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NONLINEAR COUPLING RESONANCES IN X-Y COUPLED BETATRON OSCILLATIONS NEAR THE MAIN COUPLING RESONANCE IN VEPP-2000 COLLIDER

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Abstract

In the vicinity of the linear coupling resonance where the working point of the collider is positioned, we study the effect of nonlinear coupling resonances on the single-particle phase space, beam sizes and the waveform of coherent beam motion. The latter is interesting for diagnostics of the nonlinear dynamics.

INTRODUCTION

Linear beam dynamic in accelerators is well studied, but in the modern colliders nonlinear effects are gaining importance. The example of such an innovative accelerator is the e^+e^- collider VEPP-2000 [1], which is designed to operate with round colliding beams and its working point is located on the main difference resonance.

It is easy to see from the basic resonance equation

$$l\nu_x + m\nu_y = n,$$

where l, m, n are integer, that such a working point actually stands not only on a single (1-1) resonance ($l = 1, m = -1$), but also on an infinite number of nonlinear coupling resonances, such as (2-2), (3-3) and so on. These resonances result in beam dynamics whose details and consequences are not well understood.

The operating mode of VEPP-2000 can be called "strong coupling", because two transverse dimensions whose tunes are very close are effectively mixed and form the betatron normal modes. In that situation, any perturbation in the accelerator lattice can excite two-dimensional resonances mentioned above. So it is very important to get insights into that system dynamics.

In this article the contributions of nonlinearities to coupled betatron oscillation are considered with simultaneous action of (1-1) and (2-2) resonances in the VEPP-2000 collider.

Primarily, the general Hamiltonian is derived in the variables of linear eigenvectors, then this Hamiltonian is reduced to one-dimensional difference Hamiltonian and some phase portraits are plotted. Further, the centre-of-charge motion, so as their spectra, of betatron coupled system are investigated and modelled. In the end, predictions are compared with observable spectra, and the main findings are summarized in conclusion.

NONLINEAR HAMILTONIAN

The 4-dimentional Floquet vectors, written in terms of Twiss parameters of coupling-free optics, can be introduced for (1-1) system:

$$Y_A = \begin{bmatrix} e^{i\left(\chi_x+2\pi n\frac{n_{рез}}{2}\right)} w_x \cos\gamma \\ e^{i\left(\chi_x+2\pi n\frac{n_{рез}}{2}\right)} \left(w_x' + \frac{i}{w_x}\right)\cos\gamma \\ e^{i\left(\chi_y-2\pi n\frac{n_{рез}}{2}-\alpha\right)} w_y \sin\gamma \\ e^{i\left(\chi_y-2\pi n\frac{n_{рез}}{2}-\alpha\right)} \left(w_y' + \frac{i}{w_y}\right)\sin\gamma \end{bmatrix} e^{2\pi i n(\nu_x+\nu_y+\eta)/2}$$

$$Y_B = \begin{bmatrix} -e^{i\left(\chi_x+2\pi n\frac{n_{рез}}{2}+\alpha\right)} w_x \sin\gamma \\ -e^{i\left(\chi_x+2\pi n\frac{n_{рез}}{2}+\alpha\right)} \left(w_x' + \frac{i}{w_x}\right)\sin\gamma \\ e^{i\left(\chi_y-2\pi n\frac{n_{рез}}{2}\right)} w_y \cos\gamma \\ e^{i\left(\chi_y-2\pi n\frac{n_{рез}}{2}\right)} \left(w_y' + \frac{i}{w_y}\right)\cos\gamma \end{bmatrix} e^{2\pi i n(\nu_x+\nu_y-\eta)/2}$$

They are called "normal oscillations Floquet vectors", and the general solution is the linear sum of them (with complex conjugation so that the physical variables are real).

Here $n = s/2\pi\bar{R}$ is dimensionless azimuth, $w_x = \sqrt{\beta_x}, w_y = \sqrt{\beta_y}$, $\eta = \nu_a - \nu_b$ is the difference of mode tunes and coupling angle γ introduced so that $\eta = \sqrt{\delta^2 + |C|^2}, \delta = \eta\cos 2\gamma, C = \eta \sin 2\gamma\, e^{i\alpha}$, C is the resonance (1-1) amplitude, or the coupling coefficient. Basic frequencies obey the equality $\nu_x - \nu_y = n_{res} + \delta$. In numerical simulations the dimensionless azimuth n is treated as integer variable, meaning the revolution number.

