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SIMULATION STUDY OF EMITTANCE MEASUREMENT USING A GENETIC ALGORITHM FOR SPACE CHARGE DOMINATED BEAMS

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Abstract

The quadrupole scan method is one of the traditional ways to measure beam emittance in an accelerator. The required devices are simple: several quadrupole magnets and a beam profile monitor. Beam sizes are measured from the beam profile monitor with different quadrupole settings to bring the beam through its waist and then fitted to a quadratic equation to determine the Twiss parameters. Measured data from a quadrupole scan taking the beam through its waist is fitted to a quadratic equation and this allows determining the Twiss parameters. However, with increasing beam intensity, the introduced space charge will causes a deviation of the fitted emittance from its real value, making it no longer useful. In this paper, a genetic algorithm is applied to find the optimum quadrupole scan fit in space charge dominated electron beams. Results from simulations using different space charge levels are presented and scenarios identified where this method can be applied.

INTRODUCTION

Transverse emittance is a key parameter in every accelerator. Many methods can be applied to measure the emittance such as quadrupole scan [1], multi-slit [2], pepper pot [3], etc. Among these methods, the quadrupole scan is well known in the accelerator physics community due to its simplicity and widely used in many accelerators. It utilizes the existing beam optics (normally one quadrupole magnet or two) and an additional profile monitor (could be any such as scintillating screen, secondary emission monitor, synchrotron radiation monitor, optical transition radiation monitor, etc.). Such a system could be simplified as a beam going through drifts (length D) and a quadrupole (length L and strength k) as shown in Fig. 1.

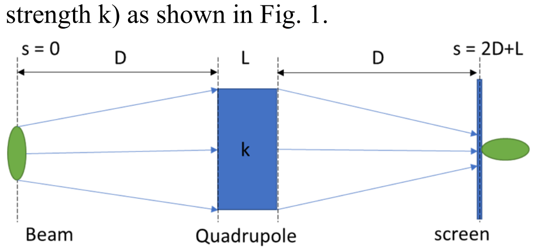

Figure 1: Illustration of a quadrupole scan system.

For such a lattice, the transfer matrix will be

$$\begin{pmatrix}1 & D\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\kL & 1\end{pmatrix}\begin{pmatrix}1 & D\\0 & 1\end{pmatrix}=\begin{pmatrix}1+2DkL & 2D\\kL & 1\end{pmatrix} \quad (1)$$

Then the beam size on the screen could be expressed as a quadratic form of the quadrupole strength k.

$$\sigma_{11}^{screen} = (4D^2L^2\sigma_{11}^{beam}) * k^2 + (4DL\sigma_{11}^{beam} + 8D^2L\sigma_{12}^{beam}) * k + (\sigma_{11}^{beam} + 4D\sigma_{12}^{beam} + 4D^2\sigma_{22}^{beam}) \quad (2)$$

When fitting the measured beam size square for the quadrupole strength in a quadratic fit, one can get the sigma elements of the beam matrix at the initial location. We rewrite the coefficients of the quadratic function as A, B, and C, where

$$A = 4D^2L^2\sigma_{11}^{beam},$$
$$B = 4Dl\sigma_{11}^{beam} + 8D^2l\sigma_{12}^{beam},$$
$$C = \sigma_{11}^{beam} + 4D\sigma_{12}^{beam} + 4D^2\sigma_{12}^{beam} \quad (3)$$

Then

$$\sigma_{11}^{beam} = \frac{A}{4D^2l^2},$$
$$\sigma_{12}^{beam} = \frac{B - 4dl\sigma_{11}^{beam}}{8D^2l},$$
$$\sigma_{22}^{beam} = \frac{C - \sigma_{11}^{beam} - 4D\sigma_{12}^{beam}}{4D^2} \quad (4)$$

And we finally can calculate the emittance as:

$$\varepsilon = \sqrt{\sigma_{11}^{beam}\sigma_{12}^{beam} - (\sigma_{12}^{beam})^2} \quad (5)$$

This method could be well applied for many beams if the space charge does not play a part. As mentioned in [4], due to the space charge effect, the measured emittance with quadrupole scan was consistently higher than multi-slit measurements and the predictions from simulation.

Earlier work [5] shows Particle Swan Optimization could be used for a solenoid scan to resolve the emittance for space charge dominated beam. In this paper, we will discuss the potential effect of space charge on the quadrupole scan measurement and propose a method to measure the emittance with quadrupole scan data for a space charge dominated beam.

