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SYSTEMATIC EFFECTS LIMITING THE SENSITIVITY OF "MAGIC ENERGY" PROTON EDM RINGS

C. Carli, M. Haj Tahar[1], CERN, Geneva, Switzerland
[1]now at Paul Scherrer Institute, CH-5232 Villigen, Switzerland

Abstract

Proposals to measure a possible Electric Dipole Moment (EDM) of protons in an electro-static storage ring are studied by a world-wide community. The machine is operated at the so-called "magic energy" to satisfy the "frozen spin" condition such that, without imperfections and with the well known magnetic moment of the particle, the spin is always oriented parallel to the direction of movement. The effect of a finite EDM is a build-up of a vertical spin component. Any effect, other than a finite EDM, leading as well to a build-up of a vertical spin limits the sensitivity of the experiment. Such "systematic effects" are caused by machine imperfections, such as magnetic fields inside the magnetic shield surrounding the ring, and misalignments of electro-static elements or of the RF cavity. Operation of the machine with counter-rotating beams helps mitigating some of the effects. The most dangerous effects are those, which cannot be disentangled from an EDM by combining measurements from both counter-rotating beams, such as an average residual radial magnetic field penetrating the magnetic shield or a combination of magnetic fields and misalignments of electric elements.

FROZEN SPIN AND MAGIC ENERGY PROTON EDM RING

Facilities to discover a possible charged particle Electric Dipole Moment (cpEDM) are studied and proposed by an international community since many years [1–4]. Most of these proposals foresee a machine operated with "frozen spin" sketched in Fig. 1. An initial longitudinal polarization of bunches is maintained in case the particles possess only a Magnetic Dipole Moment (MDM) and no EDM. This "frozen spin" condition can be derived using the Thomas-BMT equation, with additional terms to take into account a possible EDM [5, 6], describing the rotation of the spin vector $\vec{S}$ of a charged particle in an electromagnetic field:

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_s \times \vec{S} = \left(\vec{\Omega}_M + \vec{\Omega}_E\right) \times \vec{S} \qquad (1)$$

$$\vec{\Omega}_M = -\frac{q}{m}\left[\left(G+\frac{1}{\gamma}\right)\vec{B}_\perp + (G+1)\frac{\vec{B}_\parallel}{\gamma} - \left(G+\frac{1}{\gamma+1}\right)\vec{\beta}\times\frac{\vec{E}}{c}\right]$$

$$\vec{\Omega}_E = -\frac{q\eta}{2m}\left(\frac{\vec{E}_\perp}{c} + \frac{1}{\gamma}\frac{\vec{E}_\parallel}{c} + \vec{\beta}\times\vec{B}\right)$$

where e and m are the charge and the mass, γ and β the relativistic factors and G and η describe the well known MDM and the possible EDM to be identified. The frozen spin condition is fulfilled, if the vertical component of $\vec{\Omega}_s$ for a particle on the reference orbit is identical to the angular frequency $\Omega_{p,y} = (-B_y + E_x/(\beta c))e/(\gamma m)$ describing the rotation of the direction of motion. This condition leads to:

$$\vec{\Omega}_{s,y} - \vec{\Omega}_{p,y} = -\frac{e}{m}\left[GB_y + \left(G - \frac{1}{\gamma^2-1}\right)\frac{\beta E_x}{c}\right] = 0\,.$$

For protons with $G = 1.728...$ (and any particle with a positive value of G) the frozen spin condition can be met without magnetic field by operating the machine at the "magic energy". The momentum at this condition is given by $p_m = mc/\sqrt{G} = 700.7\,\text{MeV/c}$, the kinetic energy by $E_m = (\sqrt{1+1/G}-1)mc^2 = 232.8\text{MeV}$ and relativistic parameters $\beta_m = 1/\sqrt{1+G} = 0.598...$ and $\gamma_m = \sqrt{1+1/G} = 1.248....$ A typical proton magic energy EDM ring requires a circumference of about $C = 500\,\text{m}$ resulting in an average electric field of $\bar{E}_x = -5.27$ MV/m (negative as the field points towards the inside with the coordinate system as sketched in Fig. 1.

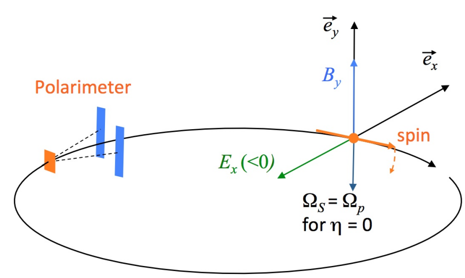

Figure 1: Principle of a frozen spin EDM ring.

