ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB226

ANALYTICAL DESCRIPTION OF THE STEERER PARAMETERS IN THE BILINEAR-EXPONENTIAL MODEL AT DELTA

S. Kötter*, Center for Synchrotron Radiation (DELTA), TU Dortmund, 44227 Dortmund, Germany

Abstract

At DELTA, a 1.5 GeV synchrotron radiation source operated by the TU Dortmund University, an analytical description of the steerer parameters in the bilinear-exponential (BE) model has been developed. The BE model describes the coupled orbit response in a storage ring. It is used in the closed-orbit bilinear-exponential analysis (COBEA) algorithm to decompose orbit response matrices into beta function, betatron phase, and a scaled dispersion. After introducing the BE model and the analytical steerer parameters, a simulation-based comparison of the BE model and another coupled orbit response model is presented.

INTRODUCTION

The bilinear-exponential (BE) model describes the orbit response in a storage ring in the approximation of coupled linear beam dynamics [1]. It is used in the closed-orbit bilinear-exponential analysis (COBEA) algorithm to decompose a measured orbit response matrix into beta function, betatron phase and a scaled dispersion at all beam position monitors (BPMs) [2, 3]. A variation of the COBEA algorithm has also been investigated to extract optical functions from orbit corrections [4].

In the absence of transverse coupling and approximated for thin steerer magnets, the BE model reduces to the most widely known orbit response model without dispersion [1]

$$r_{jk} = \frac{\sqrt{\beta_j \beta_k}}{2\sin(\pi q)} \cos\left(|\psi_j - \psi_k| - \pi q\right). \qquad (1)$$

Here, β is the beta function, ψ the betatron phase, s the longitudinal position, q the tune and θ the steerer strength. BPMs are indexed with j. Steerer magnets are indexed with k.

The addition of an analytical description of the steerer parameters makes the BE model a generalization of Eq. (1) for coupled storage rings. The steerer parameters and their simulation-based validation is presented in the following.

THE BE MODEL

According to the BE model [1], the orbit response in a storage ring without dispersion

$$r^{\mathrm{BE}}_{wjk} = \theta_k \sum_{m=0}^{M-1} \Re\left\{Z_{mwj} A^*_{mk} e^{-i\pi q_m S_{jk}}\right\} \qquad (2)$$

is determined by $M = 2$ modes of betatron motion. The plane index w refers to either the horizontal or the vertical plane. The separation of the indices m and w incorporates coupled betatron oscillations in the sense of the Mais-Ripken parametrization into the model [5]. These are not confined to a single plane. For this reason, the phasor

$$Z_{mwj} = \sqrt{\beta_{mwj}} e^{i\psi_{mwj}}$$

is indexed with both m and w. It encodes the amplitude and phase of the betatron oscillation of the m-th mode where β_{mwj} is the projection of the beta function into the w-th plane at BPM j (Fig. 1) and ψ_{mwj} is the corresponding betatron phase.

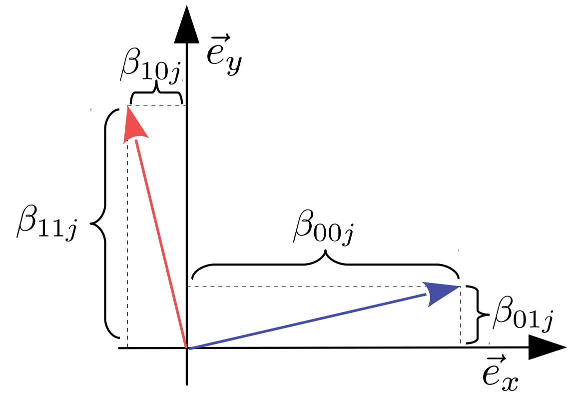

Figure 1: Beta function values β_{mwj} of coupled betatron oscillations at BPM j [4]. The index w references the horizontal ($w = 0$) or vertical plane ($w = 1$). In this example, the first mode ($m = 0$) is mostly horizontal whereas the second mode ($m = 1$) is mostly vertical.

The remaining model parameters are the tune of the m-th mode q_m, the factor S_{jk}, which is either -1 if the k-th steerer magnet is downstream of the j-th BPM or 1 otherwise, and the steerer parameters A_{mk}.

Steerer Parameters

The 4D closed orbit $\vec{r}(s_k)$ at a thin steerer magnet applying a kick $\vec{\theta}_k$ holds

$$\vec{r}(s_k) - T_{kk}\vec{r}(s_k) = \vec{\theta}_k \qquad (3)$$

where the one turn-transfer map T_{kk} propagates the phase space vector at the steerer magnet to the next turn. The closed orbit according to the BE model is a scaled betatron oscillation represented by [1]

$$\vec{r}(s_k) = \sum_m \Re\left\{\tilde{A}_{mk} \begin{pmatrix} Z_{m0k} \\ Z'_{m0k} \\ Z_{m1k} \\ Z'_{m1k} \end{pmatrix}\right\}.$$

In this ansatz, the steerer parameters take the role of complex scaling factors. They select the correct amplitudes and

* stephan.koetter@tu-dortmund.de

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB226

phases for the two modes of the betatron oscillation which is closed by the kick and therefore becomes the new closed orbit. The transition of the beam to the new closed orbit begins when the beam experiences the kick for the first time. The old orbit then becomes a betatron oscillation trajectory and the beam oscillates transversely until synchrotron radiation damping sets it on the new closed orbit.