The higher resonance is, the less it's effect, so here we proceed only with (2-2) resonance. It is corresponding to the second order Hamiltonian in action, and to fourth order in coordinates. These terms are generated by:

- quadrupole fringes,
- solenoid fringes,
- colliding beam
- general octupole perturbations.

Below only free betatron oscillations with octupole perturbation are considered, other sources can be treated in the similar way. Name "unperturbed" refers to the system with skew-quadrupole perturbation. In this paper, the impact of mentioned (2-2) sources is considered as perturbation to unperturbed system (1-1), which solution is well-known, then the new eigenvectors are the linear combination of unperturbed ones:

$$\hat{A} = A\, e^{i\,\varphi_a};\ \hat{B} = B\, e^{i\,\varphi_b}$$

$$x = \frac{1}{2}A\, a_1\left(e^{i(\psi_a+\varphi_a)}\right) + \frac{1}{2}B\left(b_1 e^{i(\psi_b+\varphi_b)}\right) + \mathrm{c.c} \quad (1)$$

$$y = \frac{1}{2}A\left(a_3\, e^{i(\psi_a+\varphi_a)}\right) + \frac{1}{2}B\, b_3\left(e^{i(\psi_b+\varphi_b)}\right) + \mathrm{c.c}$$

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Here a_i and b_i are the normal eigenvectors components of unperturbed system, ψ_i are their betatron phases; A, B are the slow amplitudes, φ_a, φ_b are the slow phases, that emerge with perturbation.

Coefficients in this linear decomposition conform with equations [2]:

$$\widehat{A}' = -i\, Y_A^{*T} SG;\ \widehat{B}' = -i\, Y_B^{*T} SG, \qquad (2)$$

where S is the unit symplectic matrix, G is the vector of perturbations in the equations of motion.

Writing down the obtained equations and switching to the difference variables ($\Phi = 2\pi n\,\eta + \varphi_a - \varphi_b, J = J_a - J_b, II = J_a + J_b$), the desired Hamiltonian is obtained after integrating over the period and fast phases averaging:

$$H = \frac{1}{2}\left((J^2 + II^2)(p + r) + (p - r)(2II\,J)\right) +$$
$$q(II^2 - J^2) + 4\pi J\eta + 2II\nu_0 + f_2(II^2 - J^2)\cos(2\Phi) \qquad (3)$$
$$+\sqrt{II^2 - J^2}\left(k_{11}(J + II) + k_{12}(II - J)\right)\cos(\alpha + \Phi)$$

The equations for the new variables can be obtained from that Hamiltonian:

$$J' = -\frac{dH}{d\Phi}, \Phi' = \frac{dH}{dJ}, \Phi'_{\text{sum}} = \frac{dH}{dII}, II' = 0$$

DIFFERENCE VARIABLES

In the difference variables the problem is one-dimensional, which leads to the possibility of 2D-phase space plotting and, therefore, allows one to analyze the system qualitatively. That analysis shows some critical condition, which lower estimation is expressed as $c \equiv \frac{|f_2|\,II}{\eta} = 1$ (so as with substitution $f_2 \to k_{11}, k_{12}$). At this point the phase portrait is altered and several additional fixed points appear.

The linear resonance moves apart the betatron frequencies. So, on the VEPP-2000, η is usually not less than 0,001. This fact, together with the estimates given below, makes it possible to assert that at the values of f_2, k_{11}, k_{12} existing at VEPP-2000, such additional autophasing domains and fixed points are either not formed or their depths are not large. In any case, this resulting in a small redistribution of particles, but non-roundness up to ~20% has almost no effect on the important parameters of the collider, as was found experimentally at the VEPP-2000.

In total, for a given η, the existence of 0 to 4 fixed points is possible, with no more than 3 centers and no more than 1 saddle.

