CONCEPTUAL DESIGN OF A MULTIPLE PERIOD STAGGERED UNDULATOR

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Abstract

In staggered undulators, a ferromagnetic pole structure paired to a solenoid generates a sinusoidal field. Interest of such insertion devices has been studied for application to FEL systems in the end of the previous century. However, the concept has never been used in synchrotron radiation sources due to the undesirable magnetic effect of the solenoid on electron beam parameters in storage rings. Advent of fourth-generation low emittance light sources is foreseen to change this situation. Indeed, consequent electron beam transverse size and divergence reduction for such new storage rings give promise for a beam less sensitive to the presence of a longitudinal solenoidal field. Relating to this, a staggered concept can be an adequate design choice for short-period undulators producing high-energy photon flux. Such undulators would have a low K value a priori limiting their photon energy tunability. Considering integration of separate magnetic arrays of distinct periods in a solenoid to compose a global assembly can help suppress this possible drawback. Magnetic design and radiative performance of such an insertion device are presented.

INTRODUCTION

Typical peak field values attainable with the staggered technology are lower than those of corresponding permanent magnet devices at equal other comparative conditions (gap, period, etc.) [1, 2]. This effect is even more considerable for short periods and leads to a relatively small K value and hence low tunability through $B_p$ at a given $\lambda_p$ as can be seen in Eq. (1) [3]:

$$\lambda_{E_n} = \frac{\lambda_p}{2n\gamma^2} \left( 1 + \frac{K^2}{2} \right).$$  (1)

Equation (1) gives the emitted on-axis radiation wavelength of the n-th harmonic $\lambda_{E_n}$, where $\gamma$ is the relativistic Lorentz factor and $K \approx 0.0934B_p[T]\lambda_p[mm]$ is the deflection parameter of the undulator.

To solve this issue one can think of ways to introduce new variation parameters for the harmonic energy. A pragmatic choice of such a parameter in a fixed-gap configuration for the staggered concept can be to implement period variation. Such an approach is presented for example in [4] where a single undulator array is mechanized for period length extension/reduction at a fixed number of periods $N_p$. In this paper a different scheme for the realization of period variability in a staggered design is proposed. It is based on an assembly composed of multiple fixed-period undulators of various $\lambda_p(n)$ and $N_p(n)$ totaling $N$ in number where $n \in [0, N - 1]$ (not to confuse with the harmonic number $n$ in Eq. (1)). The arrays are positioned adjacent to one another at a distance $d$ in the positive sens of the transverse horizontal direction $Ox$, in order of ascending $\lambda_p(n)$. In this manner a global period variation on the level of the assembly is achieved in the said direction along the pole width $w_p$ of individual arrays. Figure 1 defines such an assembly in the form of a symmetric segment destined to be a unit building block for a longer assembly.

Figure 1: Top view of an upper-pole distribution of a symmetric (defined by the mid-plane $\sigma_{center}$) module made of $N = 9$ arrays. Mirror symmetry planes $\sigma_{left}$ and $\sigma_{right}$ would serve to adjoin identical neighbouring modules at either side of the array unit and thus demonstrate the modular aspect of the final global assembly.

The period range spans from 8 to 16 mm. This small-value interval extends below the rough lower boundary of periods readily accessible to more conventional undulator technologies like those based on permanent magnets. The latter are difficult to implement for $\lambda_p < 10$ mm, typically due to permanent magnet material brittleness for magnet blocks of resulting small dimensions. The major part of current such designs have $\lambda_p \geq 14$ mm. Thus, the choice of a short-period design interval highlights and exploits a main relative practical interest of the staggered concept, another one being the absence of radiation damage experienced due to the lack of permanent magnets [4].

BASIC PHYSICAL AND OPERATING PRINCIPLE OF THE STAGGERED CONCEPT

Figure 2 defines the coordinate system used throughout the text and summarizes schematically the main design layout of a staggered undulator. An array of period $\lambda_p$ and magnetic

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Figure 2: Sketch of a staggered undulator design showing a longitudinal cut of the basic components. The solenoid bore houses the arrangement of poles used to deflect the longitudinal solenoidal field to establish a periodic on-axis undulator field \( B_z \) in the gap \( g \) visualized by magnetic flux lines.