Inserting the closed-orbit ansatz into the previous condition and absorbing half of the phase advance applied by the one-turn transfer map into the steerer parameters

$$A_{mk} = e^{i\pi q_m}\tilde{A}_{mk}$$

yields an equation system

$$-2i\sum_m \Re\left\{A_{mk}\begin{pmatrix} Z_{m0k} \\ Z'_{m0k} \\ Z_{m1k} \\ Z'_{m1k}\end{pmatrix}\right\}\sin\left(\pi q_m\right) = \begin{pmatrix} 0 \\ \theta_{0k} \\ 0 \\ \theta_{1k}\end{pmatrix}$$

for the steerer parameters. Note, that inserting the BE model from Eq. (2) directly into Eq. (3) gives the same result. The equation system is solved by [6]

$$A_{mk} = \frac{\sum_w \theta_{wk}\det(M_{mwk})}{4i\sin\left(\pi q_m\right)} \tag{4}$$

with the matrices

$$M_{11k} = \begin{pmatrix} Z^*_{11k} & Z_{21k} & Z^*_{21k} \\ Z^*_{12k} & Z_{22k} & Z^*_{22k} \\ Z'^*_{12k} & Z'_{22k} & Z'^*_{22k}\end{pmatrix}$$

$$M_{12k} = \begin{pmatrix} Z^*_{11k} & Z_{21k} & Z^*_{21k} \\ Z'^*_{11k} & Z'_{21k} & Z'^*_{21k} \\ Z^*_{12k} & Z_{22k} & Z^*_{22k}\end{pmatrix}$$

$$M_{21k} = \begin{pmatrix} Z_{11k} & Z^*_{11k} & Z^*_{21k} \\ Z_{12k} & Z^*_{12k} & Z^*_{22k} \\ Z'_{12k} & Z'^*_{12k} & Z'^*_{22k}\end{pmatrix}$$

$$M_{22k} = \begin{pmatrix} Z_{11k} & Z^*_{11k} & Z^*_{21k} \\ Z'_{11k} & Z'^*_{11k} & Z'^*_{21k} \\ Z_{12k} & Z^*_{12k} & Z^*_{22k}\end{pmatrix}$$

encoding the betatron motion at the position of the steerer magnet.

ORBIT RESPONSE FROM TRANSFER MAPS

The orbit response in a transversly coupled storage ring can also be calculated from transfer maps instead of optical functions [7]. Solving equation Eq. (3) for the orbit displacement at the steerer magnet gives the orbit response

$$\vec{r}(s_k) = \left(1 - T_{kk}\right)^{-1}\vec{\theta}_k$$

at the steerer magnet. It can be propagated through the storage ring using matrix optics to determine the orbit reponse at each BPM

$$\vec{r}^{\,\mathrm{TM}}_{jk} = \frac{T_{kj}}{1 - T_{kk}}\vec{\theta}_k. \tag{5}$$

Here, T_{kj} is the transfer map which propagates the 4D phase space vector from position s_k to position s_j.

This model has been used in the calibration of the optics model (CALIF) algorithm and the local optics from closed orbits (LOCO) algorithm to fit lattice models onto measured orbit response matrices by varying quadrupole gradients and other parameters [8, 9].

SIMULATION-BASED MODEL COMPARISON

The efficacies of the “transfer map” (TM) model and the BE model were evaluated by generating response matrices for both analytical models based on a linear optic model of the storage ring at DELTA [10] and comparing them to an orbit response matrix from simulated closed orbits. The storage ring has 54 BPMs (horizontal and vertical) and 56 steerer magnets. Each simulated matrix therefore had 108×56 elements.

The response matrix of the TM model R^{TM} was created from transfer maps accessible via the MAD-X Twiss module [11] according to Eq. (5). The response matrix for the BE model R^{BE} was created from Eq. (2) with the steerer parameters according to equation Eq. (4). The required 4D phasor vectors at all BPMs and steerer magnets were generated by propagating the complex eigenvectors of an one-turn transfer map through the lattice. The required transfer maps were also accessed via the MAD-X Twiss module. The response matrix from closed orbits R^{CO} was determined by executing an orbit response measurement for each steerer magnet in the simulation, taking the orbit after exciting each steerer magnet by 0.1 mrad as orbit-response-matrix column.

