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PRELIMINARY STUDY OF DESIGN METHOD FOR HYBRID MBA LATTICE

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Abstract

Nonlinear optimization of hybrid multi-bend-achromat (HMBA) lattice is a difficult task due to its quite limited variables of nonlinear multipole magnets. As a result, it is necessary to consider nonlinear potential of the lattice in its linear design. Nonlinear dynamics can be estimated by resonance driving terms and detuning terms. In this paper, we propose a design method for HMBA lattice. In this method, objective functions include emittance and two indicators of nonlinear dynamics, which consist of resonance driving terms and detuning terms. As an example, an HMBA lattice for a 2.2 GeV storage ring with circumference of 460.8 m was designed to demonstrate the method.

INTRODUCTION

Generally, a hybrid multi-bend-achroamt (HMBA) lattice only contains few families of nonlinear multipole magnets in the dispersion bump area. It is difficult to obtain a satisfying result for the nonlinear dynamics optimization due to limited variables after the linear lattice is fixed. As a result, considering the nonlinear potential of lattice in the preliminary design is necessary and the linear lattice should keep adjustable during nonlinear optimization.

Estimating the nonlinear performance of a lattice is not easy. Multi-objective genetic algorithm (MOGA) has been widely used for lattice design [1]. During the process of multi-objective optimization, millions of lattices would be generated and estimated. Dynamic aperture (DA) and momentum aperture (MA) are two important nonlinear parameters. However, using tracking method to calculate DA and MA is too time-consuming compared with calculating twiss parameters. For this reason, nonlinear optimization is generally separated from preliminary linear design, which leads to a condition we just mentioned: the preliminarily designed lattice is constrained by the requirements of linear parameters. Nonlinear optimization may cannot obtain good solutions for such a lattice.

To solve this problem, it is necessary to estimate and improve the nonlinear dynamics in the linear optics design with a fast method. There are two traditional strategies to control the nonlinear dynamics for synchrotrons:

1. $-I$ transformation: set the phase advances (both horizontal and vertical) between the symmetric sextupoles in a cell to $n\pi$, where n is an odd number, then many nonlinear effects can be cancelled.
2. Higher-order achromat: separate the whole ring into several superperiods, each consisting of four or more identical cells, and then adjust the phase advances of the superperiod in both transverse planes to $2n\pi$, where n is an integer.

The two strategies are effective to promote the nonlinear performance. However, they are not enough for the final design. Further nonlinear optimization must be done to obtain better nonlinear dynamics.

DESIGN METHOD

To estimate the nonlinear performance of an HMBA lattice during using MOGA, taking some related linear parameters as nonlinear indicators such as integrated strengths of sextupoles and natural chromaticities is a feasible method [2]. Instead of DA, such indicators can be calculated along with linear design. They can help to obtain lattices with better potential of nonlinear optimization.

Here we look for more appropriate parameters to be introduced as nonlinear indicators during using MOGA. The nonlinear driving terms [3] are worth trying. Each of resonance driving terms drives certain resonance. Besides, tune shifts with amplitude, also known as detuning terms, play an important role in the nonlinear dynamics as well. Acceptable resonance driving terms and detuning terms are necessary conditions for good nonlinear dynamics [4]. Most importantly, both of them can be easily calculated.

Consequently, we propose a design method based on MOGA with 3 objective functions. The three objective functions include natural emittance ϵ_{nat} and two nonlinear indicator functions f_1 and f_2, which consist of driving terms and detuning terms. The other linear design goals are treated as constraint conditions [5].

f_1 for Third-order Resonance Driving Terms

Third-order resonance driving terms drive strong resonances and must be strictly limited. In this paper, we focus on the five third-order resonance driving terms: h_{21000}, h_{30000}, h_{10110}, h_{10020} and h_{10200}. According to our experience, a lattice with large DA always has very small third-order resonance driving terms. $-I$ transformation can control the driving terms but they are always not small enough.