METHOD

For a beam with linear space charge force, the particle trajectory in the transverse direction is as follow equations.

$$x'' + k_x x - \frac{2K}{X(X+Y)}x = 0 \quad (6)$$
$$y'' + k_y y - \frac{2K}{Y(X+Y)}y = 0 \quad (7)$$

Where K is the dimensionless generalized perveance [6] which indicate a space charge level, and k_x, k_y are the external force strength in the x and y direction separately. The

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trajectories are coupled with the self-field terms which are related to the two corresponding equations for the RMS beam envelopes X(s), Y(s) which are [6]

$$X'' + k_x X - \frac{2K}{X+Y} - \frac{\varepsilon_x^2}{X^3} = 0 \quad (8)$$

$$Y'' + k_y Y - \frac{2K}{X+Y} - \frac{\varepsilon_x^2}{Y^3} = 0 \quad (9)$$

In these equations, the linear self-field is assumed to be a (K-V distribution) when deriving the space charge term, but it can be extended to other distributions in the RMS sense of equivalent beams [7, 8]. Because the trajectory is coupled with the RMS envelopes, the transfer matrix will depend on the size of the beam and there is no longer a simple quadratic relationship between the beam size square and the focusing strength. The error from a simple quadrupole scan fitting depends on the space charge force or the value of the generalized perveance K.

Since the function to describe the motion is given, the quadrupole scan data is still valuable to reconstruct the initial condition about the envelope and the edge emittance. Taking the same lattice as shown in Fig. 1 into consideration, this is an inverse problem to solve the unknown beam conditions at $s = 0$, such as envelope, divergence angle, and emittance, where the beam envelopes are measured on a screen located at $s = L+2D$ against a given varying quadrupole strengths k.

With the assumptions that the emittance does not change during beam propagation in such a short lattice, by using $M_0 = (X_0, X_0', \varepsilon_x, Y_0, Y_0', \varepsilon_y)$, the envelope equations can be solved numerically for the beam sizes (X_1, Y_1) at $s = L+2D$ for an external magnetic strength k. A hard-edge model is assumed for the quadrupole, i.e. $k_x = -k_y = k$ for $D< s <D+L$ and zero elsewhere. For a given set of initial parameters, the solved beam sizes can be denoted as $X_1^{env}(k|M_0)$, $Y_1^{env}(-k|M_0)$. Then the inverse problem can be regarded as an optimization problem to find the best set of initial conditions $M_0 = (X_0, X_0', \varepsilon_x, Y_0, Y_0', \varepsilon_y)$ that minimizes the measured beam sizes as a function of the quadrupole strength k. The optimization function is then defined as

$$J(M_0) = \frac{1}{2N}\sum_{i=1}^{N}\left\{\begin{matrix}\left(\frac{X_1^{measure}(k_i)-X_1^{env}(k_i|M_0)}{X_1^{measure}(k_i)}\right)^2 + \\ \left(\frac{Y_1^{measure}(k_i)-Y_1^{env}(k_i|M_0)}{Y_1^{measure}(k_i)}\right)^2\end{matrix}\right\} \quad (10)$$

Most of the numerical optimization methods could be applied at this point, but we will focus on the use of the genetic algorithm (GA) method because it helps avoid a local minimum compared with a classical, derivative-based optimization algorithm. Knowing the real physics meaning of each parameter, the search region can then be restricted for a quicker optimization.

RESULTS AND DISCUSSION

To test this method, a quadrupole scan of a space charge dominated beam was simulated using the particle-in-cell (PIC) code WARP [9] before the GA optimization method using MATLAB toolbox was applied to solve the reverse problem and find the initial conditions. In the simulation, an electron beam that is close to UMER [10] was used at an energy of 10 keV. Other conditions used in the simulation are listed in Table 1.

Table 1: Overview Of Beam Conditions

Beam Current (mA)	X0(Y0) (mm)	X0'(-Y0') (mrad)	ε_x (μm)
0.60	1.60	7.50	5.00
6.00	3.20	15.00	15.00
23.00	5.50	20.80	35.00

We use the slice field solver (wxy) in WARP which treats the beam as infinitely long, i.e. where the longitudinal space charge effect is not considered. The lattice is then the same as shown in Fig. 1. The quadrupole magnetic field gradient g_x (= $-g_y$) range was set from -0.40 to 0.40 T/m with an interval of 0.02 T/m. The quadrupole strength was calculated using the following equation.

$$k_x = -k_y = \frac{eg}{\beta\gamma mc} \quad (11)$$

A typical simulation at a beam current of 6 mA with a quadrupole strength of g_x = 0.4 T/m can be seen in Fig. 2 where the beam envelopes are presented with and without the space charge self-force. From this plot, it can be seen that the space charge acts as a continuous defocussing force that enlarges the beam envelopes.

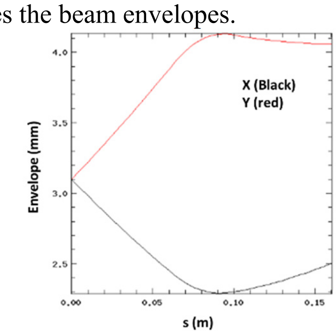

Figure 2: Envelopes of a space-charge dominated beam in the quadrupole scan lattice.

