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SIMULATIONS OF AGS BOOSTERS IMPERFECTION RESONANCES FOR PROTONS AND HELIONS

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Abstract

As part of the effort to increase the polarization of the proton beam for the physics experiments at RHIC, a scan of orbit harmonic corrector strengths is performed in the Booster to ensure polarization transmission through the $|G\gamma|$=3 and 4 imperfection resonances is optimized. These harmonic scans have been simulated using quadrupole alignment data and accurately match experimental data. The method used to simulate polarized protons is extended to polarized helions for crossing the $|G\gamma|$=5 through $|G\gamma|$=10 imperfection resonances and used to determine the corrector strength required to cross each resonance.

INTRODUCTION

Imperfection resonances result from a non-zero closed orbit that causes the particles to sample the horizontal field in quadrupoles. These resonances occur when the spin tune is equal to an integer,

$$\nu_s = |G\gamma| = k, \tag{1}$$

where k is an integer, G is the anomalous magnetic moment ($G_{helions} = -4.18415$ and $G_{protons} = 1.79285$), γ is the Lorentz factor, and ν_s is the spin tune. Polarized helions are injected into the Booster at $|G\gamma| = 4.19$ from the EBIS and are extracted at $|G\gamma| = 10.5$. In this range, polarized helions will encounter six imperfection resonances ($|G\gamma| = 5, 6, 7, 8, 9, 10$) as they are accelerated to $|G\gamma| = 10.5$. Helion imperfection resonances are separated by

$$\frac{M_o/u}{G} = 223.73 \text{ MeV/u}. \tag{2}$$

In time, these resonances are separated as shown in Fig. 1.

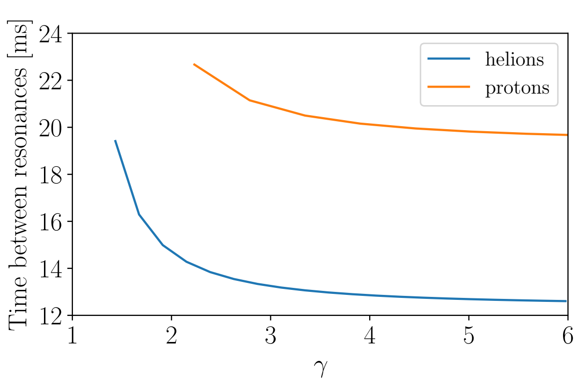

Figure 1: Imperfection resonance spacing for protons and helions.

The imperfection resonance strength is [1],

$$\epsilon_k = \frac{1+G\gamma}{2\pi} \oint \frac{\frac{\partial B_x}{\partial y}}{B\rho} y_{co} e^{iK\theta} ds, \tag{3}$$

where $\partial B_x/\partial y$ is the quadrupole gradient field, $B\rho$ is the rigidity, K is the resonance condition (which in this case corresponds to K=k), and θ is the orbital angle. It is worthwhile to note that the vertical closed orbit error is

$$y_{co} = \sqrt{\beta_y(s)} \sum_{k=-\infty}^{\infty} \frac{\nu_y^2 f_k}{\nu_y^2 - k^2} e^{ik\phi_y(s)}, \tag{4}$$

where β_y is the vertical betatron function through the ring, ν_y is the vertical betatron tune, ϕ_y is the vertical betatron phase, and f_k is the stopband integral [2]. For correcting the $|G\gamma| = k$ resonance, the h=k harmonic of the corrector dipoles is used [1]. The harmonic h=k must either be corrected so no polarization is lost, or enhanced to induce a full spin-flip.

