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ANALYSIS OF THE CHROMATIC VERTICAL FOCUSING EFFECT OF DIPOLE FRINGE FIELDS*

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Abstract

There have been questions regarding the impact of the dipole fringe field models (used by accelerator codes including ELEGANT and MADX) on vertical chromaticity. Here, we analyze the cause of the disagreement among codes and suggest a correction.

INTRODUCTION

An accurate and realistic way of modeling the dipole fringe field effects requires 3D surface field data, or on-axis vertical field shape modeling [1, 2]. However, in most cases where the bending radius is large or the vertical gap between magnet pole faces is small, such methods are numerically heavy in view of the relative importance of the fringe field. On the other hand, the analytically expressed dipole-fringe-field thin map is a computationally efficient way of modeling the dipole-fringe-field effects [3–5]. Many accelerator simulation codes adopt the thin map for dipole-fringe-field modeling. However, we observed some discrepancies in the numerically calculated vertical chromaticities of a few storage rings when different thin map models are used. In a comparison of the reference [3–5], we found that the vertical chromatic focusing terms of Ref. [3] do not agree with the Refs. [4, 5]. In this paper, we discuss the disagreement in detail and try to verify using simulations. We also illustrate the effect on the vertical chromaticity of the Integrable Optics Test Accelerator (IOTA) ring. Finally, we discuss the dipole-fringe-field effect on dispersion.

REVIEW

In this section, we review the chromatic vertical focusing term of Refs. [3–5].

Notation

For consistent comparision, we use the following notation:

- edge angle: θ
- bending radius: ρ
- vertical gap between magnet pole faces: g
- field integration parameter:

$$K = \int_{-\infty}^{\infty} \frac{B_y(z)\left(B_0 - B_y(z)\right)}{gB_0} dz, \tag{1}$$

  where B_0 is the nominal dipole magnetic field, B_y is the z dependent dipole fringe field and z is the longitudinal coordinate of the Cartesian frame perpendicular to the dipole edge so that the on-axis vertical magnetic field $B_y(z)$ is defined using single variable z.
- vertical focusing phase correction:

$$\psi = \frac{g}{\rho} K \frac{1 + \sin^2\theta}{\cos\theta}, \tag{2}$$

  which is the order of $O(g/\rho)$.

Review of Ref. [3]

The Ref. [3] is (arguably) the most widely adopted dipole fringe field thin map in various accelerator codes including ELEGANT and MAD [6, 7]. It presents the second-order Taylor map modeling the fringe field effect in terms of the matrix and the tensor elements. Among them, we would like to focus on the R_{43} and T_{436} elements that are responsible for the vertical focusing and chromatic vertical focusing respectively. It is also important to mention that the matrix and the tensor act on the 6D phase-space variables: $(x, x', y, y', t, \delta)$ where the prime indicates the differentiation along the reference orbit, t is proportional to the transit time deviation, and δ is the fractional momentum deviation.

To the first order of ψ, the matrix and tensor elements R_{43} and T_{436} are equal:

$$R_{43} = T_{436} = -\frac{\tan\theta}{\rho} - \frac{\psi}{\rho}\sec^2\theta + O\left(\psi^2\right). \tag{3}$$

These elements act on the vertical velocity, resulting in:

$$\Delta y' = \left(-\frac{\tan\theta}{\rho} - \frac{\psi}{\rho}\sec^2\theta\right)(1+\delta)\, y. \tag{4}$$

However, in terms of the canonical momentum,

$$\Delta p_y = \Delta\frac{y'}{1+\delta} = \left(-\frac{\tan\theta}{\rho} - \frac{\psi}{\rho}\sec^2\theta\right) y, \tag{5}$$

the chromatic vertical focusing vanishes.

Review of Ref. [4]

The Ref. [4] is adopted in the MAD-X PTC module. It presents the generating function-based map to the second order of dipole field strength. The Eq. (32) of the Ref. [4] shows the 2nd order of field effect (i.e. the first order of K or ψ) on vertical momentum. The canonical variables in this equation is defined in Cartesian frame perpendicular to the dipole edge. Therefore, plugging $p_x = (1+\delta)\sin\theta$, $p_y = 0$ and $p_z = \sqrt{(1+\delta)^2 - p_x^2}$ into the Eq. (32) of the Ref. [4], we get Eq. (6):

$$\Delta p_y = \left(-\frac{\tan\theta}{\rho} - \frac{\psi}{\rho}\sec^2\theta\right) y + \frac{\psi}{\rho}\sec^2\theta\, y\delta. \tag{6}$$

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Note that the chromatic vertical focusing is the first order of ψ and equal to the negative of the first order of ψ term of the on-momentum vertical focusing.

