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EFFECT OF DIFFERENT MODELS OF COMBINED-FUNCTION DIPOLES ON THE HEPS PARAMETERS*

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Abstract

The high energy photon source (HEPS) is a 6 GeV, kilometer-scale storage ring light source being built in Beijing, China. In the current ring lattice, the combined-function dipoles are used and assumed to have constant dipole field. However, in the actual magnet design, an eccentrically placed quadrupole is adopted, in which the bending field along the trajectory is not constant. In this paper, we will present the effect of the two models of combined-function dipoles on the parameters of the storage ring.

INTRODUCTION

The High Energy Photon Source (HEPS), a 6 GeV synchrotron radiation facility with ultralow emittance, is being built in Beijing, China. A hybrid 7 BA design for the HEPS, with a natural emittance of ~34 pm·rad and circumference of about 1.3 km, has been made [1-3].

The combined-function dipoles are adopted in the storage ring of HEPS. There are two possible ways to design and construct a combined-function dipole (shown in Fig. 1). One way is to design a curvilinear magnet centered at the circular orbit such that it has constant dipole field and gradient. The other way is to design a straight magnet whose central axis is straight line. It can be built as a quadrupole with a transverse offset. In such a magnet, the dipole field along the beam trajectory is not constant because the beam trajectory in the magnet is a curve rather than a straight line.

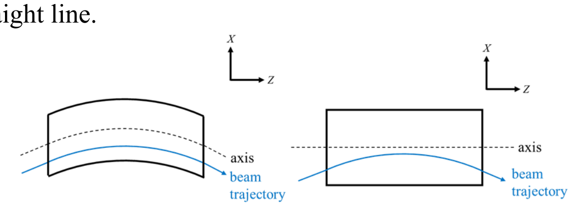

Figure 1: Curvilinear (left) and straight (right) model of combined-function dipoles.

In the evolution process of the storage ring lattice, the model of curvilinear combined-function dipole with constant dipole field is used. However, in the actual magnet design, it is considered to be a quadrupole with a transverse offset. In the following, we will compare the effect of these two models on the parameters of the storage ring.

PARAMETERS OF STRAIGHT LINE COMBINED-FUNCTION DIPOLE

For the curvilinear combined-function dipole used in the lattice, two key parameters are the length of beam trajectory, L_B and bending angle, θ_B.

We calculated the parameters of corresponding straight line combined-function dipole (x_{off} and L_Q below) to ensure the length of the beam trajectory and the total bending angle were identical. Where, x_{off} is the transverse offset at the entrance of the quadrupole, L_Q is the linear length of quadrupole in the hard-edge model. Two simplifications are made for the models:

- Considering the hard-edge model, that is, the gradient within the edge is constant $K=K_B$, and $K=0$ outside the edge;
- The beam trajectory is symmetrical with respect to the longitudinal centreline of the magnet (the black dashed line parallel to the y axis in Fig. 2).

Then, we sought for a combination of x_{off} and L_Q to make the length of the beam trajectory in the quadrupole equal to L_B and the bending angle equal to θ_B.

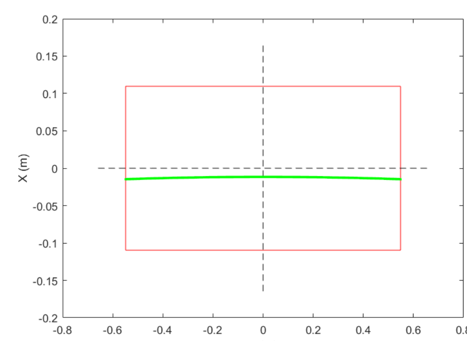

Figure 2: Schematic of the straight line combined-function dipole. The red box represents the boundary of magnet in the hard-edge model, the green curve represents the beam trajectory.

Three methods are adopted to achieve the x_{off} and L_Q, and the details are introduced as below.

Method 1: Geometric Method

In this method, the dipole field is considered to affect the particle trajectory. The method is as follows:

Assuming the center of the magnet is the starting point, and the particle coordinate at the starting point is $\mathrm{Cor}_1 = [z_1 = 0, x_1 = x_{off}]$. We divided the magnet into n slices and the dipole field of each slice can be assumed as a constant. The dipole field of the first slice equals to $G^* x_1$, and the particle rotates a very small angle $\delta\theta$ in the first

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slice, then we can get the new coordinate of the particle $Cor_2 = [z_2, x_2]$, where the dipole field changes to $G*x_2$. Repeat this process until the particle reaches the exit of the magnet. For a set of x_{off} and L_Q, the corresponding length of trajectory L_{curve} and bending angle θ_{total} can be calculated. By minimizing $[L_{curve}, \theta_{total}]-[L_B, \theta_B]$, the optimal x_{off} and L_Q can be obtained.

