

# A SUPERCONDUCTING UNDULATOR FOR CompactLight: RESISTIVE WALL WAKEFIELD ANALYSIS

K. Marinov<sup>†</sup>, STFC Daresbury Laboratory, ASTeC, Daresbury, Warrington, UK

## Abstract

The CompactLight project is an advanced X-ray FEL light source, with high-frequency, high-gradient linacs and compact undulators. Lower electron energies give higher energy efficiency and a smaller environmental footprint. The extremely short bunch lengths (few fs) and narrow undulator gaps (4 mm) drastically increase the impact of resistive wall wakefields on the lasing process. The longitudinal resistive wall wakefield impedance is calculated in accordance with anomalous skin effect (ASE) theory. The dependence of the electron energy loss factor and the energy spread of the bunch on the residual resistivity ratio (RRR) for both copper and aluminium is much stronger for long (100 fs) than for ultra-short (6 fs) bunches. This is due to a known property of the longitudinal resistive wakefield - the field acting on a single particle traversing a resistive vessel does not depend on the conductivity of the vessel. The wakefields generated by the ultra-short bunch are already close to that of a single-particle regime and this leads to interesting consequences which are discussed in the present work.

## INTRODUCTION

The anomalous skin effect has been experimentally discovered by Heinz London in 1940. It was noticed that the electric properties of a cold metallic sample at microwave frequencies are drastically different from its DC conductivity properties. In particular the variation of the surface resistance of the sample (a quantity used in waveguide attenuation and resonator Q-factor calculations) with temperature displayed an “anomalous” behaviour. As the temperature was gradually lowered the surface resistance reached a lower limit followed by no further variation at even lower temperatures. In contrast, the bulk resistivity of the sample was found to decrease continuously with temperature even beyond the point where the surface resistance had stopped changing. This behaviour is impossible to explain if the validity of Ohm’s law is maintained. The latter predicts that the surface resistance is proportional to the square root of the resistivity. Brian Pippard [1, 2] pointed out that at high frequencies and low temperatures the applicability of Ohm’s law becomes questionable. Indeed, as the sample temperature is lowered the main electron scattering mechanism in “pure” metals (lattice phonons) gradually disappear and this increases the mean free path of the electrons and, consequently, increases the DC conductivity of the sample. The latter in turn decreases the skin depth. Ohm’s law breaks down when the mean free path of the electron exceeds the skin depth. Therefore, Pippard argued, a new theory of charge transport in metals, valid at arbitrary

<sup>†</sup> kiril.marinov@stfc.ac.uk

temperatures and frequencies was needed. The latter was created in 1948 by Reuter and Sondheimer [3, 4].

The application of this theory to the calculation of resistive wall impedances and wakes in the cold beam pipes of superconducting undulators can be found in the literature [5, 6]. The main parameter of the theory is the ratio between the conductivity at the cryogenic temperature and that at room temperature. The latter ratio is known as the residual resistivity ratio (RRR).

CompactLight [7] aims to produce a design of a hard X-ray FEL that can outperform today’s stay-of-the-art lasers. It will rely on high-gradient accelerator structures and will benefit from lower electron energies thus producing a smaller environmental footprint. The latter will be achieved through the installation of efficient superconducting undulators. Electron bunches as short as 6 fs have been considered in combination with undulator vessel apertures of 4 mm. Both these factors favour the generation of strong resistive wall wakes that need to be analysed and their impact on the undulator performance needs to be carefully considered. This is the aim of the present study.

## RESULTS AND DISCUSSION

In what follows undulator vessels made of copper and aluminium have been considered (see Table 1). The mathematical model of [5] and the standard definitions of a loss factor and r.m.s. energy spread have been adopted [6]. The calculations have been performed for both an ultra-short 6 fs and a more standard 100 fs Gaussian bunches. Two types of vessel geometries – a cylindrical one (“Round”) and set a two parallel plates (“Flat”) have been considered, although in practice only the cylindrical vessel is relevant. The “Flat” geometry can be seen as a limiting case of a high-ellipticity elliptical vessel as the wakes generated by the two structures closely approximate each other.