A full scan of this beam with space charge is presented in Fig. 3. A quadratic fitting was applied and shows a clear difference between the fit and the scanned data. Using Eqs. (3)-(5), the resulting emittance yields an imaginary value, which indicates that the quadrupole scan method cannot be applied in this case. Figure 4 shows a result from the reconstruction where a population size of 100 with an error function [Eq. (10)] of 3.5e-5 was applied for the GA algorithm, ending the optimization after 196 generations. The total reconstruction time is about 1 hour which can be further optimized by using a smaller population size and a better exit criterion. The reconstructed values for the three cases are shown in Table 2 and the percentage errors are shown in Table 3. The envelope sizes are best matched for all cases where the errors are less than 3%. The error for the emittance decreases when the beam current increases.

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Except for the 0.6 mA case, the errors are smaller than 1%, showing that the reconstruction method works. The errors for the slopes do not show any clear trend.

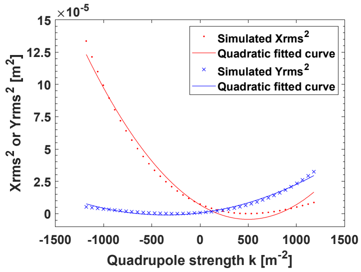

Figure 3: Quadrupole scan of a space charge dominated beam, showing beam size squared vs quadrupole strength k.

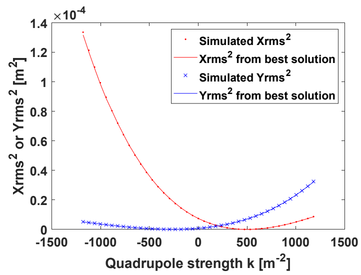

Figure 4: GA optimization of the reverse problem for initial conditions of 6 mA with an emittance of 15 μm.

Table 2: Reconstructed Values

I (mA)	X_0 (Y_0) (mm)	X_0' (Y_0') (mrad)	ε_x (ε_y) (μm)
0.60	1.64 (1.65)	7.38 (-8.04)	5.28 (5.99)
6.00	3.12 (3.12)	11.97 (-12.15)	15.05 (15.15)
23.00	5.57 (5.63)	21.09 (-21.34)	35.03 (35.15)

Table 3: Error Analysis

I(mA)	X_0(Y_0) (%)	X_0'(Y_0') (%)	ε_x (ε_y) (%)
0.60	2.38 (2.87)	1.64 (7.25)	5.52 (19.78)
6.00	2.40 (2.36)	20.19 (19.01)	0.31 (0.97)
23.00	1.28(2.34)	1.39 (2.59)	0.08 (0.44)

To further investigate the applicable range of this reconstruction method, the emittance values are varied in the PIC simulation. The reconstruction procedure is the same as above, and results are presented in Table 4. For an emittance larger than 15 μm, this method gives a good reconstruction from quadrupole scan data. However, when the emittances are smaller, the errors begin to grow and can be significant.

Further investigations into the PIC simulation show that for a space-charge dominated beam that started with a small emittance, the emittance is no longer constant or within a small variation - even over a short propagation distance. This breaks the basic assumption for the reconstruction methodand can be seen from Fig. 5 where the emittance blows up during the propagation in the lattice for a 6 mA beam with an initial emittance of only 0.1 μm. It should be noted that these conditions are created in a simulation environment where the beam is initially confined within a small emittance and that this does not reflect realistic condition, making emittance growth unavoidable. Note that for the 15 μm beam, the slight emittance decreases is due to an initial semi-Gaussian beam distribution used in the simulation.

Table 4: Emittance Reconstruction For 6 mA Beams

ε_{simu} (μm)	$\varepsilon_{x,\,rec}$ (μm)	$\varepsilon_{y,\,rec}$ (μm)	$Error_x$ (%)	$Error_y$ (%)
100	100.32	105.32	0.32	5.32
50	50.97	50.81	1.94	1.61
15	15.05	15.15	0.31	0.97
1	2.86	2.06	186.07	105.76
0.5	4.59	1.51	817.21	202.02
0.1	32.21	2.31	32108.80	2207.19

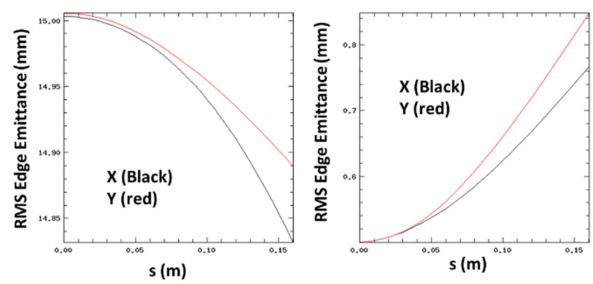

Figure 5: Edge emittance changes in the lattice.

CONCLUSION

In this paper, we presented a novel method to measure the emittance of a space-charge dominated beam. Data from quadrupole scans is used which is widely applied in many accelerators. Further investigation will be required to identify the applicable range of this method, focusing on questions such as acceptable space charge levels and quadrupole scanning range. Future experiments will need to target a comparison with other techniques used for space-charge dominated beam, such as pepper pot and multi-slit measurements. Other optimization techniques and algorithms can also be tested, especially machine learning techniques, to reduce the optimization time. In this way, this method could be implemented in an environment where real-time diagnostics is required.

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