABB) using the octupoles installed in the LHC machine, without first order amplitude detuning. Given the BPM noise level discussed previously, 5% horizontal amplitude beta-beating at 0.01 m is used as target value. To this purpose, we solve the system $\vec{h} = \mathbf{A}\vec{K}$ where:

- $\vec{h}$: a vector of target Hamiltonian contributions which act on Amplitude detuning and RDTs (at a reference BPM (b)), i.e. $(h_{2200}, h_{1111}, h_{0022}, \Re(f_{3100}^{(b)}), \Im(f_{3100}^{(b)}), \dots)$;
- $\mathbf{A}$: a matrix of respectively Amplitude detuning and RDTs coefficients between the source of the b_4 (octupole correctors o_i) and the BPM (b);
- $\vec{K}$: a vector of octupole o_i integrated strengths.

In $\vec{h}$, we fix $h_{2200}, h_{1111}, h_{0022}$ to be zeros. The terms $\Im(f_{3100}^{(b)})$ is such that it generates the 5 % horizontal amplitude beta-beating, using the expressions for Ξ_u in Eq. (8). The other terms can be either fixed to zeros or free parameters. As the target values of $\vec{h}$ are fixed at a Reference BPM, the system has a unique solution.

In the LHC, there are three types of octupoles: the main octupoles in the Arc cells used for Landau damping (named MO); the octupole correctors attached to every-other dipole in the Arc cells (named MCO); and the octupole correctors in the Interaction Regions (named MCOX). We start considering the octupole correctors (MCOX) in the Interaction regions (IR1, IR2, IR5 and IR8) only.

In this case, called HABB case 1, the vector $\vec{h}$ is composed of:

- h_{2200}, h_{1111} and h_{0022} equal to zeros such that they do not generate direct or cross amplitude detuning;
- the RDT term $f_{3100}^{(b)}$ with its real and imaginary parts respectively equal to zeros and $5/12\,\mathrm{m}^{-1}$ such that the direct horizontal amplitude beta-beating increases by 5 % when the action reaches 0.01 m;
- the RDT term $f_{0031}^{(b)}$ is also set to zero such that the direct vertical amplitude beta-beating is trivial.

All the other RDTs are unconstrained in the inversion of $\vec{h} = \mathbf{A}\vec{K}$. The reference BPM, called “BPM.34R8.B1”, is at the position S = 18 325.04 m. It is located near the middle of the arc between the IP8 and IP1. Figure 2 shows only the values of the RDTs related to the direct beta-beating when the action increases in the x-plane. Those values are shown for each BPM around the LHC and the reference BPM is marked in red. The vector $\vec{h}$ has size 7 (3 for Direct and Cross Amplitude Detuning terms and 2×2 for the RDTs terms), with the 7 octupole correctors in the IRs, it gives an invertible system. Their respective integrated strengths are reported in Table 1.

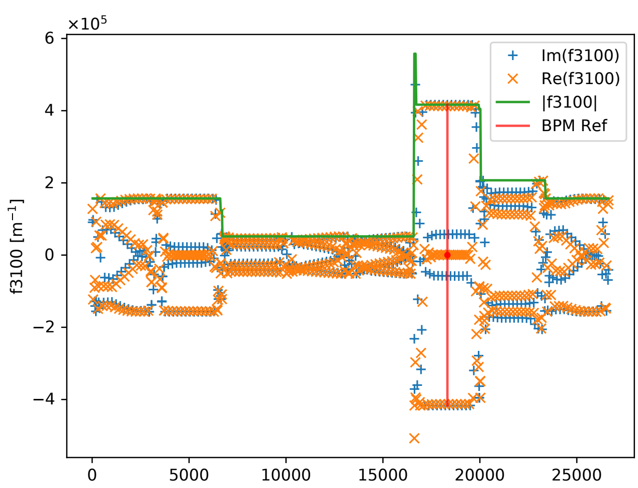

Figure 2: Prediction for case 1 of the values of the RDTs f_{3100} at all the LHC BPM positions. The reference BPM “BPM.34R8.B1” (S= 18 325.04 m) is marked in red.

