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# TOWARDS DETERMINISTIC DESIGN OF MBA-LATTICES*

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*Abstract*

Since the pioneering work of MAX IV, multi-bend achromat (MBA) lattices have become the standard in lattice design for 4th generation lights sources as well as upgrades of 3rd generation storage rings. The distribution of the bending angle to many weak dipoles enables to reach unprecedented low emittance und highest brightness. In their most basic form, MBA-lattices consist of a repetitive unit cell and two identical matching cells on either end of the achromatic arc. The simplicity of the unit cell allows for a unique determination of the linear lattice parameters in dependence on boundary conditions defined by the project goals. Those might be the emittance, momentum compaction factor, chromaticity, as well as phase advances with respect to achieving higher order achromatic structures. A scan of optional lattice prototypes is quickly obtained. We demonstrate this concept and apply it to design an optional lattice of BESSY III, a greenfield 4th generation storage ring being currently planned at HZB, Berlin, Germany.

## INTRODUCTION

The most recently commissioned storage rings utilizing MBA lattices are Sirius, Brazil [1], and ESRF-EBS, France [2], but many more projects are in the planning phase.

The natural emittance is proportional to the cube of the bending angle of the individual dipoles, which suggests to maximize the number of dipoles per super period (SP), as well as the number of super periods in the lattices of low emittance rings. The costs, increasing with circumference, and the vanishing dispersion function are the limiting constraints of this approach. Composing the dipoles together with the necessary focusing and higher order corrections in standardized repetitive unit cells (UC) eases the engineering efforts. Towards the straight sections the dispersion needs to be suppressed and the beta functions are adjusted either to ease injection, or to achieve highest brilliance and coherence in the respective insertion device. The so called matching cell (MC) covers all elements needed for the fitting. It includes the focusing elements in the straight section and extends into the dispersive structure, usually as a modification of the last (half) unit cell.

As the unit cell is the "necessary evil" to reach small emittance, it should be as compact as possible to keep the circumference short. This paper starts from the most simple UC and investigates the implications of different magnet options. Potential matching cells are briefly described and a candidate lattices for BESSY III is introduced [3]. All calculations have been performed with OPA [4], a quick, interactive optics program, optimized for MBA lattices.

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## THE UNIT CELL

### *The Basic Unit Cell*

The most basic unit cell utilizes separated function magnets: a bend and quadrupoles and sextupoles, acting on both planes. The hybrid MBA lattice, where the sextupoles are exclusively placed towards the end of the achromat, is not treated here, but could be analyzed in an analog way. Unit cells are symmetric and end with symmetry points, i.e. $\alpha_x = \alpha_y = \eta' = 0$ at the center and at the end of the UC. Thus, it is sufficient to investigate half a unit cell, hUC, see also [5].

A quick estimate of the necessary bending angle per dipole is achieved by testing a few $m$-bend super periods with identical dipoles, where the two outer (half) bends are counted as one. 4° is a good starting point for 16, 18 or 20 SP with 7, 6 or 5 bends respectively.

As analyzed in [5–7], the minimal theoretical emittance (TME) arising from a homogeneous dipole depends only on the values of the dispersion function, $\eta$, and the horizontal beta function, $\beta_x$, at the center of the dipole.

$$\beta_x^{0,TME} = L/\sqrt{15}, \eta^{0,TME} = \theta L/6, \quad (1)$$

$$\epsilon^{TME} \propto 2\theta^3/(3\sqrt{15}), \quad (2)$$

with L and $\theta$ being the length and deflection angle of the dipole. Treating the hUC as a transfer line, the TME values fix the initial conditions at the dipole center, except for $\beta_y^0$. The two gradients are used to fit $\alpha_{x,y} = 0$, at the end. $\eta' = 0$ is achieved, either by adjusting the drift lengths, or by using a reverse bend, RB, as proposed in [5]. The RB focuses the dispersion with negligible effects on the beta functions and the emittance and keeps the UC short.