Method 2: Transfer Matrix Method

Dividing the magnet into n slices with the same beam trajectory length (a small length l_s) satisfying $n * l_s = L_B$, and the dipole field $B=G*x$ is assumed to be constant within each slice, where x is the transverse position at the entrance of each slice. Considering that at the entrance of the first slice the orbit $x_0= x_{off}$ and the angle $x_0' = \theta_B/2$, and assuming the length of beam trajectory in the first slice is l_s, and then the bending angle is expressed as $angle_1 = \frac{l_s*B_1}{B\rho} = \frac{l_s*G*x_0}{B\rho}$. From the geometric relationship (as shown in Fig. 3), we got that the linear length of the first slice $l_{d1} = 2 * \frac{ls}{angle_1} * \sin\left(\frac{angle_1}{2}\right) * \cos(x_0' - \frac{angle_1}{2})$. Moreover, the orbit and angle at the exit of the first slice (x_1, x_1') can be obtained from the transfer matrix of quadrupole. By iterating this process, we can achieve the orbit and angle of the beam at the exit of each slice. Scanning the value of x_{off} such that the angle at the midpoint of the magnet is as close to 0 as possible, then we can get the optimal x_{off} and the total linear length L_Q, along with the linear length and bending angle of each slice.

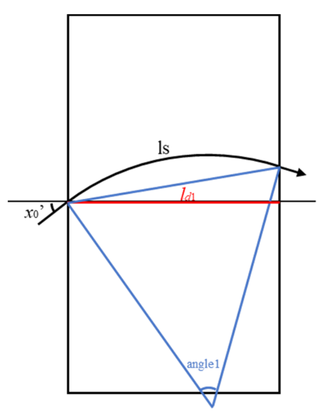

Figure 3: Geometric schematic of the beam trajectory and linear length of each slice.

Method 3: Differential Equation Method

Since the Lorentz force does not do any work, the speed of the particle remains unchanged and can be approximated as the speed of light c. The acceleration of the particle can be expressed as $a=Bec/m$, where $B=G*x$ is the dipole field, e is the electron charge, m=6 GeV/c^2 is the mass of electron. Assuming that the angle between the electron velocity and the positive direction of the z-axis is θ, and letting $K=Ge/mc$, since the Lorentz force is perpendicular to the direction of velocity, the second-order differential equations of the particle position with respect to time t can be obtained:

$$x''[t] = K * x[t] * c^2 * (1 - (x'[t])^2/c^2)^{0.5}$$
$$z''[t] = K * x[t] * c^2 * x'[t]/c$$

The initial conditions are that at t=0, the particle is located at the center of the magnet and the velocity is parallel to the z axis, that is $x'[0]=0$, $z'[0]=c$, $x[0]= dx$, $z[0]=0$. The beam trajectory can be obtained by solving this differential equation. With the bending angle as the target, we can get the corresponding dx by using the inverse function method, then $x_{off}= x[L_B/2/c]$, $L_Q=2*z[L_B/2/c]$.

We got the values of x_{off} and L_Q with the above three methods. The results are listed in Table 1. From the table, we can see that the results of different methods are close.

Table 1: Parameters of Straight Line Combined-function Dipole with Different Methods

Parameter	Method 1	Method 2	Method 3
x_{off}	-14.713 mm	-14.714 mm	-14.714 mm
L_Q *	1.0972 m – 22 μm	1.0972 m – 24 μm	1.0972 m – 24 μm

* 1.0972 m is the nominal arc length of the combined-function dipole.

EFFECTS ON THE PARAMETERS OF STORAGE RING

To compare the effects of the two different models of combined-function dipole on the parameters of storage ring, we substituted the model of quadrupole with transverse offset (with parameters in Table 1) into the V3.0 lattice of the HEPS storage ring [3] and compared the main parameters with those with the model of curvilinear magnet.

Model 1 from Method 1

This model is based on Accelerator Toolbox (AT) [4]. Firstly, we compared the transfer matrix of straight line and curvilinear combined-function dipole.