The resonance strengths are calculated by simulation, and Eq. (3) using optics outputs, and listed in Table 1. The resonance crossing speed for protons is $\alpha_{protons} = 5.105 \times 10^{-6}$ and for helions is $\alpha_{helions} = 7.961 \times 10^{-6}$. The Froissart-Stora formula allows predicting the final polarization at a given resonance, k, and harmonic, h=k, as a function of corrector current using [1],

$$\frac{P_f}{P_i} = 2e^{-\frac{(I_{k,S}-I_{k,oS})^2}{2\sigma_{k,S}^2}} e^{-\frac{(I_{k,C}-I_{k,oC})^2}{2\sigma_{k,C}^2}} - 1, \tag{5}$$

where P_i and P_f are the asymptotic values of the polarization before and after crossing the resonance, $I_{k,S}$ and $I_{k,C}$ are the corrector dipole current for the sine and cosine components, $I_{k,oS}$ and $I_{k,oC}$ are the optimal corrector currents for the sine and cosine components, and $\sigma_{k,S}$ and $\sigma_{k,C}$ are the RMS widths for the two families. The currents I_S and I_C for harmonic h will be referred to as sinhv and coshv.

The Booster has 24 vertical orbit correctors, placed adjacent to vertically focusing quadrupoles, and are used for creating and correcting orbit harmonics. These corrector magnets are 10 cm long with an excitation of 0.975 G · m/A, where the supplies have a maximum current of ±25 A [3].

These correctors are powered according to [2]

$$B_{j,h} = a_h \sin(h\theta_j) + b_h \cos(h\theta_j), \tag{6}$$

where j is corrector number, θ_j is the location in the ring, a_h and b_h are the amplitudes for harmonic h.

The primary source of these orbit errors that require correction are from alignment errors of the quadrupoles, causing

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Table 1: Summary of Imperfection Resonance Strengths for Protons and Helions With Quadrupole Alignment Based on Fig. 2a)

Species	**k**	**Bρ [T · m]**	**k**	
			Simulation	**Eq. 3**
Protons	3	4.198	0.000714	0.000644
	4	6.240	0.002367	0.002396
Helions	5	3.064	0.004605	0.004492
	6	4.814	0.000701	0.000716
	7	6.282	0.001299	0.001158
	8	7.633	0.003582	0.003834
	9	8.920	0.000226	0.000239
	10	10.167	0.006252	0.006646

particles to sample the depolarizing horizontal fields. These alignment errors, Fig. 2a), were placed into the zgoubi input files using pyzgoubi, and with the CHANGREF keyword, to allow comparison of simulation data with experimental data [4].

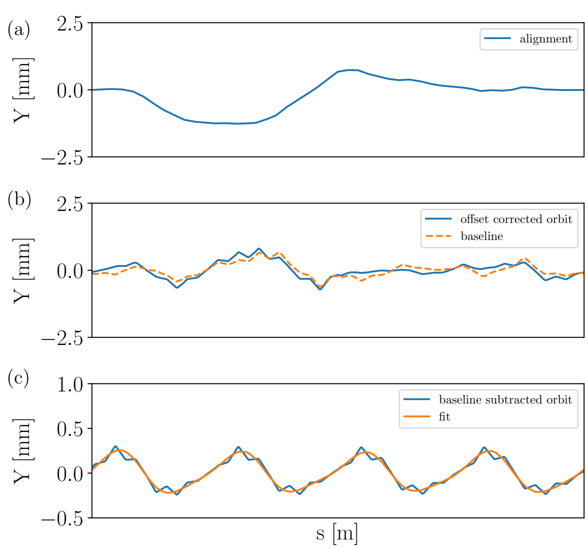

Figure 2: a) Vertical quadrupole misalignments in the Booster are scaled to 65% to match h=4 data, Fig. 4; b) Orbit after incorporating misalignments; c) Baseline subtracted orbit for helions crossing the $|G\gamma| = 8$ resonance where harmonic correction is the same except the baseline orbit has sin4v=6.97 A and the other has sin4v=5.22 A. This example has a corrector current with respect to h=8 of cos8v=5 A, sin8v=13 A. The components of the fit results are: [sin4, cos4, sin5, cos5, sin8, cos8]=[0.1997 mm, 0.07796 mm, 0.01137 mm, 0.00031849 mm, -0.01263 mm, -0.04177 mm].