An approximate short-period analytical expression for the on-axis undulator peak field \( B_p \), is given by [2,4,9]:

\[
B_p = \frac{2 B_s}{\sinh \left( \frac{\pi g}{\lambda_p} \right)} \left( \sin \left( \frac{\pi f}{\lambda_p} \right) \right). \tag{2}
\]

In Eq. (2) \( f \) is the ratio of pole spacing to undulator period (Fig. 2), \( f = \frac{d_p}{\lambda_p} \), linked to the ratio \( \alpha = \frac{l_p}{\lambda_p} \) of pole length \( l_p \) to period through \( f = 1 - \alpha \), and \( B_s \) is the solenoid field at the center of the solenoid. Variation of \( B_z \) through \( j \) to act on \( B_p \) for other parameters fixed, namely gap \( g \), is the main conceptual means to tune the harmonic energy of the emitted radiation in a staggered undulator [2,9], as alluded to by Eqs. (1) and (2).

**SHORT-PERIOD MAGNETOSTATIC OPERATIONAL LIMITS**

Table 1 presents results on optimized peak field \( B_p \) values and corresponding central solenoid field \( B_s \) for optimized pole width \( w_p = 13.63 \) mm and longitudinal geometric ratio \( \alpha \approx 0.57 \) at different periods \( \lambda_p \). Underlying calculations are done with the Radia 3D magnetostatic code [10] at a fixed magnetic gap \( g = 4 \) mm and pole height \( h_p = 20 \) mm. Simulative setup conditions are those of a pole array of \( N_p = 20 \), centered in the homogenous-field region of a solenoid of length \( L_{s} = 2.5 \) m \( \gg N_p \lambda_p \) and radii \( r_1 = 25 \) mm, \( r_2 = 30 \) mm. A simple cylindrical model of a solenoid is used. Chosen pole material is 49Fe-49Co-2V Vanadium Permendur of saturation flux density \( B_{sat} = 2.3-2.4 \) T.

<table>
<thead>
<tr>
<th>( \lambda_p [\text{mm}] )</th>
<th>( B_p [\text{T}] )</th>
<th>( B_s [\text{T}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.8724</td>
<td>0.4100</td>
</tr>
<tr>
<td>9</td>
<td>0.8400</td>
<td>0.4854</td>
</tr>
<tr>
<td>10</td>
<td>0.8206</td>
<td>0.5556</td>
</tr>
<tr>
<td>11</td>
<td>0.8136</td>
<td>0.6203</td>
</tr>
<tr>
<td>12</td>
<td>0.7710</td>
<td>0.6802</td>
</tr>
<tr>
<td>13</td>
<td>0.7502</td>
<td>0.7350</td>
</tr>
<tr>
<td>14</td>
<td>0.7304</td>
<td>0.7852</td>
</tr>
<tr>
<td>15</td>
<td>0.7082</td>
<td>0.8312</td>
</tr>
<tr>
<td>16</td>
<td>0.6954</td>
<td>0.8733</td>
</tr>
</tbody>
</table>

**MAIN DESIGN CRITERIUM FOR CONTINUOUS ENERGY TUNABILITY**

A particular period interval is ultimately determined by a dedicated parametrized design criterium. The latter is the functionally essential part of a valid set of main geometric specifications for an example module like the one shown in Fig. 1. The criterium reflects by construction magnetostatic results on staggered peak field operational limits obtained beforehand. It is derived from the setting of the requirement for continuity on the harmonic energy tunability interval in the case of the fundamental harmonic.