### *Choice of Boundary Conditions*

Smallest emittance, minimal length and technically feasible fields and gradients, are suitable initial boundary conditions. This study uses 0.1 m for drifts, 1.3 T for homogeneous dipoles and 65 T/m for quadrupoles. $\beta_y^0$ is chosen such, that $\beta_y > \beta_x$ at the defocusing sextupole. At 2.5 GeV and a dipole length of 0.25 m, $\eta^{0,TME}$ =1.6 mm and $\beta^{0,TME}$ = 0.064 m. The resulting hUC is shown in Fig. 1. This UC has an emittance of 87 pm rad, a vanishingly small momentum compaction factor, $\alpha = 8 \times 10^{-6}$, and a large horizontal chromaticity, $\xi_x = -1.5$.

Small $\alpha$ implies short bunches, significant intra-beam scattering, short lifetime and impedance problems. Large chromaticity leads to strong sextupoles and highly non-linear behavior. It is easily checked, that relaxing $\beta_x^0$ and $\eta^0$ can be used to improve on $\alpha$ and $\xi_x$. But leaving the TME conditions necessarily increases the emittance, in the case of BESSY III to unacceptably large values. Notice, that the

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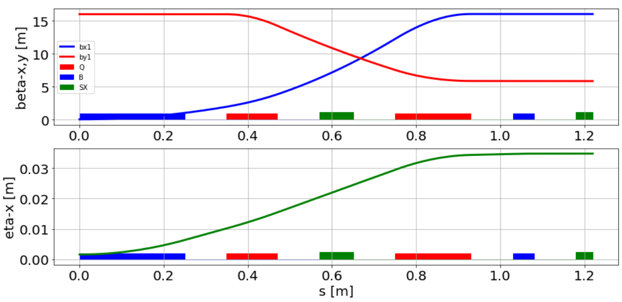


Figure 1: $\beta_{x,y}$ (top; blue, red) and $\eta$ (bottom, green) of the most basic (half) unit cell seeking minimal emittance and length and obeying given field constraints. Dipoles are blue, quadrupoles red and sextupoles green.

contribution of the matching cell to $\alpha$ is negligible due to the vanishing dispersion. The value for the unit cell therefore needs to largely exceed the desired final value.

### *Impact of Different Hardware Solutions*

Combined function magnets can be used to reach better values of $\alpha$ and $\xi$ without sacrificing on emittance. These approaches need longer bends for their technical realization.

**Increased Dipole Length** The TME-emittance is independent of the dipole length, L, while $\beta_x^{0,TME}$ and $\eta^{0,TME}$ are proportional to L, Eq. (1).

For L/2 = 0.4 and 0.6 m, Figure 2 shows $\epsilon$ and $\alpha$ as a function of $\eta^0$ in the matched UC. $\beta_x^0 = \beta_x^{0,TME}$. $\eta_x^{0,TME}$ (red dots) doesn't result in minimal $\epsilon$. This is an effect of the RB, that starts to impact the emittance with the increasing dispersion. The quadratic dependence of $\epsilon$ on $\eta^0$ is shifted to higher $\eta^0$ values for longer dipoles. Therefore, larger $\alpha$ values can be reached at similar emittance values. The average dispersion increases and dominates the dipole length, $\alpha \approx \theta\,\overline{\eta}\,/\mathrm{L}$. In addition, longer bends significantly reduce the chromaticity. Both aspects relax the necessary sextupole strength. The drawback is the lengthening of the circumference.

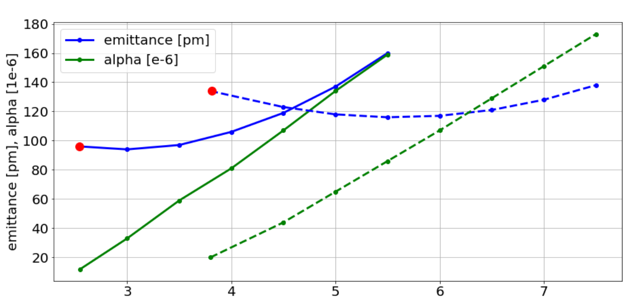


Figure 2: Longer dipoles: $\epsilon$ and $\alpha$ as a function of $\eta_0$ for a half dipole length of 0.4 m (line) and 0.6 m (dashed). Red dots indicate the values for $\eta^{0,TME}$, clearly off $\epsilon_{min}$.