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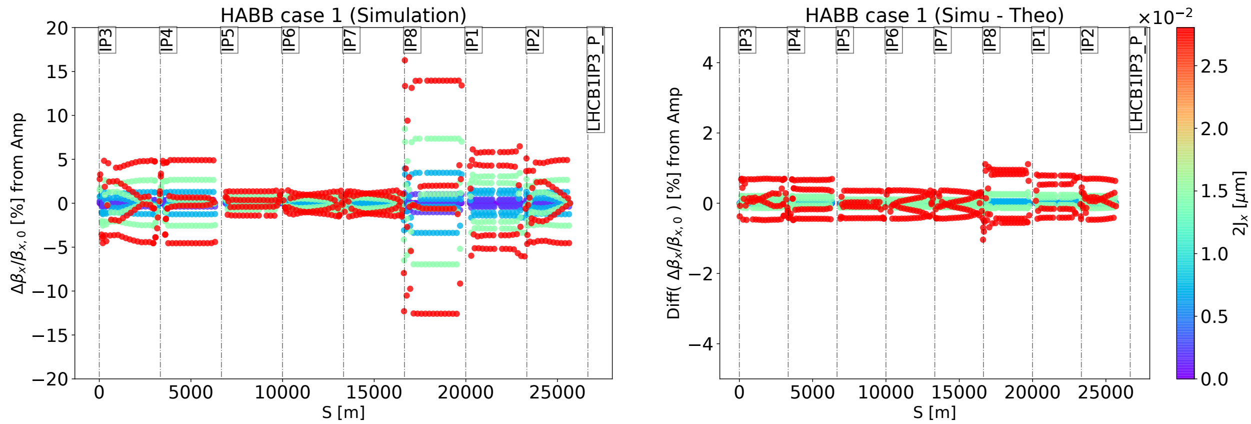

Figure 3: Direct ABB from spectral line amplitude. On the left are the results of the simulations, and on the right the difference between the simulations and Ξ_x. The action (noted $2J_x$) is the average over BPMs measured from spectral line amplitude.

Table 1: Integrated Strength (K in m^{-3}) of Octupole Correctors

MCOX3.L5	MCOX3.R5	MCOX3.L8	
1739.11	-207.69	1468.17	
MCOX3.R8	MCOX3.L1	MCOX3.R1	MCOX3.R2
-2142.16	-148.01	-1384.10	680.42

Figure 3 shows how the horizontal beta-beating vary along the LHC when the horizontal action increases using the configuration of octupolar correctors mentioned before (see Table 1). The tracking simulations where made using SixTrack and the particle positions at each BPMs are analysed with the BetaBeat.src code, an analysis code developed at the CERN (Ref. [6]). The color of the dots corresponds to the particle actions given by BetaBeat.src. As the action is computed using a linear approximation, so it is noted as $2J_x$.

With this octupole configuration, the model and the simulation agree pretty well for the direct ABB ($\Delta\beta/\beta_x$) even at high action. Three regions of the LHC have visible different behaviour (corresponding to three direct ABB values): from Interaction Point IP8 to IP1, from IP1 to IP5 and from IP5 to IP8. The first region has strongest direct ABB, the smallest value is in the third region. It is interesting to note that IP1, IP5 and IP8 are the regions where the strongest correctors are located.

The simulated cross ABB ($\Delta\beta_y/\beta_y$) also agrees well with the theory for low action, but a discrepancy appears at high action (see Ref. [5]). The three regions observed previously, are not clearly defined here. But it can still be noted that the cross ABB is smaller between IP1 and IP2. As a reminder, f_{2011} is not minimized in this case.

As the beta-beating measured from the phase is based on a different approximation, the difference between those two methods could at least indicate the approximated location of the strongest source of non-linearities.

CONCLUSION

This paper presents the expression of the beta-beating varying with the particle action, when measurement is computed from the spectral line amplitude. This expression is developed for normal octupoles and it is compared to tracking simulations using the LHC optics at injection energy. The octupole correctors strengths are selected such that they do not generate first order direct or cross amplitude detuning and such that the beta-beating increases by a least 5 % at one specific BPM, when the action increases by 0.01 m on the horizontal axis. One possible configuration, using the octupole correctors in the Interaction Region only, is presented in this paper.

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