Review of Ref. [5]

The Ref. [5] is adopted in ELEGANT [6]. It presents the single exponent Lie map (to the 4th order of canonical variables) and corresponding Taylor map (to the 3rd order of canonical variables). Recently, ELEGANT factorized the Lie map to implement an explicit symplectic map. The Eqs. (43) and (47) of Ref. [5] show the action of the Taylor map on the vertical momentum. Writing the vertical focusing terms only:

$$\Delta p_y = \left(-\frac{\tan\theta}{\rho} - \frac{\psi}{\rho\,(1+\delta)}\sec^2\theta\right) y. \qquad (7)$$

Note that it is equivalent to Eq. (6) to the first order of δ.

SIMULATIONAL VERIFICATION

Taylor Coefficient

In order to verify the chromatic vertical focusing term, we use various simulation tools:

- Lorentz force tracking with an automatic differentiation library
- Hamiltonian (Eq. (16) of Ref. [5]) tracking with an automatic differentiation library.
- MaryLie/Impact code that uses Lie map tracking over soft fringe field region and is capable of producing Taylor coefficients [8].

The automatic differentiation library [9] is used to calculate the Taylor coefficients.

We model fringe field strength using the sigmoid function. We also set the edge angle to $\pi/4$. Figure 1 shows the comparison between the simulation results and the theory from Refs. [4, 5] for various bending radii and vertical gaps. The theory and simulation results agree well especially for large bending radius and small vertical gap. Recall that the chromatic vertical focusing in term of canonical momentum is zero in the Ref. [3].

IOTA Vertical Chromaticity

IOTA is a storage ring built as a testbed for novel accelerator physics technologies [10]. We took a version of the IOTA design lattice that is for the nonlinear integrable optics test. Recall that as the bending radius become smaller the fringe field effects become more important. IOTA ring has two types of sector dipoles whose bending radii are 0.822, 0.773 (m). Assuming 2.5 cm vertical half gap, and the field integration parameter $K = 0.5$, the optics parameter using Ref. [3] and Refs. [4, 5] model of the dipole fringe field are shown in the Table 1. Note that relative difference in the vertical chromaticity may not be neglectable.

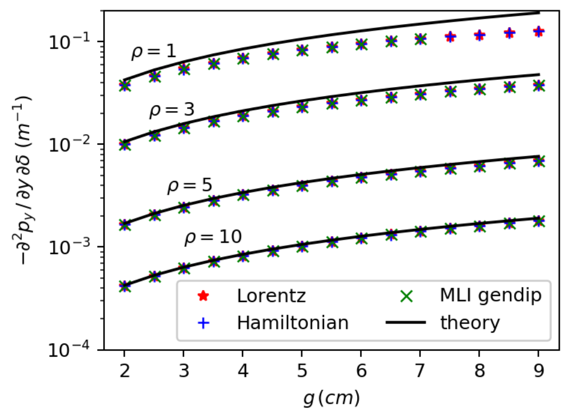

Figure 1: Taylor coefficient of the chromatic vertical focusing term of the dipole fringe field thin map.

Table 1: IOTA Tunes and Chromaticities

Model	ν_x	$\partial\nu_x/\partial\delta$	ν_y	$\partial\nu_y/\partial\delta$
Ref. [3]	0.3	4.072	0.3	0.487
Refs. [4, 5]	0.3	4.072	0.3	0.581

CONCLUSION AND REMARKS

We compared the chromatic vertical focusing of dipole fringe field thin map from three references that are often adopted in accelerator simulation codes. We conclude using three different simulations that the vertical chromatic focusing term of Ref. [3] needs to be corrected. We saw that for a compact ring like the IOTA, the difference of the vertical chromaticity by this correction is not ignorable.

It is also worth mentioning that the dispersion error due to the dipole fringe field is often ignored in most accelerator simulations. However, it can be more important than the chromatic vertical focusing effect as it introduces the energy-dependent closed orbit deviation. Ref. [5] indicates that the off-energy closed orbit deviation is order of $x_{closed} \sim O\left(\delta g^2/\rho\right)$.

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