**Transverse Gradient Dipoles** An increased dipole length allows for saving space by including the defocusing gradient in the bend. Field limits for combined function magnets were set to 0.8 T and 30 T/m. The values of $\beta_x^0$ and $\eta^0$ to achieve lowest emittance change due to the gradient. Again, we scan $\eta^0$ and fit $\eta' = 0$ with the RB angle, using $\beta_x^0 = \beta_x^{0,TME}$, see Fig. 3. Due to the gradient, the damping partition number rises, $J_x \approx 1.3$, but the achievable minimal emittance is larger than that of the basic UC (87 pm), due to the additional bend length and the $\epsilon$ contribution of the RB. The gradient can reduce the chromaticity by up to 60%, but opposite to the homogeneous long bend, the dispersion decreases. For $\eta^0 > 7.5$ mm, $\alpha$ values of $\approx 2 \times 10^{-4}$ can be reached for L/2 = 0.6 m and $\epsilon \approx 110$ pm.

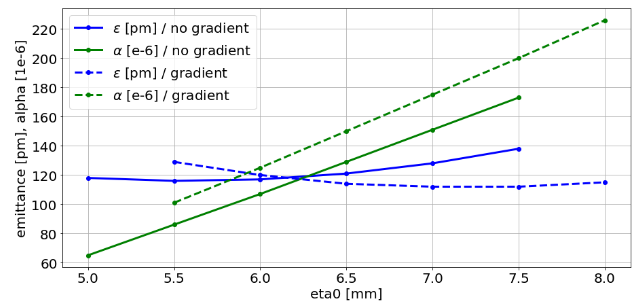


Figure 3: $\epsilon$ and $\alpha$ as a function of $\eta_0$ for a homogeneous dipole (line) and a gradient dipole (dashed) with L/2 = 0.6 m. The $\epsilon$ gain due to the gradient becomes relevant for large $\eta^0$.

**Longitudinal Gradient Dipole** The idea of optimizing the Twiss parameters in many short bends in MBA lattices is taken one step further by the introduction of bends in longitudinal slices. The larger fraction of the bending is placed where $\eta$ and $\beta_x$ are small. 7 slices can reduce the emittance by over 50%, tough accompanied by an unavoidable reduction of $\alpha$. Unfortunately, a considerable part of the emittance reduction is lost for the complete UC, due to the RB.

**Transverse Gradient in Reverse Bend** The focusing gradient can be included in the reverse bend. Often a quadrupole, displaced from its central position to integrate the (small) deflection angle, is used. While the TME-conditions in the main dipole remain untouched, the damping partition number, $J_x$, raises due to the gradient, leading to a considerable emittance reduction. This shifts a significant part of the burden to reach small $\epsilon$ to engineering efforts and should always be utilized.

Figure 4 shows the $\epsilon - \alpha$ combinations achievable with the discussed separate hardware modification. Longer dipoles with or without transverse gradient push $\alpha$ towards higher values. This is accompanied by a considerable chromaticity reduction. Reverse bends with gradient and longitudinal gradient dipoles reduce the emittance at constant $\alpha$. By combining different magnet options, UCs can be tailored to fulfill the project goals and boundary conditions. For BESSY III, a combination of a long homogeneous dipole (needed for metrology applications) and a gradient RB yields

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$\epsilon$ = 77 pm, $\alpha$ = $2.2 \times 10^{-4}$ and $\xi_x$ = -0.37 for a UC length of 2.74 m.

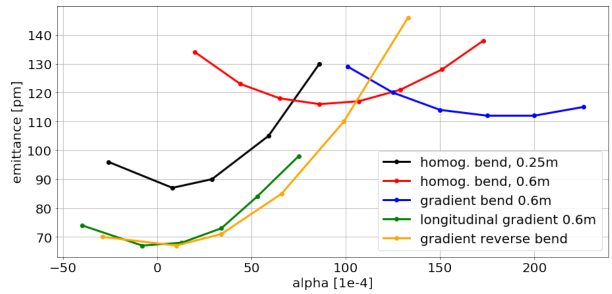


Figure 4: $\epsilon$ - $\alpha$ pairs achievable with 5 UCs, utilizing different magnets. Different hardware solutions support either large $\alpha$ or small $\epsilon$. Combinations are possible.