For the curvilinear combined-function dipole, the transfer matrix can be obtained easily from analytical formula method or AT numerical calculation, the result is:

$$\begin{pmatrix} 2.2318 & 1.5185 & 0 & 0 \\ 2.6215 & 2.2318 & 0 & 0 \\ 0 & 0 & 0.1288 & 0.7548 \\ 0 & 0 & -1.3029 & 0.1288 \end{pmatrix}.$$

For the straight line combined-function dipole, it is divided into many slices. In principle, if the length of each slice is short enough, the transverse orbit of the particle changes very little when it passes this slice and the dipole field in this slice can be assumed to be constant. This slice can be described by the transfer matrix of the curvilinear combined-function dipole. After multiplying the transfer matrices of all slices, we can achieve the transfer matrix of the whole straight line combined-function dipole. We scanned the number of slices to calculate the transfer matrix based on the beam trajectory obtained with the geometric method in Section II. Figure 4 shows the value of [m_{11}, m_{12}, m_{33}, m_{34}] of transfer matrix with respect to the number

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of slices. One can see that when the number of slices exceeds 100, the value of [m_{11}, m_{12}, m_{33}, m_{34}] tends to be converging. Here, arbitrarily considering the case that the number of slices is 116, the transfer matrix is:

$$\begin{pmatrix} 2.2319 & 1.5187 & 0 & 0 \\ 2.6217 & 2.2319 & 0 & 0 \\ 0 & 0 & 0.1287 & 0.7548 \\ 0 & 0 & -1.3029 & 0.1287 \end{pmatrix}.$$

The transfer matrixes of straight line and curvilinear combined-function dipole are close. Then the slice model is substituted into the lattice and some main parameters are calculated.

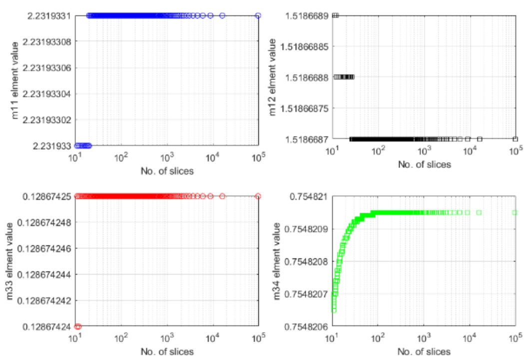

Figure 4: The value of [m_{11}, m_{12}, m_{33}, m_{34}] of transfer matrix with respect to the number of slices.

Model 2 from Method 2

Divide the straight line combined-function dipole into 100 slices which is enough to get the convergent results. Each slice has the same arc length $l_s = L_B/100$ and the bending angle can be obtained based on method 2 in Section II. We replaced the combined-function dipole with these 100 magnets in the lattice, then calculated the parameters.

Model 3 based on ELEGANT

The element 'CCBEND' in ELEGANT [5] represents a straight dipole magnet which can be adjusted to have multipole strength. By specifying the quadrupole field gradient, arc length and bending angle, the program can match the x_{off} and L_Q automatically. In the ELEGANT lattice file, we replaced the combined-function dipole with 'CCBEND' and calculated the parameters of the lattice.

The results of different models are summarized in Table 2. The parameters of curvilinear combined-function dipole model are also listed for comparison. We can see that there are not many differences in the main parameters for the straight line and curvilinear combined-function dipole models. The emittance of the straight line model is a little bit higher than that of the curvilinear model, about 1 pm·rad, and other parameters related to the bending radius changed slightly.

SUMMARY

As described above, we studied the difference of straight line and curvilinear combined-function dipole. The differences come from that the dipole field along the beam trajectory in the straight line combined-function dipole is not constant while that in the curvilinear combined-function dipole is constant. This leads to the differences of bending radius which result in the differences of the parameters related to the bending radius. Among them, the most concerned is the horizontal natural emittance, which increases by about 1 pm·rad.

Table 2: Main Parameter of Storage Ring Lattice of Different Model of Combined-function Dipole

Parameters	**Curvilinear model (AT)**	**Straight line model (model 1)**	**Straight line model (model 2)**	**Straight line model (model 3)**
Circumference (m)	1360.4	1360.4013 *	1360.4	1360.4
Net bending angle (deg.)	360	360–0.0071*	360+1.3E-5	360
Horizontal natural emittance (pm·rad)	34.8271	35.8311	35.8132	36.0510
Working point (x/y)	115.1521/ 104.2905	115.1521/ 104.2908	115.1520/ 104.2903	115.1618/ 104.2169
Beta functions at the center of high-beta sections (x/y) (m)	8.1757/ 4.9976	8.1754/ 4.9978	8.1756/ 4.9970	8.2038/ 4.9540
Momentum compaction (10^{-5})	1.8311	1.8621	1.8621	1.8476
Energy loss per turn, U_0 (MeV)	2.6412	2.6461	2.6473	2.5888
Energy spread (10^{-3})	1.0031	1.0095	1.025	1.0280

* The difference between the length and bending angle comes from the calculation error of the trajectory length in the straight line combined-function dipole by the numerical method.

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