Table 1: Room-temperature Parameters of Copper (Cu) and Aluminium (Al)

Material	Conductivity [S/m]	Mean Free Path [nm]	Relaxation Time [fs]
Cu	$5.7 \times 10^7$	35.6	22.3
Al	$3.7 \times 10^7$	14.5	7.25

Figures 1 and 2 show the dependence on RRR, or, equivalently, the temperature, of the loss factor and the r.m.s. energy spread of a 6 fs Gaussian bunch. As can be seen the “Flat” vessel results in lower wakefield impact on the bunch. In addition, the aluminium slightly outperforms the copper, despite having a lower conductivity. Interestingly, the temperature, or RRR, plays only a minor role beyond

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2021). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI

small variation for aluminium at low RRR values. To understand the reason for this outcome the same calculation has been repeated for a much longer 100 fs bunch and the result is shown in Fig. 3. As can be seen there is more than a factor of two variation of the loss factor between a room temperature vessel and a cold one in accordance with what is expected intuitively.

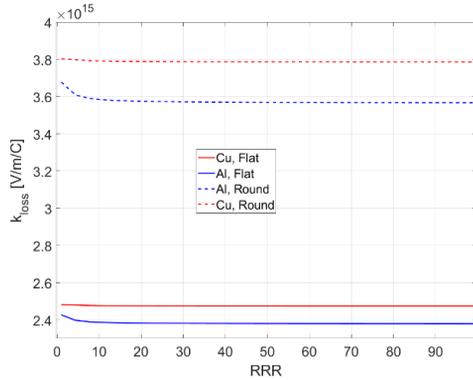


Figure 1: Loss factor  $k_{\text{loss}}$  vs the residual resistivity ratio (RRR) for an aluminium and copper undulator vessels in “Flat” (two parallel plates) and “Round” (cylinder) geometries for a 6 fs r.m.s. length Gaussian bunch.

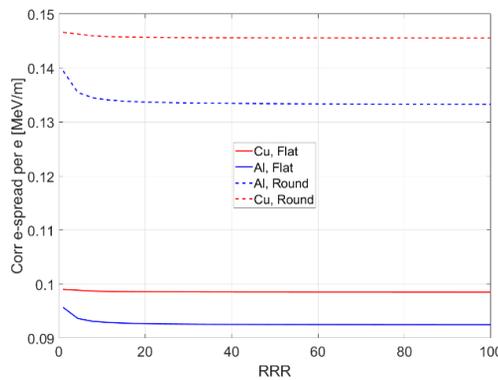


Figure 2: R.M.S. energy spread (per electron) vs the residual resistivity ratio (RRR) for an aluminium and copper undulator vessels in “Flat” (two parallel plates) and “Round” (cylinder) geometries for a 6 fs r.m.s. length Gaussian bunch.

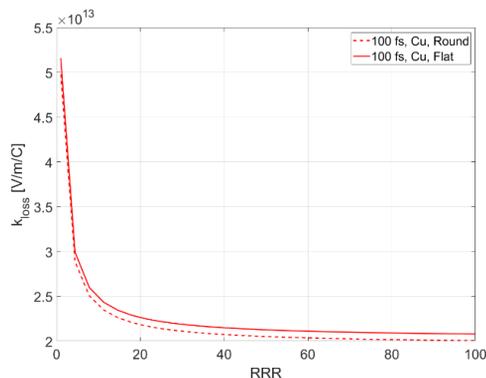


Figure 3: Loss factor  $k_{\text{loss}}$  vs the residual resistivity ratio (RRR) for a copper undulator vessels in “Flat” (two parallel plates) and “Round” (cylinder) geometries for a 100 fs r.m.s. length Gaussian bunch.