For helions, the h=4 orbit is corrected at $|G\gamma|$=5 and the correction from the h=5 harmonic scan is scaled to all higher order resonances by the ratio of rigidity. That is

$$I(h=5, |G\gamma|=k) = I(|G\gamma|=5)\frac{B\rho(|G\gamma|=k)}{B\rho(|G\gamma|=5)}. \quad (7)$$

This allows all helion imperfection resonances to be studied with the same orbit seen in Fig. 2b). The resulting orbit after introducing h=8 into the corrector dipoles and correcting the h=4, 5 orbits is shown in Fig. 2c). These currents are [sin4v, cos4v, sin5v, cos5v]=[2.797 A, 0.669 A, 0.520 A, 4.296 A] and scaled up appropriately to the resonance being simulated.

The total current on corrector j is

$$I_j = \sum_h I_{h,S}\sin(h\theta_j) + I_{h,C}\cos(h\theta_j), \quad (8)$$

where $I_{h,S}$ and $I_{h,C}$ are the equivalent of sinhv and coshv. This is used to determine the total current on each of the correctors, where the maximum current of all correctors is

$$I_{max} = \max[|I_j|]. \quad (9)$$

This is an important parameter so as to avoid exceeding the maximum current of the supplies.

PROTONS CROSSING $|G\gamma|$=3,4

Results from these simulation and comparison to experimental data is shown in Figs. 3 and 4. The harmonic scan data is fit with a Gaussian defined as

$$S_y(I) = B\exp\left[-\frac{(I_{of}-\mu_f)^2}{2\sigma_f^2}\right] - 1, \quad (10)$$

with f is the sine or the cosine corrector family, μ_f is the location of the peak corresponding to the optimal corrector current to correct the harmonic, σ_f is the width of the response, and B is the normalization. These fit parameters are used to determine the shape of each corrector currents and the optimal corrector currents for correction summarized in Table 2. There is a small difference in comparison of μ and σ between simulations and data with the largest difference of 0.27 A which is about 0.5% of the range of the power supplies.

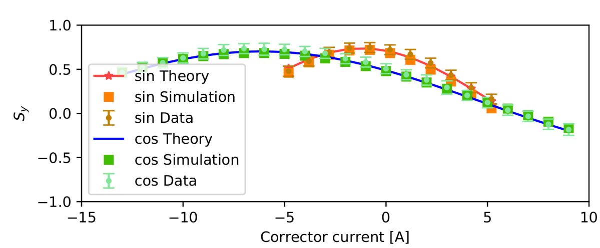

Figure 3: Harmonic scan of protons crossing $|G\gamma| = 3$ with a comparison between theory, simulations, and experimental data.

A scan using zgoubi at various initial currents of the sine and cosine corrector families shows the reliance on initial currents in the scan shown in Fig. 5.

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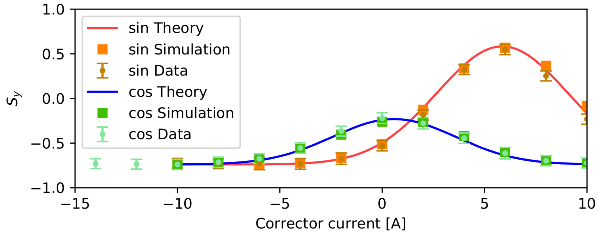

Figure 4: Harmonic scan of protons crossing $|G\gamma| = 4$ with a comparison between theory, simulations, and experimental data.

Table 2: Summary of Fit Data to Proton Harmonic Scans

k	Source	μ_{sin}	σ_{sin}	μ_{cos}	σ_{cos}
3	scan data	-1.1821	3.8390	7.5322	6.1607
3	simulation	-1.2997	3.5643	7.8536	6.2140
4	scan data	5.8646	3.2160	0.5740	3.2160
4	simulation	6.0330	3.2482	0.7019	3.3593

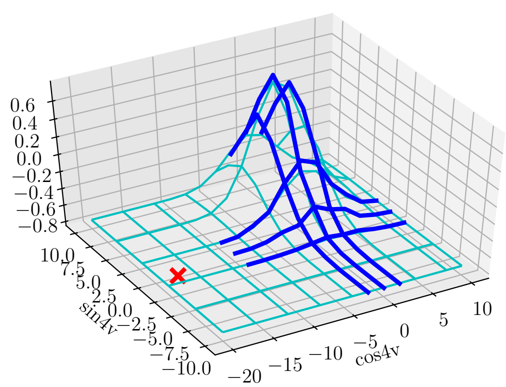

Figure 5: Harmonic scan of protons crossing the $|G\gamma| = 4$ resonance with initial currents at [-2,0,2] for the sine and cosine components. Dark blue lines are resulting simulation data. The light blue surface grid is reconstructed by fitting to the experimental data and extrapolating it to a larger range. The red 'X' marks corrector currents used in Run17, [sin4v,cos4v]=[0, -18].