The baseline proton EDM magic energy ring proposal foresees to install the ring in a state-of-the-art magnetic shielding allowing to reduce residual fields to the nT range. Operation with beams rotating simultaneously in opposite direction is foreseen to mitigate some of the systematic effects.

The spin rotation from the horizontal plane into the vertical direction due to an EDM of $d = 10^{-29}\, e\cdot\text{cm}$, often quoted as the smallest EDM that can be discovered with the proposed facility, and corresponding to $\eta = 1.9\cdot 10^{-15}$ can be computed as well using Eq. (1):

$$\Omega_{E,x} = -\frac{q\eta}{2m}\frac{\bar{E}_x}{c} = 1.6\,\text{nrad/s}.$$

AVERAGE RADIAL MAGNETIC FIELD - A SYSTEMATIC EFFECT PROPORTIONAL TO THE PERTURBATION

The acceleration due to an average residual radial magnetic field $\bar{B}_x$ is compensated by a vertical average electric

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field $\bar{E}_y = -\beta c \bar{B}_x$ from the focusing structure. The spin rotation from the horizontal plane into the vertical direction due to a residual radial magnetic field is obtained using Eq. (1). For an average residual radial magnetic field as low as $\bar{B}_x = -9.3 \cdot 10^{-18}$ T one obtains:

$$\Omega_{M,x} = -\frac{q}{m}\frac{G+1}{\gamma^2}\bar{B}_x = 1.6\,\text{nrad/s}.$$

One observes that a residual radial magnetic field about eight orders of magnitude lower than feasible inside a state-of-the-art magnetic shield gives an effect that cannot be disentangled from the smallest EDM to be detectable.

Thus, another essential element of the magic energy baseline EDM concept is to operate the ring with a small vertical tune Q_v to enhance the vertical separation of the two simultaneously counter-rotating beams and to use this separation to estimate the average radial magnetic field. The orbit separation at a longitudinal position s due to an integrated radial magnetic field $B_x d\hat{s}$ at a position $\hat{s}$ is given by:

$$\Delta y(s) = 2\frac{\sqrt{\beta_v(s)}}{2\sin(\pi Q_v)}\frac{B_x d\hat{s}}{p_m/q}\sqrt{\beta_v(\hat{s})}\cos(|\mu_v(s)-\mu_v(\hat{s})|-\pi Q_v)$$

With β_v the vertical Twiss betatron function and μ_v the vertical betatron phase. Furthermore, the beam separation pick-ups have to be positioned very regularly in terms of betatron phase advance. For the following, we assume that n pick-ups are located at phases $\mu = 2\pi Q_v k/n$ with $1 \leq k \leq n$ and the radial magnetic stray field is located at a phase $0 \leq \mu(\hat{s}) \leq 2\pi Q_v/n$ and consider the quantity:

$$\sum_{k=1}^{n}\frac{\Delta y_k}{\sqrt{\beta_{v,k}}} = \frac{\sqrt{\beta_v(\hat{s})}}{\sin(\pi Q_v)}\frac{B_x d\hat{s}}{p/q}\sum_{k=1}^{n}\cos\left(k\frac{2\pi Q_v}{n} - \mu(\hat{s}) - \pi Q_v\right)$$

$$= \frac{\sqrt{\beta_v(\hat{s})}}{\sin(\pi Q_v/n)}\frac{B_x d\hat{s}}{p/q}\cos\left(\pi Q_v/n - \mu_n(\hat{s})\right) \qquad (2)$$

With Δy_k the vertical separation at the k−th pick-up where the vertical betatron function is $\beta_{v,k}$. One notes that the argument of the cosine function in the second line is the phase advance between the perturbing magnetic field and the center between the adjacent pick-up. This term can be rewritten and generalized to a perturbation anywhere around the circumference as $\cos(\pi Q_V/n - \Delta\mu_n(\hat{s}))$, where $\Delta\mu_n$ is the phase advance between the perturbing magnetic field and the closest pick-up. The equation can only be inverted approximately as the contribution of the perturbation to the average beam separation depends on the position. Replacing the betatron function at the location of the perturbation by its average $\bar{\beta}_v$ around the circumference and the term $\cos(\pi Q_v/n - \Delta\mu_n)$ by its average $\sin(\pi Q_v/n)/(\pi Q_v/n)$ over betatron phases leads to the estimate of the average radial magnetic field:

$$\bar{B}_{x,est} = \frac{1}{C}\frac{(p/q)\,\pi Q_v}{\sqrt{\bar{\beta}_v}}\frac{1}{n}\sum_{k=1}^{n}\frac{\Delta y_k}{\sqrt{\beta_{v,k}}}. \qquad (3)$$