Due to the preservation of the sum action, it is convenient to plot phase trajectories on a II radius sphere. In these portraits the I/II ratio is plotted along the z-axis, and the polar angle represents the difference phase Φ. Several typical cases of such portraits are shown in Fig. 1: the symmetric cases with 2 and 3 fixed points, and general non-symmetric case with saddle point. $\alpha = 0$ in each portrait for ease of observation.

Also the phase portraits without (a), with slight (b), and with large (c) nonlinearities are presented in Fig. 2 in order to show the complications caused by resonances.

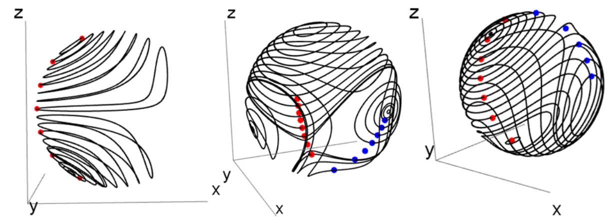

Figure 1: Phase portraits of free betatron oscillations in 1-1+2-2 system.

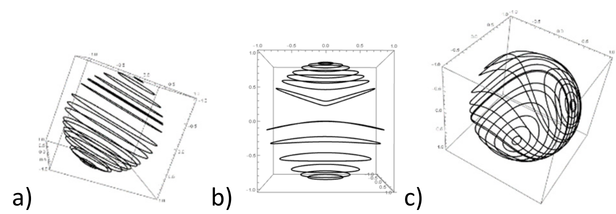

Figure 2: Phase portraits without (a) and with a growing (b, c) nonlinearities.

SIGNAL MODELING

The presence of the nonlinearities described above is indicated by the decoherence in the signals from BPMs – the beam dipole moment disappears after ~ 1000 revolutions. The decoherence theory is well described in [3, 4] for uncoupled oscillations, and it is applicable to mode signals.

Assuming that nonlinearities strength is small ($c = \frac{|f_2|\,II}{\eta} \ll 1$), we obtain expressions for phases φ_a and φ_b in the first order of smallness for Hamiltonian (3):

$$\varphi_a(n) = (q\, b_2^2 + p\, b_1^2)\, n,\quad \varphi_b(n) = (q\, b_1^2 + r\, b_2^2)\, n$$

Where $b_1 = \sqrt{2J_a}, b_2 = \sqrt{2J_b}$ – average oscillation amplitudes. Resonances appear only in the second order, so in the current situation $A, B = const$.

The comparison with BPM data reveals that the constant-nonlinearity coefficients are not small, $p, q, r \sim \eta/II$ in order of magnitude. Assuming that all nonlinear coefficients in Hamiltonian are of the same order, we can conclude, that coefficient c is also not small in general case, and the system is approximately on the verge of additional fixed points emergence. One of the modeled BPM signals is shown in Fig. 3.

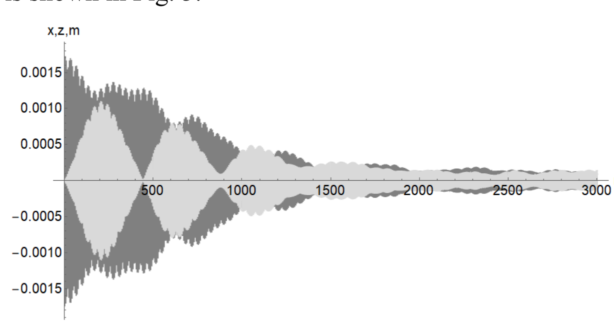

Figure 3: Betatron history modeling example.

In the presence of $2-2$ resonance, the actions, obviously, cease to be constants, and oscillate at the double frequency 2η as well as phases. The amplitude of these oscillations and frequencies shifts are proportional to c^2. The

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same effect is produced by $1-1$ resonance, exciting oscillations with a single η. When passing to the observable x-y coordinates, the spectra of the modes are added with the weights that are the components of the vectors of normal oscillations in accordance with the linear expansion of the coordinates into modes.

For different parameters, the mode frequencies are shifted, increasing or decreasing the distance between the main peaks, denoted below as η_1. As a result, in the Fourier analysis of the dipole moment oscillations, two shifted fundamental frequencies ν_x and ν_y are visible, and one or more equidistant satellites responsible for phase oscillations in the autophasing domains on single and double η_1.