The analytical expressions for the five third-order resonance driving terms are as follows [3]:

$$\mathrm{h}_{21000} = -\frac{1}{8}\sum_{1}^{N}(b_{3i}L)\beta_{xi}^{\frac{3}{2}}e^{i\mu_{xi}}, \tag{1}$$

$$\mathrm{h}_{30000} = -\frac{1}{24}\sum_{1}^{N}(b_{3i}L)\beta_{xi}^{\frac{3}{2}}e^{i3\mu_{xi}}, \tag{2}$$

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$$h_{10110} = \frac{1}{4}\sum_{1}^{n}(b_{3i}L)\beta_{xi}^{\frac{1}{2}}\beta_{yi}e^{i\mu_{xi}}, \quad (3)$$

$$h_{10020} = \frac{1}{8}\sum_{1}^{n}(b_{3i}L)\beta_{xi}^{\frac{1}{2}}\beta_{yi}e^{i(\mu_{xi}-2\mu_{yi})}, \quad (4)$$

$$h_{10200} = \frac{1}{8}\sum_{1}^{n}(b_{3i}L)\beta_{xi}^{\frac{1}{2}}\beta_{yi}e^{i(\mu_{xi}+2\mu_{yi})}. \quad (5)$$

During the linear lattice design, beta functions and phase advances have been calculated. To obtain the integrated strengths of three families of sextupoles, we can first determine the final corrected chromaticities ξ_x and ξ_y, and then integrated strength of one sextupole family can be treated as a free knob of MOGA. After chromaticity correction, integrated strengths would be determined so the values of resonance driving terms can be calculated.

In addition, nonlinear driving terms change along the ring. Figure 1 shows the changes of the five third-order resonance driving terms with the cell number. For nonlinear driving terms, considering the average values over different number of cells are more representative rather than the only value of the whole ring.

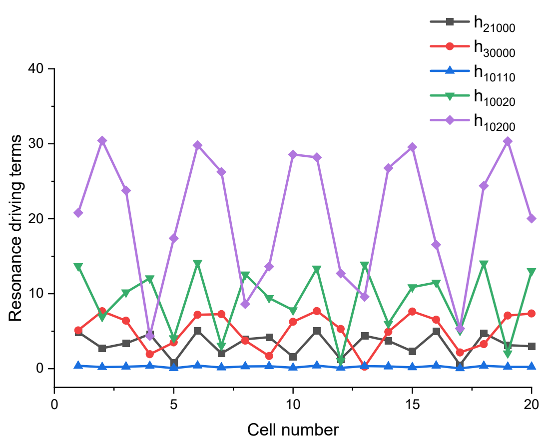

Figure 1: Changes of the third-order resonance driving terms with the cell number.

Considering the balance of resonance driving terms, the average value of square of resonance driving terms is chosen as the first nonlinear indicator function:

$$f_1 = \frac{1}{N_{cell}}\sum_{i=1}^{N_{cell}}\sum h_{abcde}{}^2, \quad (6)$$

where N_{cell} is the number of cells, h_{abcde} only include h_{21000}, h_{30000}, h_{10110}, h_{10020} and h_{10200}.

f_2 for Detuning Terms

Detuning terms linearly increase along with the number of cells. Detuning terms also have numerical formula [3]. It should be noted that octupoles contribute to detuning terms. We have tried to treat the integrated strength of one octupole family as a free knob. But the integrated strength is hardly optimized to an appropriate value, which will make the indicator function inaccurate. So we adopt a simple method: enumerate the integrated strength of octupole for the minimum indicator function.

The sum of square of detuning terms is chosen as the second nonlinear indicator function:

$$f_2 = \left(\frac{\partial\nu_x}{\partial J_x}\right)^2 + \left(\frac{\partial\nu_x}{\partial J_y}\right)^2 + \left(\frac{\partial\nu_y}{\partial J_y}\right)^2. \quad (7)$$

Usually, $\partial\nu_x/\partial J_x$ is more important in actual design since we usually focus on the horizontal DA. A weight coefficient may be useful.

APPLICATION

An HMBA lattice for a 2.2 GeV storage ring was taken as an example. The ring with a circumference of 460.8 m consists of 20 identical cells. Longitudinal gradient bends (LGBs) and reverse bends (RBs) were employed to reduce natural emittance.

We adopted NSGA-III [6] with a population of 30000 to optimize the lattice for 300 generations. Figure 2 shows the linear optical functions and magnet layout of the selected lattice. The main bends in dispersion bumps and in the middle are five LGBs and the bends near the middle main bend are two RBs. The natural emittance ϵ_{nat} was optimized to 62 pm.rad and chromaticities were corrected to (4, 4). The transverse tunes are (48.24, 17.34).