For a smooth focusing ring with $\beta_{v,k} = \bar{\beta}_v = C/(2\pi Q_v)$, the well known expression

$$\bar{B}_{x,est} = \frac{2\,(p/q)\,(\pi Q_v)^2}{C^2}\Delta\bar{y} \quad \text{with} \quad \Delta\bar{y} = \frac{1}{n}\sum_{k=1}^{n}\Delta y_k$$

is obtained. Note that for $Q_v = 0.1$ proposed in reference [2] and $Q_v = 0.44$ from the "strong focusing" proposal [7], the average separation between the two counter-rotating beams with an average magnetic field of $\bar{B}_x = 9.3$ aT becomes $\Delta\bar{y} = 5$ pm and $\Delta\bar{y} = 0.26$ pm. Thus, the determination of the orbit difference with sufficient precision is very challenging even with very long averaging times and many SQUID based pick-up stations installed.

The approximations implemented to obtain the average radial magnetic field estimate given in Eq. (3) from Eq. (2) lead to systematic errors of the former:

- Limited number of orbit difference pick-ups: the effect of a residual magnetic field on the average magnetic field estimate depends on the vertical phase advance to the nearest pick-up as already pointed out earlier [7] based on a different analysis. Mitigation measures are to increase the number of orbit difference pick-ups and a proposal to implement a modulation of the vertical tune.

- Variations of the betatron functions around the circumference: the effect of a perturbing magnetic field on the observed vertical orbit difference is proportional to $\sqrt{\beta_v(\hat{s})}$ and, thus, depends on the location of the perturbation. This results in a difference between the real average radial magnetic field and its estimate computed with Eq. (3), which can be orders of magnitude larger than the magnetic field giving the same effect than the smallest EDM to be detectable for "strong focusing" lattices [7] with intentional variations of the betatron functions. The effect can be somewhat mitigated by designing a smooth focusing lattice. Such observations were the trigger for the proposal of a "hybrid ring" [8], where the beam is focused with magnetic quadrupoles.

An average magnetic field is the only first order systematic effect in the sense that a spin rotation proportional to the perturbation is generated[1]. The effect mimics an EDM and cannot be disentangled from an EDM by combining observations from the two counter-rotating beams.

SECOND ORDER EFFECTS

In many cases, two (or more) deviations of the real machine from the theoretical design lead to systematic effects. A typical example is a combination of vertical magnetic fields rotating the spin around the vertical axis combined with longitudinal magnetic fields described, e.g., in reference [9, 10]. The vertical spin build up is generated by "geometric phase effects" caused by the fact that rotations do not commute. In this case, as for many other second order effects, the resulting spin rotation into the vertical direction

[1] In the strict sense, gravitational effects lead as well to spin rotations from the horizontal plane proportional to the gravitational constant. These effects are not treated here as they can be precisely predicted and they do not mimic EDM in the sense that they can be disentangled from an EDM combining observations of both counter-rotating beams.

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does not mimic an EDM in the sense that combining observations from both counter-rotating beams allow to disentangle the effect from the one generated by a finite EDM.

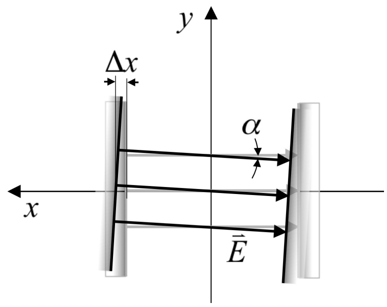

Figure 2: Electrostatic bend with misaligment.

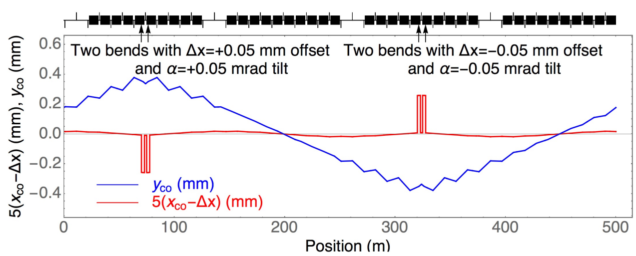

Figure 3: Vertical and horizontal closed orbit with two pairs of bendings with a horizontal offset of ±50 µm and a tilt of ±50 µrad.