Figure 4 shows the spectrum of the 100 fs bunch (arbitrary units) plotted with the real part of the wakefield impedance of a cold vessel and a vessel at room temperature (see Eq. (26) in [6]). As Fig. 4 shows the spectrum of the 100 fs bunch is entirely confined to the frequency range (below 5 THz) where lowering the temperature lowers the real part of the impedance. This is in full accordance with the trend observed in Fig. 3. It can be seen that there is a frequency range around the impedance peak in Fig. 4 where lowering the temperature increases the impedance. In addition, beyond 10 THz lowering the temperature does not seem to have an impact at all. The ultrashort bunch has a very broad spectrum with non-zero frequency components up to 80 THz thus covering all three frequency regions in Fig. 4. It is, therefore, clear that the ultrashort bunch displays a qualitatively different behaviour. To illustrate this further Fig. 5 shows the wakepotentials (Green’s functions) of a cylindrical beam pipe in three cases: aluminium at low temperature, aluminium at room temperature and low-temperature copper (see Figs. 2 and 4 in [5]). The point-like source charge is located in the origin of the co-ordinate system. The field that this charge “sees” does not seem to depend on the temperature, conductivity and the charge transport mechanism (classical or anomalous). A formula for the loss factor of a point-like charge in a resistive vessel in the Ohmic (room-temperature) regime can be found in the literature [8] and the loss factor in this case does not depend on the conductivity. It can be concluded, therefore, that the wakes generated by the of ultra-short 6 fs bunch resemble that of a point-like charge with only weak dependence on the temperature, conductivity and charge-transport mechanism as Figs. 1 and 2 show.

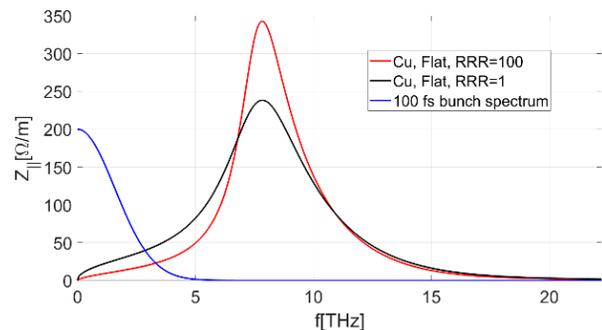


Figure 4: Real part of the longitudinal wakefield impedance of a “Flat” copper vessel (two parallel plates) at cryogenic temperature (red) and room temperature (black).

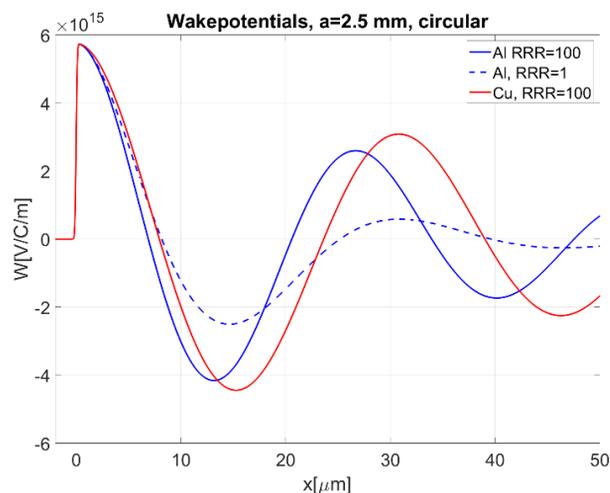


Figure 5: Wakepotentials (Green's functions of the problem) of a 2.5 mm radius cylindrical beam pipe made of aluminium at cryogenic temperature (blue line) and room temperature (dashed blue line) and of room-temperature copper pipe (red line) of the same shape and radius.