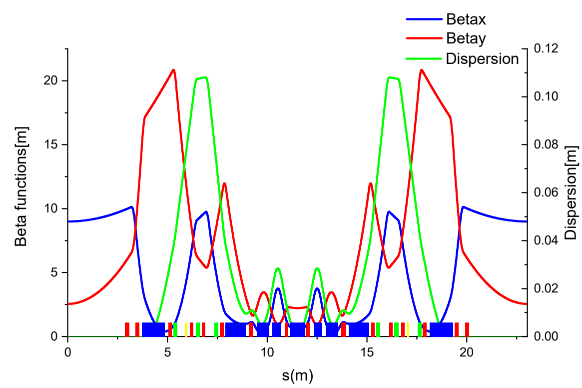

Figure 2: Linear optical functions and magnet layout of the selected lattice.

Table 1: Third-Order Resonance Driving Terms of the Selected Lattice

Resonance Driving Terms	Value (Cell)	Value (Ring)
h_{21000}	5.22	3.92
h_{30000}	5.38	5.19
h_{10110}	6.80	5.11
h_{10020}	13.72	15.58
h_{10200}	17.37	9.22

Resonance driving terms of a cell and the whole ring are shown in Table 1. Detuning terms of a cell and the whole ring are shown in Table 2. Both selected resonance driving terms and detuning terms were well optimized after optimization.

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Table 2: Detuning Terms of the Selected Lattice

Detuning Terms	Value (Cell)	Value (Ring)
$d\nu_x/dJ_x$	-101.98	-2038.90
$d\nu_x/dJ_y$	-184.29	-3685.77
$d\nu_y/dJ_y$	439.94	8798.73

Figure 3 shows the on-momentum DA of the selected lattice. It can be seen that the horizontal DA is large. The method is effective for nonlinear optimization. Tune shifts with momentum are shown in Fig. 4.

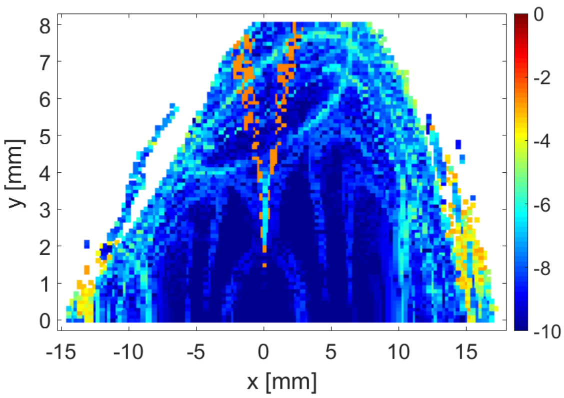

Figure 3: On-momentum DA of the selected lattice, tracked for 1024 turns.

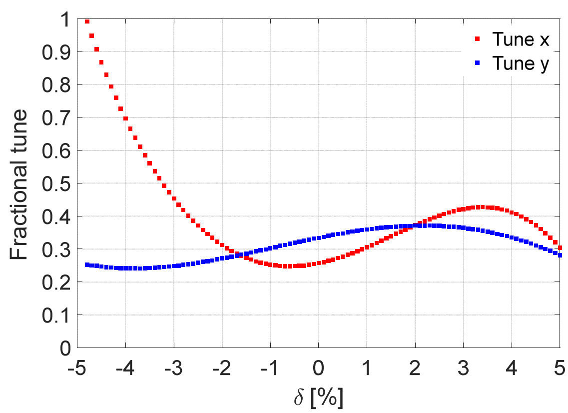

Figure 4: Transverse tune shifts with momentum, tracked for 1024 turns.

CONCLUSION

In this paper, we preliminarily studied a design method for HMBA lattice, which adopts resonance driving terms and detuning terms as nonlinear indicators of a storage ring, and it was applied to the design of an HMBA lattice with a large DA. Controlling resonance driving terms and detuning terms using MOGA is feasible, but we still look for more efficient ways to control them. Small nonlinear driving terms are necessary for good nonlinear dynamics. Finding the sufficient conditions for good nonlinear dynamics is also our further goal.

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