Another example of a second order effect is generated by electro-static bending elements that have a horizontal offset and a tilt (rotation around the longitudinal axis) as depicted in Fig. 2. To explain the basic mechanism, a special case with two pairs of bendings of the "strong focusing" lattice [7] misaligned as indicated in Fig. 3 is considered. The resulting closed orbit distortions are plotted in Fig. 3. Due to the radial misalignment $\Delta x = \pm 50\,\mu\text{m}$ and the resulting closed orbit x_{co}, the kinetic energy of the beam is slightly shifted from the magic energy. The relativistic γ-factor of the beam center is given by:

$$\gamma = \gamma_m + \frac{qE_x}{mc^2}(x_{co} - \Delta x) = \gamma_m - \frac{\beta_m^2 \gamma_m}{\rho}(x_{co} - \Delta x)$$

where ρ denotes the bending radius. As the particles do (in average) not have magic energy any more, the rotation of the spin due to the vertical electric field due the tilt $\alpha = \pm 50\,\mu\text{rad}$ does not follow the rotation of the direction. The difference of the angular frequencies is given by:

$$\Delta\Omega_x = \Omega_{M,x} - \Omega_{p,x} = -\frac{qE_y \beta}{mc}\left(G - \frac{1}{\gamma^2 - 1}\right) \approx 2\frac{\beta_m c}{\gamma_m}\frac{\Delta x - x_{co}}{\rho^2}\alpha\,.$$

With the bending radius $\rho = 52.3$ m and averaging over the circumference (rotation taking place inside four bends, each with length $L = 2.74$ m) one obtains $\Delta\bar{\Omega}_x \approx 5.8\,\mu\text{rad/s}$. In addition, spin rotations around the vertical axis are generated by the electric potential shifting the energy away from the magic one as plotted in Fig. 4. These spin oscillations in the horizontal plane, together with the slope of the vertical orbit generating a rotation around the longitudinal axis, lead to geometric phase effects and vertical spin build up, which is small compared to the main contribution described in detail.

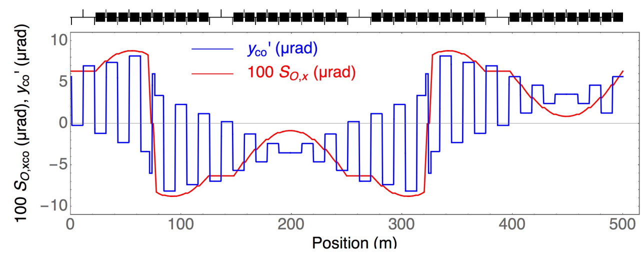

Figure 4: Vertical slope of closed orbit and radial spin with perturbations described in Fig. 3.

The effect described gives a particularly large rotation of the spin into the vertical plane, but can in principle be disentangled from an EDM by combining observations made with both counter-rotating beams. Nevertheless, this requires to measure the spin rotations for both beams with high accuracy or to implement a feedback system counteraction spin rotations not compatible with the signature of a finite EDM. However, there are many second order effects. Many of these second order effects cannot be disentangled from spin rotations due to a finite EDM by combining observations from both counter-rotating beams and, thus, mimic an EDM.

SUMMARY

The main systematic effect of baseline magic energy proton EDM rings is caused by residual radial magnetic fields penetrating the magnetic shield. A measurement of the vertical separation of the two counter-rotating beams is foreseen to estimate the average radial magnetic field and to mitigate the effect. Variations of the vertical betatron function around the circumference limit the efficiency of the scheme, in particular for the "strong focusing" lattice proposal. Effects not yet studied are unintentional variations of the vertical betatron function ("beta-beating") and a horizontal orbit separation due to vertical magnetic field transferred into the vertical plane by betatron coupling.

Second (and in principle as well higher) order effects generate additional spin rotations from the horizontal plane into the vertical direction. Some, but not all, of these effects can be disentangled from an EDM by combining observations with both counter-rotating beams.

In conclusion, further systematic and thorough studies are needed to understand limitations and, finally, come to a realistic estimate of the sensitivity of cpEDM rings.

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