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ON NONLINEAR ELECTRON BEAM DYNAMICS IN A PLASMA ENVIRONMENT

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Abstract

The nonlinear dynamics of an electron beam propagating in a low-density plasma is investigated. The beam envelope equation obtained analytically with a model approximation close to the Kapchinsky-Vladimirsky model is studied, the case of continuous beam with axial symmetry being considered. Solutions of the envelope equation are presented for various initial beam parameters.

INTRODUCTION

Numerous tasks of accelerator physics require careful study of an electron beam propagation in a plasma environment, among them traditional accelerators as well as novel machines based on plasma-wakefield or hybrid methods of acceleration, some types of plasma and ion sources, some classes of generators of electromagnetic radiation. Hardness of such a study is caused by the fact that in general the behavior of a charged particle beam in a plasma is significantly nonlinear, and the beam propagation depends on a lot of initial beam and plasma parameters (see, for instance [1]).

As a rule nonlinear dynamics of a beam in a plasma is studied by means of the methods of numerical modeling, but it is an attractive way to use analytical methods that can provide the best understanding of the physical processes during the beam particle propagation in complicated beam-plasma system.

The analytical model first proposed in [2, 3] is under development now. This model describes an electron beam dynamics in a rarefied plasma in approximation of a quasi-neutral regime. The model was first developed for the analysis of relativistic electron beam behavior in the nodes of conventional accelerators, but it is now extended for the application in plasma wake-field accelerators too. It is also applicable for the research and development of ion and plasma sources based on a beam-plasma discharge. Note here, that similar analytical approach was successfully applied while the dynamics of a ribbon relativistic electron beam in a rarefied plasma was investigated [4].

The character of the beam interaction with a rarefied plasma depends on the relations between the densities of the beam and plasma background, on the energy of the beam, the beam geometry, the beam phase configuration.

Here we consider the case of the beam current before the Alfven limit. Mathematical model [2] is based on the method of Vlasov equation solution which represents the kinetic distribution function as a function of integrals of particle motion. In this report modified model [3] is used to study the conditions of the beam transport. We would like to mention here that this method (or method of envelopes) for the description of the beams with essential own fields was first proposed by I. M. Kapchinsky and V.V. Vladimirsky [5], and now this model (KV-model) and its modifications are successfully applied in various tasks [6-13].

MODEL EQUATION FOR THE BEAM RADIUS AND ITS SOLUTIONS

We will consider a quasi-neutral regime of an electron beam propagation in a low-density plasma here. In approximation of a continuous beam with axial symmetry and uniform density of the beam particles over the beam cross-section the following equation for the transverse oscillations of the beam particle can be written:

$$\frac{d\vec{q}}{dz} = -\omega^2(z)\vec{r},$$

where

$\vec{q} = \frac{\vec{r}'}{\sqrt{1+\vec{r}'^2}}$, $\vec{r}' = \frac{d\vec{r}}{dz}$, $\vec{r}$ - the radius-vector of the beam particle position in the coordinate system, connected with the beam cross-section, z - longitudinal coordinate, $\omega^2(z) = \frac{i}{R^2}$, R - the radius of the beam, $i = \frac{J}{J_A}$, J - full beam current, J_A - the Alfven current, $J_A = \frac{mc^3}{e}\gamma_0\beta_0$, γ_0 and β_0 - the beam relativistic factors, m and e - the mass and the charge of the electron respectively, c - the light velocity.

In the case of $i << 1$ one can obtain $\vec{r}' << 1$. It corresponds to the oscillations of the particles characterized by variable frequency and weak nonlinearity.

Another case characterized by $i \to 1$ corresponds to strong nonlinearity of the particle oscillations.

We can write the beam current density j_z as follows:

$$j_z = -ec\kappa \int \frac{cp_z}{H} F dp.$$

Here p_z - longitudinal component of the particle momentum, p is full momentum of the particle, H is full energy

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of the particle, F is kinetic distribution function, κ - the normalization constant.

If we suppose that all the particles over the beam have the same full energy:

$$F = f\delta(H - H_0),$$

where δ is delta-function, and assuming that for all the particle the relation $p_z > 0$ is fulfilled, we can find the invariant of the Eq. (1) in the next form:

$$I = \frac{\left(\dot{\vec{q}}u - \vec{q}\dot{u}\right)^2}{\varepsilon^2} + \frac{\vec{q}^{\,2}}{2},$$

where ε - the constant, corresponding to the beam transverse emittance, *u(z)* - the function, corresponding to the beam radius [3]. Note here that at first the invariant *I* was used in [2]. Assuming that the distribution function *f* looks as

$$f = \delta(I - 1),$$

for the radius dependence on longitudinal coordinate we can find the following equation:

$$\sqrt{1 + \frac{R'^2}{2} + \frac{\varepsilon^2}{2R^2}}\,\frac{d}{dz}\left(R'\sqrt{1 + \frac{R'^2}{2} + \frac{\varepsilon^2}{2R^2}}\right) + \\ + \frac{i}{R\sqrt{1 + \frac{R'^2}{2} + \frac{\varepsilon^2}{2R^2}}} = \frac{\varepsilon^2}{R^3}. \qquad (1)$$

Equation (1) is valid for the uniform axisymmetric relativistic electron beam in a rarefied plasma up to critical current corresponding to the beam pinch. All the beam parameters in the equation are dimensionless. The Eq. (1) is essentially nonlinear and should be integrated numerically. Solutions of Eq. 1 are obtained with the help of MATLAB. At Fig. 1 the part of the code used for the numerical integration is given.