The considered perspective of period variation by switching among undulator arrays of different periods \( \lambda_p \) is practically realized through a corresponding discrete period increment value \( \delta \lambda_p(n) = \lambda_p(n + 1) - \lambda_p(n) \geq 0 \) between neighbouring arrays. In such an operational scheme the aforementioned researched property of continuity for the energy variation interval translates into a necessary overlapping of individual array tuning curves in a more or less controlled manner. This would allow an expanded effective global continuous tuning range to be constituted for the assembly considered as a whole. In the hypothesis of independent magnetostatic operation of separate arrays one can formulate a design choice condition resulting in an interval of allowed switching step values \( \delta \lambda_p(n) \) for each \( n \). This is done in terms of a criterium function \( C \) depending on the corresponding period couple \( (\lambda_p(n), \lambda_p(n + 1)) \):

\[
\delta \lambda_p(n) \leq C(\gamma B_p, \lambda_p(n), \lambda_p(n + 1)) \tag{3}
\]

In Eq. (3) \( \gamma B_p \) is assumed identical for all arrays and is defined by \( B_{p,\text{min}}(n) = \gamma B_p B_{p,\text{max}}(n) \) where \( B_{p,\text{min}}(n) \) is the lower boundary of the peak field variation range of array \( n \) and \( B_{p,\text{max}}(n) \) is the upper one, practically corresponding to the optimized value of \( B_p(n) \). The function \( C \) in the case of the first harmonic \( \lambda_{E_1} \) of prime interest to a staggered
undulator, is precisely given by:
\[
C(\gamma B_p, \lambda_p(n), \lambda_p(n + 1)) = \frac{e^2 B^2_p}{8 \pi^2 m_e^2 c^2} \times \\
\left( \lambda^2_p(n) \exp \left( - \frac{2 \pi g}{\lambda_p(n)} \right) - \gamma^2 B^2_p \lambda^3_p(n + 1) \exp \left( - \frac{2 \pi g}{\lambda_p(n + 1)} \right) \right) \tag{4}
\]

In Eq. (4) \( B_0 \) [T] is an exponential gap-over-period fit constant over the period interval of interest, \( m_e \) is the electron rest mass and \( c \) is the speed of light. Successive application of Eq. (3) and a straightforward geometric condition for the modular symmetry illustrated in Fig. 1 guarantees the strict inequality in Eq. (3) for all \( n \).

Table 2: Resulting Individual Array \( \lambda_p(n) \) and \( N_p(n) \) for a Modular Assembly Yielded by the First Harmonic Energy Tunability Adjustment Criterion in the Period Interval [8 mm, 16 mm]

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \lambda_p(n) ) [mm]</th>
<th>( N_p(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
<td>29</td>
</tr>
<tr>
<td>1</td>
<td>8.28</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>8.57</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>8.89</td>
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<td>6</td>
<td>10.91</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>12.63</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>

For initial parameter values \( \gamma B_p = 0.091 \), \( N = 9 \), \( \lambda_p(0) = 8 \) mm, \( N_p(0) = 29 \), \( B_0 = 1.9329 \) T (calculated with the data in Table 1) the sequence of \( \lambda_p(n) \) and \( N_p(n) \) given in Table 2 is yielded.

Figure 3 shows on-axis tuning curves for flux and brilliance of the array module specified in Table 2 calculated with the SRW code [11]. The results are acquired for array lengths \( L \approx 2 \) m and the Extremely Brilliant Source (EBS) parameter beam given in Table 3 [12]. A resulting continuously covered global energy tuning range is observed with slight overlapping of the individual-array ranges illustrating a successful application of the specification method proposed.

Figure 4 shows corresponding power density plots for the minimal-period array of \( \lambda_p(0) = 8 \) mm from Table 2 for an array length \( L \approx 2 \) m taking into account a main field component \( B_p = 0.3740 \) T.

**CONCLUSION**

A basic conceptual approach for the specification of a continuously tunable variable-period modular undulator assembly of interest to staggered array designs is presented. Magneto-static as well as radiative performance of such an assembly with an ultra-low horizontal emittance fourth-generation storage ring beam is investigated in respective terms of field optimization and flux, brilliance and power density. The study is focused on short undulator periods for improved first harmonic operation, reinforcing interest in the concept.

**REFERENCES**