The error of the analytical matrices

$$\sigma_R = \frac{\left\|R^{\mathrm{CO}} - R^{\mathrm{TM/BE}}\right\|_2^2}{\left\|R^{\mathrm{CO}}\right\|_2^2}, \tag{6}$$

where $\|...\|_2^2$ is the l2 matrix norm, was calculated for variations of the base lattice model where each variation used a different skew angle for the quadrupoles in the arcs of the storage ring. The skew angles were deliberately added to the lattice model to introduce transverse coupling. The results are given in Fig. 2.

The predictions of the TM model and the BE model are very close to the simulated outcomes for all skew angles and coincide very well.

CONCLUSION

The BE model in thin-steerer approximation predicts the orbit response in presence of transverse coupling very well. In addition, it practically makes the same predictions as another well-known orbit response model used in the LOCO and CALIF algorithms [8, 9].

The BE model in Eq. (2) with the steerer parameters in Eq. (4) can therefore be regarded as generalization of Eq. (1) for transversly coupled storage rings: a formula to

ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-MOPAB226

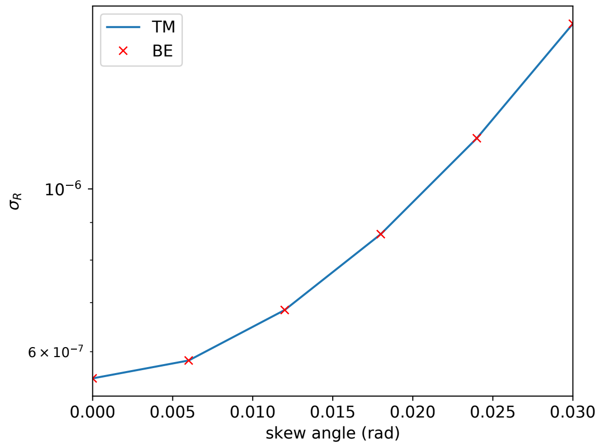

Figure 2: Response matrix errors for a range of quadrupole skew angles.

determine the orbit response of a thin steerer magnet from beta function and betatron phase (and their derivatives) in the Mais-Ripken parametrization.

ACKNOWLEDGEMENTS

The author thanks his working group for its support and B. Riemann (PSI) for his comments regarding the BE model and COBEA. Also, the author expresses his gratitude towards the scientists and technicians who maintain and operate the storage ring at DELTA.

REFERENCES

[1] B Riemann, "The Bilinear-Exponential Closed-Orbit Model and its Application to Storage Ring Beam Diagnostics", Ph.D. thesis, TU Dortmund University, Dortmund, Germany, 2016.

[2] B. Riemann, S. Kötter, S. Khan, and T. Weis, "COBEA - Optical Parameters from Response Matrices without Knowledge of Magnet Strengths", in *Proc. 8th Int. Particle Accelerator Conf. (IPAC'17)*, Copenhagen, Denmark, 2017, pp. 676–679. doi:10.18429/JACoW-IPAC2017-MOPIK066

[3] B. Riemann, S. Kötter, B.D. Isbarn, S. Khan, and T. Weis, "Validating the COBEA Algorithm at the DELTA Storage Ring", in *Proc. 9th Int. Particle Accelerator Conf. (IPAC'18)*, Vancouver, Canada, 2018, pp. 4851–4854. doi:10.18429/JACoW-IPAC2018-THPML084

[4] S. Kötter and T. Weis, "Towards an Adaptive Orbit-Response-Matrix Model for Twiss-Parameter Diagnostics and Orbit Correction at DELTA", in *Proc. 8th Int. Beam Instrumentation Conf. (IBIC'19)*, Malmö, Sweden, 2019, pp. 485–489. doi:10.18429/JACoW-IBIC2019-WECO02

[5] V. A. Lebedev and S. A. Bogacz, "Betatron motion with coupling of horizontal and vertical degrees of freedom", *J. Instrum.*, vol. 5, no. 10, pp. P10010–P10010, Oct. 2010. doi:10.1088/1748-0221/5/10/p10010

[6] S. Kötter, unpublished dissertation.

[7] M. Beckmann and V. Ziemann, "How well do we now the circumference of a storage ring?", *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 771, pp. 115-120, 2015. doi:10.1016/j.nima.2014.10.044

[8] W. J. Corbett, M. J. Lee, and V. Ziemann, "A Fast Model-Calibration Procedure for Storage Rings", in *Proc. 15th Particle Accelerator Conf. (PAC'93)*, Washington D.C., USA, 1993, pp. 108–111. doi:10.1109/pac.1993.308999

[9] W. J. Corbett, D. Robin, J. Safranek, and V. Ziemann, "Debugging Real Accelerators", *Part. Accel.*, vol. 58, pp. 193–200, 1997.

[10] D. Schirmer, private communication, TU Dortmund University (DELTA), 2020.

[11] F. Schmidt, "MAD-X PTC Integration", in *Proc. 21st Particle Accelerator Conf. (PAC'05)*, Knoxville, TN, USA, 2005, paper MPPE012.