The series of signal modeling were made for collective (see Fig. 3), as well as for single particle dynamics. Simulations were made based on numerical solutions of averaged Hamiltonian (3) equations. Further only single particle simulations are considered due to their superior accuracy.

Some results are shown in Fig. 4. Blue and red lines mean B and A mode tunes respectively. The spectra correspond to different initial parameters on the first phase portrait from the Fig. 1, showed by dots.

As was expected, the obtained Fourier spectra show several peaks, the distance η_1 between them depends on the proximity of the trajectory to the separatrix – it is inverse to the period of revolution around the fixed point. For trajectories located far from fixed points and separatrices, the spectrum shows the main mode oscillations and small satellites. At fixed points, the actions and the difference phases are constant, and only the line with the frequency of the given mode is visible. When a particle trajectory passes close to the saddle point, the frequency of the phase oscillations decreases, and the spectrum shows many lines that are close to each other.

In Fig. 5 the mode oscillations near a separatrix (1st graph), and near a fixed point (2nd graph) are shown. It can be seen from them that, indeed, near the separatrix, many harmonics are observed. The periodicity of the envelope clearly indicates that the spectrum of such oscillations is linear.

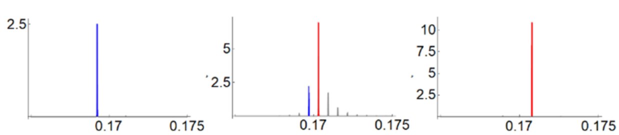

Figure 4: Fourier spectra in terms of a-b modes with k_{11} and k_{12} close to each other.

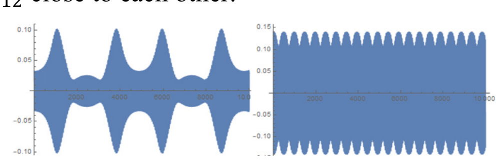

Figure 5: Particle mode oscillations near the separatrix.

EXPERIMENTAL OBSERVATIONS

Predicted spectrum patterns were prepared to compare with the experimental observables. Presently but a little time was available for this experiment.

The measured spectra (two of them are shown in Fig. 6) are not rich for satellites, and comparing with the series of predicted spectra, it can be concluded, that the resonance terms are weaker or slightly above the critical values, as was estimated from the decoherence time, and conclusions from the measurements are preliminary.

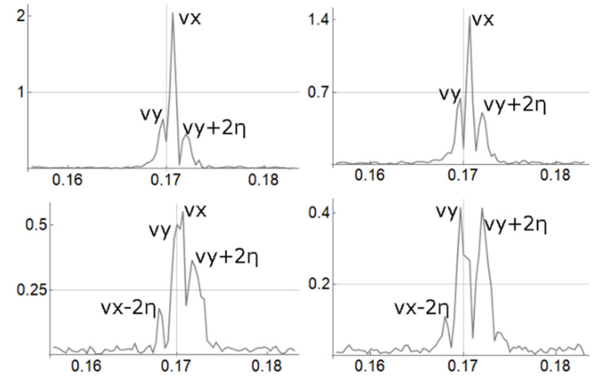

Figure 6: x (top), and y (bottom) spectra, observed after kicks on the VEPP-2000.

Sure, more experimental data is needed to prove the resonance $2-2$ influence on the particle motion on the VEPP-2000 collider.

CONCLUSION

A system with several closely spaced resonances can be reduced to integrable form using fast-phase averaging, so that its dynamics is regular.

The simultaneous action of resonances 1-1 and 2-2 was considered. The phase portraits were plotted and analyzed, the qualitative features, auto-phasing domains and fixed points were found.

Strengths of VEPP-2000 cubic nonlinearities resulting in the resonance term amplitude were estimated. The comparison with decoherence time showed the qualitative agreement of estimations and measurements.

Extensive series of modeling the betatron motion were performed with different phase-space patterns. Fourier spectra of those oscillations were predicted in both normal-mode and observable coordinates.

Series of spectra from the routine operating mode of the collider were compared with predicted ones. The comparison showed a fairly good agreement, but a dedicated validation experiment is required to get more data.

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