## CONCLUSION

The aim this work is to assess the impact of the resistive wall wakes in the CompactLight superconducting undulator on the FEL performance and to help devise mitigation strategies. The outcome from the FEL performance analysis is presented elsewhere. It turns out, however, that the wakes generated by the ultra-short 6 fs bunch are qualitatively different from that of the more standard bunches that are tens or hundreds of femtoseconds long and this makes the ultra-short bunch case an interesting one. The 6fs bunch is similar to a “point-like” charge: the way its wakes impact the bunch depends weakly on the vessel material, temperature and conductivity mechanism (ASE or Ohmic), as predicted by the standard theory. Naturally, this conclusion depends on the applicability of the Leontovich boundary condition that the wakefield impedance derivation is based on [5, 6, 8]. If the wakes generated by the 6 fs bunch result in a tolerable FEL performance degradation the material for the undulator vessel can be chosen from mechanical engineering considerations; aluminium alloy instead of “pure” aluminium can be used as conductivity and RRR values attainable do not seem to be critical for ultra-short bunches. In addition, reducing the bunch length further will lead to a negligible additional wakefield-induced performance degradation. Indeed, the latter is equivalent to adding beam current at frequencies where there is no wakefield response. Naturally, an alloy with very low RRR is not optimal if the same undulator is operated with much longer bunches as in the latter case higher RRR is beneficial to some extent (see Fig. 3). A room-temperature undulator with an ultrashort bunch should naturally favour NEG as a vacuum solution because the additional wakefield-induced performance degradation due to the lower conductivity of NEG would be relatively small. In contrast, in a recent study NEG coating was rejected for CLARA (Daresbury Laboratory, UK), as NEG was predicted to increase the

bunch energy spread by as much as a factor of the order of 100 [9, 10] compared to a copper vessel with no coating. The CLARA bunch length considered in that study, however, was of the order of 250 fs.

## REFERENCES

- [1] A. B. Pippard, “The surface impedance of superconductors and normal metals at high frequencies II. The anomalous skin effect in normal metals”, *Proc. R. Soc. A*, vol. 191, pp. 385-399, 1947. doi:10.1098/rspa.1947.0122
- [2] A. B. Pippard, “The surface impedance of superconductors and normal metals at high frequencies III. The relation between impedance and superconducting penetration depth”, *Proc. R. Soc. A*, vol. 191, pp. 399-415, 1947. doi:10.1098/rspa.1947.0123
- [3] G. E. H. Reuter and E. H. Sondheimer, “The theory of the anomalous skin effect in metals”, *Proc. R. Soc. A*, vol. 195, pp. 336-364, 1948. doi:10.1098/rspa.1948.0123
- [4] R. Dingle, “The anomalous skin effect and the reflectivity of metals I”, *Physica*, vol. 19, pp. 311-347, 1953. doi:10.1016/S0031-8914(53)80035-2
- [5] G. Stupakov, K. L. F. Bane, P. Emma, and B. Podobedov, “Resistive wall wakefields of short bunches at cryogenic temperatures”, *Phys. Rev. Accel. Beams*, vol. 18, p. 034402, 2015. doi:10.1103/PhysRevSTAB.18.034402
- [6] B. Podobedov, “Resistive wall wakefields in the extreme anomalous skin effect regime”, *Phys. Rev. Accel. Beams*, vol. 12, p. 044401, 2009. doi:10.1103/PhysRevSTAB.12.044401
- [7] CompactLight Project, <http://www.compactlight.eu>
- [8] B. Zotter and S. A. Kheifets, *Impedances and Wakes in High-Energy Particle Accelerator*. Singapore:World Scientific, 1998.
- [9] O. B. Malyshev *et al.*, “RF surface resistance study of non-evaporable Getter coatings”, *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 844, pp. 99-107, 2017. doi:10.1016/j.nima.2016.11.039
- [10] O. B. Malyshev, K. B. Marinov, K. J. Middleman, N. Thompson, R. Valizadeh, and P. H. Williams, “The Applicability of NEG Coated Undulator Vessels for the CLARA FEL Test Facility”, in *Proc. 8th Int. Particle Accelerator Conf. (IPAC'17)*, Copenhagen, Denmark, May 2017, pp. 3181-3183. doi:10.18429/JACoW-IPAC2017-WEPIK100