### *Higher Order Achromat*

Many MBA lattices try to realize higher order achromatic arcs (HOA), as described in [8]. The idea is, to include criteria for the non-linear behavior already in the linear design. For MBA lattices with repetitive UCs this basically means to achieve phase advances in the UC, that add up to $2\pi$ over the dispersive section. This will lead to the intrinsic cancellation of all $3^{rd}$ order chromatic and geometric driving terms. The phase advances of most UCs under consideration for BESSY III are $\phi_x/2\pi \approx 0.4$ and $\phi_y/2\pi \approx 0.1$, suiting well to a HOA with 4 full UCs and two half UCs on either end, composing a 6 bend MBA. On the other hand it is costly in terms of $\epsilon$ and $\xi$ to push the phase advances to built a 5- or 7-bend HOA.

## THE MATCHING CELL

The matching cell consists of two parts. The dispersion suppressor, DS, usually a modification of the last half UC, and the straight section itself. The quadrupoles in the matching cell are the only adjustable knobs of the machine in operation, as the UC is completely fixed in order to deliver the design goals. A careful consideration of how much flexibility is needed in the respective project, is indispensable. Principally, 6 free parameters are needed for full linear control. This can be realized f.e. with 2 quadrupoles and the adjustable RB in the last hUC and 3 quadrupoles in the straight section. Dispersion suppression with the length and gradient of the last bend is the most compact option, but only one beta function can be adjusted in the straight section. Usually these restricted MCs depend on compromises in the UC, to find an adequate interface. It is absolutely non-trivial to find a matching cell with low contributions to emittance and chromaticity, but sufficient degrees of freedom. Examples of matching cells are shown in Fig. 5.

## LATTICE CANDIDATES

Initial lattices can be constructed by merging the desired number of UCs with the MCs. Minor adaptions of the bending angles are necessary. Note, that different combinations of UCs and SPs will result in similar circumferences. A

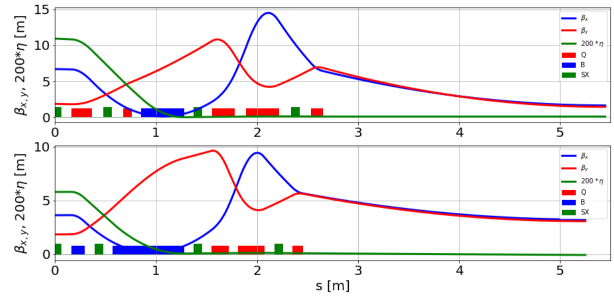


Figure 5: Dispersion matching with 2 quads and the bend gradient (top); and with the length and gradient of the dipole, (bottom). Full linear control depends on 6 free parameters.

20-period 5-bend and a 16-period 7-bend MBA, both occupy ≈ 350 m, with a 25% increase in straight sections in the former. Fig. 6 is an example for a BESSY III lattice, with 16 periods on a circumference of 340 m. It utilizes homogeneous dipoles for metrological applications. $\epsilon$ = 106 pm, $\alpha$ = $1.5 \times 10^{-4}$, phase advances are HOA-compatible and it has a promising non-linear behavior. It is a qualified starting point for further optimizations, as outlined in [9].

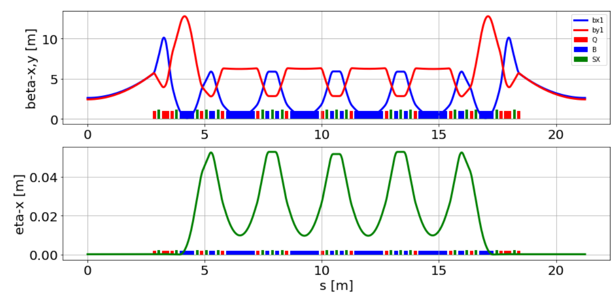


Figure 6: An example lattice for BESSY III, with gradient-free, long dipoles in the UC, but gradient bends in the MC.

## CONCLUSION

It has been shown, how the unit cell of an MBA lattice can be carefully optimized to satisfy given project boundary conditions. The understanding of the unit cell allows to consciously construct the MBA lattice. This is academically more pleasing and more sustainable, than computing time intensive optimization algorithms. The understanding of the impact of the different magnet options on the lattice parameters is indispensable to find an optimal solution in the vast parameter space. Further work is needed to also categorize different realizations of the matching cell. A optional lattice for BESSY III has been introduced, that can now be further improved in terms of non-linear behavior, life times and technical realization.

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