HELIONS CROSSING $|G\gamma|$=5 TO $|G\gamma|$ =10

With the experimental data of protons being well matched with simulation, the treatment is extended to helions. Figure 6 shows a harmonic scan for helions crossing $|G\gamma| = 5$.

Harmonic scans for the remaining imperfection resonances and fit results are summarized in Table 3 and corrector family currents and corresponding I_{max} in Table 4. Note more corrector current is available for the h=k resonances if the h=4, 5 harmonics are not corrected. The μ_f and σ_f are used to determine the optimal sinhv and coshv while remaining under the 25 A limit.

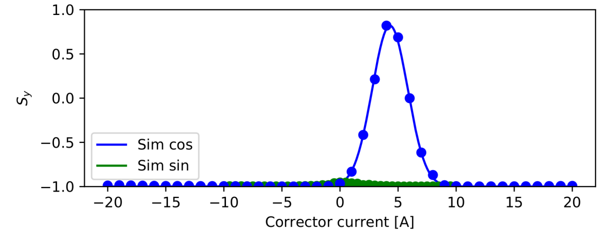

Figure 6: Helions crossing the $|G\gamma| = 5$ resonance. Correction of harmonic is insufficient and needs to be enhanced to spin-flip. To correct the orbit harmonic h=4, [sin4, cos4]=[2.797, 0.669] and to spin-flip: h=5 currents are [sin5, cos5]=[10.0, -18.0] and I_{max}=23.086.

Table 3: Summary of Fit Data for Helion Harmonic Scans

k	μ_{sin}	σ_{sin}	μ_{cos}	σ_{cos}
5	0.5200	1.5750	4.2955	1.5288
6	1.2226	3.6268	-0.2896	2.7384
7	3.1077	4.4358	1.8801	4.5166
8	-4.8460	4.8366	10.6646	5.5313
9	-1.1232	5.2331	-0.3165	3.9495
10	-23.6518	5.5783	-0.4287	5.4708

Table 4: Current Scaling Factor To Correct the Two Major Orbit Harmonics (h=4, 5), Current To Correct or Amplify the Orbit Harmonic Corresponding to the Resonance (h=k), and the Resulting Maximum Current on Any Single Dipole Corrector

Species	k	$\frac{B(k)}{B(5)}$	sinkv	coskv	I_{max} [A]
Protons	3	-	0.9	-6.468	6.530
	4	-	0.0	18.0	24.306
Helions	5	-	10.0	-18.0	23.086
	6	1.571	15.0	-10.0	24.672
	7	2.051	-10.0	-10.0	24.246
	8	2.491	4.0	-13.0	24.491
	9	2.911	-1.1	0.0	17.924
	10	3.318	10.0	10.0	23.459

CONCLUSION

Simulations of imperfection resonances using misaligned quadrupoles from survey data match experimental scan data. This method was extended to simulate these resonances for helions. These simulations determined there is sufficient corrector current to correct each of the imperfection resonances crossed as helions are accelerated to $|G\gamma|$=10.5.

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REFERENCES

[1] S. Y. Lee, *Spin Dynamics and Snakes in Synchrotrons*, Singapore: World Scientific Publishing Company Incorporated, 1997.

[2] S. Y. Lee, *Accelerator Physics*, Singapore: World Scientific Publishing Company Incorporated, 2012.

[3] R. Thern, "Booster Ring Correction Magnets", Brookhaven National Lab. (BNL), Upton, NY, USA, Booster Tech Note BNL-105266-2014-IR, 1994.

[4] C. Yu *et al.*, "AGS Booster Adjustment Report", C-AD Survey Reports 2015, unpublished.