At Figs. 2 and 3 the results of the numerical integration of envelope equation are given. The dependence of the radius is found for various initial parameters of the beam.

At Fig. 2 the radius R is shown versus longitudinal coordinate z for the case of the dimensionless beam current value i=1. The values of dimensionless transverse emittance of the beam are accepted equal to 0.0, 0.5 and 1.0. Initial dimensionless radius value R_0 is chosen equal to 1. As one can see from Fig. 2, the nonzero beam transverse emittance significantly affects the character of the beam radius behavior.

```
f1=@(x,y,yy) yy;
f2=@(x,y,yy)((u^2)*(1+(1/2)*yy^2)-
2*I*y^2/((4+2*yy^2+2*u^2/(y^2))^(1/2)))/(y*(y^2+y
^2*yy^2+(u^2)/2));
a=c1;
b=c2;
n=c3
h=(b-a)/n;
x(1)=a;
y(1)=c4;
yy(1)=0;
for i=1:n
  m1=h*f1(x(i),y(i),yy(i));
  k1=h*f2(x(i),y(i),yy(i));
  m2=h*f1(x(i)+0.5*h,y(i)+0.5*m1,yy(i)+0.5*k1);
  k2=h*f2(x(i)+0.5*h,y(i)+0.5*m1,yy(i)+0.5*k1);
  m3=h*f1(x(i)+0.5*h,y(i)+0.5*m2,yy(i)+0.5*k1);
  k3=h*f2(x(i)+0.5*h,y(i)+0.5*m2,yy(i)+0.5*k1);
  m4=h*f1(x(i)+h,y(i)+m3,yy(i)+k3);
  k4=h*f2(x(i)+h,y(i)+m3,yy(i)+k3);
  x(i+1)=x(i)+h;
  y(i+1)=y(i)+(1/6)*(m1+2*m2+2*m3+m4);
  yy(i+1)=yy(i)+(1/6)*(k1+2*k2+2*k3+k4);
 end
```

Figure 1: Code for numerical integration of the envelope equation.

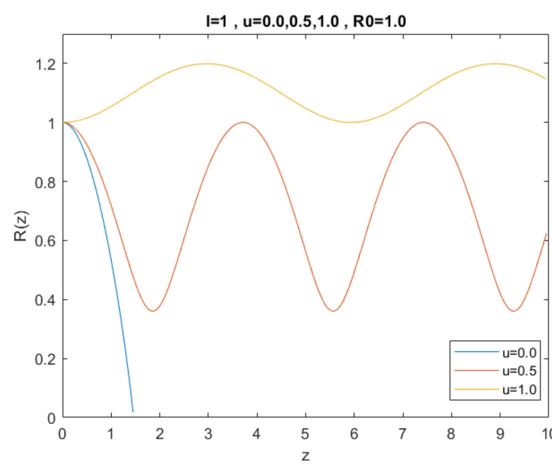

Figure 2: Dependence of REB radius on longitudinal coordinate z in the case of nonzero transverse emittance.

Figure 3 illustrates the beam dynamics for various values of the beam current in the case when the beam particles are cold (there is no transverse emittance). The beam current relatively small and is decreased from 0.1 to 0.0. The plots at Fig. 3 confirm the physical correctness of the model applied for the Eq. (1) derivation at least qualitatively.

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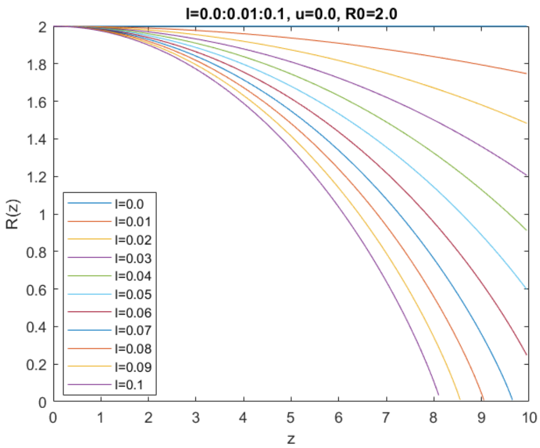

Figure 3: Dependence of REB radius on longitudinal coordinate z in the case of zero transverse emittance.

CONCLUSION

The dynamics of an electron beam in a plasma environment has essentially nonlinear character due to self-consistent fields. In the case of a beam with axial symmetry it may be described by model envelope equation, that may be applied up to the beam current equal to Alfven one. The results of the numerical integration of the envelope equation show the dependence of the beam behavior on the relations between the initial beam parameters such as beam current, beam radius and beam transverse emittance, particularly, the parameters affect the possibility of the beam transport.

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